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Distributed and Fair Beacon Power and Beaconing Rate Adaptation Based on Game Theoretic Approach for Connected Vehicles

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ABSTRACT

In vehicular ad hoc networks, vehicles regularly transmit information through beacons to raise awareness among nearby vehicles about their presence. However, as the number of beacons increases, the wireless channel becomes congested, resulting in packet collisions and the loss of numerous beacons. This paper addresses the challenge of optimizing joint beaconing power and rate in VANETs. A joint utility-based beacon power and rate game is formulated, treated both as a non-cooperative and a cooperative game. To compute the desired equilibrium, three distributed and iterative algorithms (Best Response Algorithm, Cooperative Bargaining Algorithm) are introduced. These algorithms simultaneously update the optimal values of beaconing power and rate for each vehicle in each step. Extensive simulations showcase the convergence of the proposed algorithm to equilibrium and offer insights into how variations in game parameters may affect the game's outcome. The results demonstrate that the Cooperative Bargaining Algorithm is the most efficient in converging to equilibrium.

KEYWORDS

Beacon Power, Beacon Rate, Cooperative Game, Game Theory, Nash Bargaining Solution, Nash Equilibrium, Non-Cooperative Game, VANETs

1. INTRODUCTION

Vehicular Ad Hoc Networks (VANETs) represent a cutting-edge approach to wireless communication, leveraging advancements in device technology to facilitate intelligent communication between vehicles. Over recent decades, the emergence of VANETs has captured significant attention within the traffic research community. This novel communication paradigm offers promising avenues for enhancing Intelligent Transportation Systems, as evidenced by its potential applications in public

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transport management (Paquet 2010). Additionally, VANETs play a crucial role in bolstering transportation security, thereby mitigating the occurrence of accidents and disasters. To this end, various safety mechanisms have been devised for VANETs, encompassing functionalities such as emergency alerts, accident notifications, curve warnings, file-sharing, internet connectivity, and targeted advertisements.

Improving security in VANETs is primarily accomplished through the exchange of Basic Safety Messages (BSMs), commonly referred to as beacons, between vehicles. These beacons serve as vital communication tools, with vehicles regularly broadcasting them to relay essential information such as their position, speed, and direction within the network. In critical situations like collisions, accidents, or road surface collapses, vehicles also transmit emergency beacons or safety messages to alert nearby vehicles. However, in densely populated vehicular environments, the sheer volume of beacons can lead to congestion in the communication channel, resulting in an increased likelihood of message loss and delays. This congestion not only hampers vehicles' awareness but also diminishes the accuracy of safety-related information. The growing rate of beacon transmissions exacerbates this issue, raising concerns about the channel's capacity to handle the escalating data load effectively. Given these challenges, the development of robust congestion control strategies for VANETs has garnered significant attention in recent years. Effectively managing channel congestion is crucial for ensuring timely and reliable message delivery, particularly as vehicular density continues to rise.

The endeavor to model vehicle behavior in VANETs analytically has become a focal point of research interest, with increasing attention from scholars. Numerous analytical models have been proposed to scrutinize VANET performance and offer viable solutions tailored to the unique challenges encountered in these networks. Among these challenges, congestion control stands out as a significant concern in computer networks. Metrics commonly employed to assess congestion control include fairness among vehicles, convergence time, and oscillation size (Chiu and Jain 1989). In the context of VANETs, congestion control must operate in a decentralized manner, without relying on any centralized infrastructure. This decentralized approach is essential to accommodate the dynamic nature of VANETs. Additionally, the convergence time of the control mechanism must be minimized to swiftly adapt to changing network conditions and ensure efficient traffic management.

Several work used game theory in wireless networks (Outanoute et al. 2019) (Omar et al. 2019a) (Omar et al. 2019b) (Garmani et al. 2019) (Garmani et al. 2018) (Ait Omar et al. 2019). The authors in (Le et al. 2011) introduced a beacon power control algorithm. This algorithm involves each participant computing the optimal beaconing power required to attain maximum communication effectiveness while ensuring that the Channel Busy Ratio (CBR) remains below a predefined threshold. In (Li and Huang 2018), the authors investigate the efficacy of a multi-hop broadcast protocol within VNETs for enhancing safety. They achieve this by developing a versatile probabilistic forwarding scheme and introducing an analytical model to assess the performance of their proposed approach. The authors in (Luong et al. 2017) present a method for determining the most effective beacon rates, focusing on maximizing a utility function. They explore how varying beacon rates influence network performance and illustrate the significant impact these rates can have on overall network effectiveness. In (Qureshi et al. 2018), the author investigates a dynamic congestion control mechanism aimed at facilitating the broadcast of Basic Safety Messages (BSMs). Their objective is to ensure the dependable and punctual delivery of messages to all neighboring nodes within the network. The authors in (Ishaq et al. 2018) used the tabu search algorithm with multi-channel allocation capability to reduce the time delay and jitter for improving the quality of service in VANET. In (Li and Huang 2018), the authors introduce a method for adapting beacon rates based on vehicle mobility prediction. Here, each vehicle utilizes a prediction module to assess the real-time situation of its neighboring vehicles. The authors in (Goudarzi and Asgari 2018) studied the competition among vehicles in beaconing power as a non-cooperative game. In (Goudarzi and Asgari 2017) the authors employed a non-cooperative game to devise a mechanism for controlling beacon rates. They demonstrated the uniqueness of the Nash equilibrium point and introduced a distributed method to determine this equilibrium. Expanding on this approach, our paper delves into the utilization of both non-cooperative and cooperative games to analyze the simultaneous control of beaconing rate and beaconing power in VANETs. We present three algorithms aimed at learning the optimal joint beaconing rate and beaconing power, considering both Nash equilibrium and Nash bargaining solutions.

This paper addresses the challenge of achieving fair and stable control over joint beaconing power and rate in VANETs, employing a combination of non-cooperative and cooperative game theory. The aim is to devise a distributed approach that minimizes beacon losses by determining vehicle beaconing power and rate effectively. Utilizing the principles of supermodular games and the Nash bargaining solution, we formulate and solve the optimization problem associated with this joint control. The existence of a Nash equilibrium point within the non-cooperative game framework is established. We introduce three learning algorithms that iteratively adjust beaconing rates and powers, aiming to reach equilibrium in a distributed manner. Performance evaluation demonstrates the convergence of these algorithms towards equilibrium beaconing power and rate. Moreover, it elucidates the impact of system parameters on vehicle strategies. Our findings reveal the superiority of the proposed cooperative game algorithm in effectively controlling beaconing rate and power, making it the preferred choice for vehicles in VANETs.

The rest of this paper is organized as follows. In Section 2, we describe the proposed model. In Section 3, we present the non-cooperative game formulation and the price of anarchy. In Section 4, we present a cooperative game. Then, we present the Performance evaluation in Section 5. Finally, in Section 6 conclusions.

2. SYSTEM MODEL

The utility function of each vehicle is the difference between revenue and fees. Accordingly, the payoff of the vehicle i can be written as:

$$U_i = a_i \log\left(r_i + p_i + 1\right) - c_i p_i CBR_i\left(r, p\right) - \left(C_{s_i} + C_{p_i} p_i + C_{r_i} r_i\right) \tag{1}$$

where a_i and c_i are two positive parameters. $CBR\left(p_i, r_i, p_{-i}, r_{-i}\right)$ is the channel busy ratio that vehicle i senses, and it is a function of all vehicle beaconing rates and beaconing power. The term $a\log\left(r_i + p_i + 1\right)$ is the revenue of vehicle i; it is an increasing function with respect to beaconing rate and beaconing power. A logarithmic function has been used because it is increasing and has excellent concavity properties. Thus, the vehicle with lower beaconing power and their beaconing rate has more incentive to increase their beaconing power and their beaconing rate. The second term $C_{s_i} + C_{p_i} p_i + C_{r_i} r_i$ is the energy consumed to send beacons and to switch the state of the transceiver. C_{s_i} is the energy consumed for switching the state of the transceiver, C_{p_i} is the energy consumed for sending beacons with power p_i, and C_{r_i} is the energy consumed for sending beacons with a rate r_i. The third term $c_i p_i CBR_i\left(\mathbf{p}, \mathbf{r}\right)$, is the congestion cost. It indicates that a vehicle should pay higher costs at higher congestions, which discourages the vehicles from using a high beacon rate and high beacon power.

Then, we define $CBR_i\left(\mathbf{p}, \mathbf{r}\right)$ as that in (Chen et al. 2011) by:

$$CBR_i\left(\mathbf{p}, \mathbf{r}\right) = \sum_{j=1}^{N} h_{ij} r_j \tag{2}$$

where:

$$h_{ij} = T_{frame} \times \frac{\Gamma\left(m, m\frac{C_{Tt}}{\Omega_{ij}}\right)}{\Gamma\left(m\right)} \tag{3}$$

$$\Omega_{ij} = \frac{p_j \lambda^2}{(4\pi)^2 d_{ij}^{\gamma}} \tag{4}$$

Γ is the gamma function, $\Gamma\left(.,.\right)$ is the upper incomplete gamma function, C_{Tt} is the threshold power level of carrier sense, p_j is the BSM transmit power of vehicle j, d_{ij} is the distance between j th and i th vehicles, m is Nakagami fading parameter, λ is the wavelength, γ is the path loss exponent, r_j is the beaconing rate of vehicles j, and T_{frame} is the time needed to transmit a beacon message.

Equation (2) indicates that the channel load experienced by vehicle i is the weighted sum of the beaconing rate of all the other vehicles $\sum_{j=1}^{N} h_{ij} r_j$. The channel load also depends on various parameters such as channel fading, the time needed to transmit a beacon message, and the distance of other vehicles. The coefficients h_{ij} defined in (3), represents the action of these parameters in the channel load sensed by vehicle i.

3. A NON-COOPERATIVE GAME FORMULATION

Let $G = \left[\mathcal{N}, \left\{R_i, P_i\right\}, \left\{U_i\left(.\right)\right\}\right]$ denote the non-cooperative beaconing rate and beaconing power game (NRPG), where $\mathcal{N} = \left\{1, \ldots, N\right\}$ is the index set identifying the vehicle, P_i is the beaconing power strategy set of vehicle i, R_i is the beaconing rate strategy set of vehicle i, and $U_i\left(.\right)$ is the utility function of vehicle i defined in Equation (1). We assume that the strategy spaces R_i and P_i of each vehicle i are compact and convex sets with maximum and minimum constraints, for any given vehicle i we consider as strategy spaces the closed intervals $R_i = \left[\underline{r}_i, \overline{r}_i\right]$ and $P_i = \left[\underline{p}_i, \overline{p}_i\right]$. Let the beaconing power vector $\mathbf{p} = (p_1, \ldots, p_N)^T \in P^N = P_1 \times P_2 \times \ldots \times P_N$, beaconing rate vector $\mathbf{r} = (r_1, \ldots, r_N)^T \in R^N = R_1 \times R_2 \times \ldots \times R_N$.

Definition 1. The strategy vector $\left(\boldsymbol{p}^*, \boldsymbol{r}^*\right) = \left(p_1^*, p_2^*, \ldots, p_N^*, r_1^*, r_2^*, \ldots, r_N^*\right)$ is a Nash equilibrium of the NRPG $G = [\mathcal{N}, \left\{R_i, P_i\right\}, \left\{U_i\left(.,.\right)\right\}]$ if:

$$\forall\left(i, r_i, p_i\right) \in \left(\mathcal{N}, R_i, P_i\right),\ U_i\left(p_i^*, r_i^*, \mathbf{p}_{-i}^*, \mathbf{r}_{-i}^*\right) \geq U_i\left(p_i, r_i, \mathbf{p}_{-i}^*, \mathbf{r}_{-i}^*\right)$$

Definition 2. The game G is submodular if she satisfies the following conditions:

- $S_i = P_i \times R_i$ is a compact subset of Euclidean space.
- $U_i\left(p_i, r_i\right)$, $p_i \in P_i$, $r_i \in R_i$ is smooth and:
  - submodular in $\left(p_i, r_i\right)$ for fixed $\left(\mathbf{p}_{-i}, \mathbf{r}_{-i}\right)$ i.e.,

$$\frac{\partial^2 U_i}{\partial p_i \partial r_i} \leq 0 \tag{5}$$

  - Has non-increasing differences in $\left\{\left(p_i, r_i\right), \left(\mathbf{p}_{-i}, \mathbf{r}_{-i}\right)\right\}$, i.e.,

$$\frac{\partial^2 U_i}{\partial r_i \partial r_j} \leq 0, \ \forall j \neq i \tag{6}$$

given that:

$$\frac{\partial^2 U_i}{\partial r_i \partial p_j} = 0, \ \forall j \neq i \tag{7}$$

Theorem 1. The utility function $U_i\left(\boldsymbol{p}, \boldsymbol{r}\right)$ is submodular in $\left(p_i, r_i\right)$ for fixed $\left(\boldsymbol{p}_{-i}, \boldsymbol{r}_{-i}\right)$.
Proof: The second-order partial derivative utility function is written as:

$$\frac{\partial^2 U_i}{\partial p_i \partial r_i} = -\frac{a_i}{\left(1 + r_i + p_i\right)^2} - c_i h_{ii} \leq 0 \tag{8}$$

then the utility function $U_i\left(\mathbf{p}, \mathbf{r}\right)$ is submodular in $\left(p_i, r_i\right)$ for each fixed $\left(\mathbf{p}_{-i}, \mathbf{r}_{-i}\right)$.

Theorem 2. The utility function $U_i\left(\boldsymbol{p}, \boldsymbol{r}\right)$ has non-increasing differences in $\left\{\left(p_i, r_i\right), \left(\boldsymbol{p}_{-i}, \boldsymbol{r}_{-i}\right)\right\}$.
Proof: The second partial derivative of the utility function is:

$$\frac{\partial^2 U_i}{\partial r_i \partial r_j} = 0 \tag{9}$$

and:

$$\frac{\partial^2 U_i}{\partial r_i \partial p_j} = 0 \tag{10}$$

Then the utility function $U_i\left(\mathbf{p},\mathbf{r}\right)$ has non-increasing differences in $\left\{\left(p_i,r_i\right),\left(\mathbf{p}_{-i},\mathbf{r}_{-i}\right)\right\}$.

Based on theorems 1, theorems 2, and definition 2, we conclude the following theorems.

Theorem 3. The NRPG G is submodular in $\left(p_i,r_i\right)$ for all $i\in\mathcal{N}$.

Based on theorem 3, the game G is a submodular game, and the set of its Nash equilibrium points is nonempty. Therefore, the following holds:

Theorem 4. The NRPG game $G=\left[\mathcal{N},\left\{R_i,P_i\right\},\left\{U_i\left(\mathbf{p},\mathbf{r}\right)\right\}\right]$ has at least one Nash equilibrium [6], which is defined as:

$$\left(p_i^*,r_i^*\right)=\arg\max_{p_i\in P_i,r_i\in R_i} U_i\left(\mathbf{p},\mathbf{r}\right) \tag{11}$$

The following theorem proves the uniqueness of the Nash equilibrium point.

Theorem 5. The unique Nash equilibrium point of the NRPG G is given by:

$$\left(p_i^*,r_i^*\right)=\arg\max_{p_i\in P_i,r_i\in R_i} U_i\left(\mathbf{p},\mathbf{r}\right) \tag{12}$$

s.t.

$$\left.\frac{\partial U_i\left(\mathbf{p},\mathbf{r}\right)}{\partial p_i}\right|_{p_i=p_i^*}=0\ and\ \left.\frac{\partial U_i\left(\mathbf{p},\mathbf{r}\right)}{\partial r_i}\right|_{r_i=r_i^*}=0 \tag{13}$$

and:

$$\left(p_i,r_i\right)J\left(p_i,r_i\right)(p_i,r_i)^T\leq 0,\ \forall p_i\in P_i,\ \forall r_i\in R_i \tag{14}$$

where:

$$J=\begin{pmatrix}\frac{\partial^2 U_i}{\partial p_i^2} & \frac{\partial^2 U_i}{\partial p_i\partial r_i}\\ \frac{\partial^2 U_i}{\partial p_i\partial r_i} & \frac{\partial^2 U_i}{\partial r_i^2}\end{pmatrix}$$

is the Hessian matrix at point $\left(p_i,r_i\right)$.

Proof: The conditions of the first-order partial derivatives (13) determine the stationary points of the utility function $U_i\left(\mathbf{p},\mathbf{r}\right)$, which can either be a maximum, a minimum or a saddle point. The condition (14) is necessary to find the global maximum of the utility function.

$$J=\begin{pmatrix} \frac{\partial^2 U_i}{\partial p_i^2} & \frac{\partial^2 U_i}{\partial p_i \partial r_i} \\ \frac{\partial^2 U_i}{\partial p_i \partial r_i} & \frac{\partial^2 U_i}{\partial r_i^2} \end{pmatrix} \tag{15}$$

$$=\begin{pmatrix} -\frac{a_i}{\left(1+r_i+p_i\right)^2} & -\frac{a_i}{\left(1+r_i+p_i\right)^2}-c_i h_{ii} \\ -\frac{a_i}{\left(1+r_i+p_i\right)^2}-c_i h_{ii} & -\frac{a_i}{\left(1+r_i+p_i\right)^2} \end{pmatrix} \tag{16}$$

Thus,

$$\begin{aligned}\left(p_i,r_i\right)J\left(p_i,r_i\right)\left(p_i,r_i\right)^T &= -\frac{a_i p_i^2}{\left(1+r_i+p_i\right)^2}-\frac{a_i r_i^2}{\left(1+r_i+p_i\right)^2} \\ -\frac{a_i p_i^2}{\left(1+r_i+p_i\right)^2}-c_i h_{ii} p_i^2-\frac{a_i r_i^2}{\left(1+r_i+p_i\right)^2}-c_i h_{ii} r_i^2 &\le 0\end{aligned} \tag{17}$$

Then, the Hessian matrix J is negative definite.

Since it is hard to get the analytical result of the system (13), we use an iterative and distributed algorithm that finds the unique Nash equilibrium point $\left(\mathbf{p}^*,\mathbf{r}^*\right)$. This algorithm is defined as follows.

3.1 Iterative Nash Equilibrium Algorithm

In this section, building upon our earlier analysis, we present two distributed and iterative learning methodologies designed to converge towards the Nash equilibrium point of the Non-cooperative Reinforcement Power Game (NRPG). The best response algorithm, well-established for S-modular games, is employed here, leveraging the monotonicity of the best response functions. Each player in the game determines its optimal strategies to maximize its individual utility. Subsequently, players assess the strategies adopted by their counterparts in previous iterations, incorporating this information into their decision-making processes to adjust their own strategies accordingly. Consequently, the Nash equilibrium emerges as the natural convergence point of the game.

Algorithm 1 outlines the steps involved in the best response learning process, detailing the iterative actions that each player undertakes to identify its Nash equilibrium strategy.

Algorithm 1. Best Response Algorithm

1: Initialize vectors $\mathbf{p}\left(0\right)=\left[p_1\left(0\right),\ldots,p_N\left(0\right)\right]$ and $\mathbf{r}\left(0\right)=\left[r_1\left(0\right),\ldots,r_N\left(0\right)\right]$ randomly;

2: **For** each vehicle i at round t computes:

$$p_i\left(t+1\right)=\underset{p_i\in P_i}{\operatorname{argmax}}\left(U_i\left(\mathbf{p},\mathbf{r}\right)\right)$$

$$r_i\left(t+1\right)=\underset{r_i\in R_i}{\operatorname{argmax}}\left(U_i\left(\mathbf{p},\mathbf{r}\right)\right)$$

3: **If** $\left|r_i\left(t+1\right)-r_i\left(t\right)\right|<\varepsilon$ and $\left|p_i\left(t+1\right)-p_i\left(t\right)\right|<\varepsilon$, then **STOP**.

4: **Else** make $t\leftarrow t+1$ and go to step (2).

4. COOPERATIVE GAME

The Nash bargaining game (Nash Jr 1950) is a cooperative game in which players have a mutual agreement for cooperation in order to obtain a higher payoff compared to the non-cooperative case. Let $\mathcal{U}$ be a closed and convex subset of $\mathbb{R}^N$ that represents the set of feasible payoff allocations that the players can get if they all cooperate. Suppose $\{U_i\in\mathcal{U}\mid U_i\geq U_i^{min},\forall i\in\mathcal{N}\}$ is a nonempty bounded set. Define $\mathbf{U}^{min}=\left(U_1^{min},U_2^{min},\ldots,U_N^{min}\right)$, then the pair of $\left(\mathcal{U},\mathbf{U}^{min}\right)$ constructs a $K-$ player bargaining game. Here, we define the Pareto efficient point (Fudenberg and Tirole, J. 1993), where a player can not find another point that improves the utility of all the players at the same time.

Definition 3. A strategy profile $\left(\boldsymbol{p}^*,\boldsymbol{r}^*\right)=\left(p_1^*,p_2^*,\ldots,p_N^*,r_1^*,r_2^*,\ldots,r_N^*\right)$ is Pareto-optimal if and only if there is no other strategy profile $\left(\mathbf{p},\mathbf{r}\right)$ such that $U_i\left(\mathbf{p},\mathbf{r}\right)\geq U_i\left(\mathbf{p}^*,\mathbf{r}^*\right)$, $\forall i\in\mathcal{N}$, and $U_i\left(\mathbf{p},\mathbf{r}\right)>U_i\left(\mathbf{p}^*,\mathbf{r}^*\right)$, $\exists i\in\mathcal{N}$, i.e., there exists no other strategies that lead to superior performance for some players without causing inferior performance for some other players (Fudenberg and Tirole, J. 1993).

There may be an infinite number of Pareto optimal points in a game of multi-players. Thus, we must address how to select a Pareto point for a cooperative bargaining game. We need a criterion to select the best Pareto point of the system. A possible criterion is the fairness of resource allocation. Notably, the fairness of bargaining games is a Nash bargaining solution, which can provide a unique and fair Pareto optimal point under the following axioms.

Definition 4. $\overline{r}$ is a Nash bargaining solution in $\mathcal{U}$ for $\mathbf{U}^{min}$ i.e., $\overline{r}=\mathcal{H}\left(\mathcal{U},\mathbf{U}^{min}\right)$, if the following axioms are satisfied (Fudenberg and Tirole, J. 1993).

- Individual rationality: $\overline{r}_i\geq U_i^{min}$, $\overline{r}_i\in\overline{r}$, $i\in\mathcal{N}$.
- Feasibility: $\overline{r}\in\mathcal{U}$.
- Pareto Optimality: $\overline{r}$ is Pareto optimal.
- Independence of Irrelevant Alternatives: If $\overline{r}\in\mathcal{U}'\subset\mathcal{U}$, $\overline{r}=\mathcal{H}\left(\mathcal{U},\mathbf{U}^{min}\right)$, then $\overline{r}=\mathcal{H}\left(\mathcal{U}',\mathbf{U}^{min}\right)$.

- Independence of Linear Transformations: For any linear scale transformation Θ, $\Theta\left(\mathcal{H}\left(\mathcal{U},\mathbf{U}^{min}\right)\right) = \mathcal{H}\left(\Theta\left(\mathcal{U}\right),\Theta\left(\mathbf{U}^{min}\right)\right)$.
- Symmetry: If $\mathcal{U}$ is invariant under all exchanges of players, that is $\mathcal{H}_i\left(\mathcal{U},\mathbf{U}^{min}\right) = \mathcal{H}_j\left(\mathcal{U},\mathbf{U}^{min}\right)$, $\forall i,\ j$.

Theorem 6. A unique and fair Nash bargaining solution $\mathbf{x}^* = \left(\mathbf{p}^*,\mathbf{r}^*\right)$ that satisfies all the axioms in Definition 4 can be obtained by maximizing a product term as follows:

$$\boldsymbol{x}^* = \underset{p_i\in P_i, r_i\in R_i}{\operatorname{argmax}} \prod_{i=1}^{N} U_i\left(\mathbf{p},\mathbf{r}\right) \tag{18}$$

Proof: The proof of the theorem 6 is omitted due to space limitations. A similarly detailed proof can be found in (Nash Jr 1950).

Our work aims to maximize utility functions while decreasing the number of losses beacons. Therefore, the corresponding cooperative Nash bargaining game-theoretic power and rate control problem for vehicle underlying the communication system can be formulated as:

$$\mathbf{P}1: \max_{p_i\in P_i, r_i\in R_i} \prod_{i=1}^{N} U_j\left(\mathbf{p},\mathbf{r}\right) \tag{19}$$

$$s.t. \begin{cases} C1: 0 \le p_i \le p_i^{max} \\ C2: 0 \le r_i \le r_i^{max} \end{cases}$$

where constraint $C1$ limits the beaconing power of vehicle i to be below p_i^{max} and $C2$ limits the beaconing rate of vehicle i to be below r_i^{max}.

Lemma 1. Define $V_i\left(\mathbf{p},\mathbf{r}\right) \triangleq ln\left(U_i\left(\mathbf{p},\mathbf{r}\right)\right)$, $i \in \mathcal{N}$. These objective functions are concave and injective, which satisfy all the Nash axioms in Definition 4.

Proof: The proof of theorem 5 shows that the Hessian matrix of the utility function $U_i\left(\mathrm{p},\mathrm{r}\right)$ is negatively define. Then, the utility function $U_i\left(\mathrm{p},\mathrm{r}\right)$ is strictly concave with regard to the 2-tuple $\left(p_i,r_i\right)$. Subsequently, $V_i\left(\mathbf{p},\mathbf{r}\right) = ln\left(U_i\left(\mathbf{p},\mathbf{r}\right)\right)$ is also concave in $\left(p_i,r_i\right)$. Therefore, $V_i\left(\mathbf{p},\mathbf{r}\right)$ defined above satisfies all the axioms required by Definition 4 and Theorem 6.

According to Theorem 6 and Lemma 1, the unique Nash bargaining equilibrium with fairness can be found over the strategy space. Then, taking advantage of the increasing property of the logarithmic function, the optimization problem **P1** can be rewritten as:

$$\mathbf{P}2: \max_{p_i\in P_i, r_i\in R_i} \sum_{i=1}^{N} V_i\left(\mathbf{p},\mathbf{r}\right) = \max_{p_i\in P_i, r_i\in R_i} \sum_{i=1}^{N} U_i\left(\mathbf{p},\mathbf{r}\right) \tag{20}$$

$$s.t. \begin{cases} C1: 0 \le p_i \le p_i^{max} \\ C2: 0 \le r_i \le r_i^{max} \end{cases}$$

4.1 Solution of the Cooperative Game

Herein, we derive the unique equilibrium by solving the constrained optimization problem in (20) utilizing the method of Lagrange multipliers (Shi et al. 2017). Introducing Lagrange multipliers $\{\chi_i^{ite}\}_{i=1}^N$ and $\{\psi_i^{ite}\}_{i=1}^N$ for the multiple constraints, the Lagrangian of problem (20) can equivalently be solved by maximizing the following expression:

$$\mathcal{F}(\mathbf{p},\mathbf{r},\{\chi_i^{ite}\}_{i=1}^N,\{\psi_i^{ite}\}_{i=1}^N) = \sum_{i=1}^N \Big(a_i \log\big(r_i + p_i + 1\big) - c_i p_i CBR_i\left(\mathbf{p},\mathbf{r}\right) - \left(C_{s_i} + C_{p_i} p_i + C_{r_i} r_i\right) - \chi_i p_i - \psi_i r_i\Big) \tag{21}$$

Based on the standard optimization methods and the Karush–Kuhn–Tucker conditions, the beaconing power of vehicle i can be obtained by taking the first derivative of (21) with respect to p_i, which is expressed as follows:

$$\frac{\partial \mathcal{F}}{\partial p_i} = \frac{a_i}{1 + p_i + r_i} - c_i CBR\left(\mathbf{p},\mathbf{r}\right) - C_{p_i} - \chi_i \tag{22}$$

Letting $\frac{\partial \mathcal{F}}{\partial p_i} = 0$ we get,

$$p_i^* = \frac{a_i}{c_i CBR\left(\mathbf{p},\mathbf{r}\right) + C_{p_i} + \chi_i^*} - 1 - r_i^* \tag{23}$$

Meanwhile, the beaconing rate of vehicle i can be obtained by taking the first derivative of (21) with respect to r_i as:

$$\frac{\partial \mathcal{F}}{\partial r_i} = \frac{a_i}{1 + p_i + r_i} - c_i h_{ii} - C_{r_i} - \psi_i \tag{24}$$

Let (24) equals to zero, then we get:

$$r_i^* = \frac{a_i}{c_i h_{ii} + C_{r_i} + \psi_i^*} - 1 - p_i^* \tag{25}$$

In this work, we employ the fixed-point technique to derive an iterative procedure that updates the beaconing rate and beaconing power control decisions, which can be given as:

$$p_i^{ite+1} = \left[\frac{a_i}{c_i CBR\left(\mathbf{p},\mathbf{r}\right) + C_{p_i} + \chi_i^{ite}} - 1 - r_i^{ite}\right]_0^{p_i^{max}} \tag{26}$$

$$r_i^{ite+1} = \left[\frac{a_i}{c_i h_{ii} + C_{r_i} + \psi_i^{ite}} - 1 - p_i^{ite} \right]_0^{r_i^{max}} \quad (27)$$

4.2 Update of the Lagrange Multipliers

The Lagrange multipliers $\{\chi_i^{ite}\}_{i=1}^N$ and $\{\psi_i^{ite}\}_{i=1}^N$ need to be updated to guarantee the fast convergence property. Several practical approaches can be employed in the update of Lagrange multipliers. In this paper, the sub-gradient technique is utilized to update the multipliers, as formulated as follows:

$$\begin{cases} \psi_i^{ite+1} = \left[\psi_i^{ite} - \alpha^{ite} p_i^{ite+1}\right]^+ \\ \chi_i^{ite+1} = \left[\chi_i^{ite} - \alpha^{ite} r_i^{ite+1}\right]^+ \end{cases} \quad (28)$$

where $(x)^+ = max\left(0, x\right)$, β denotes the step size of iteration ite $(ite \in \left\{1, 2, \ldots, L_{max}\right\}$ and L_{max} denotes the maximum number of iterations.

4.3 Iterative Nash Bargaining Algorithm

In this section, a distributed algorithm is proposed as an implementation of our cooperative bargaining beaconing rate and beaconing power control solution. The proposed iterative Algorithm 2 will guarantee convergence by using the subgradient method.

5. PERFORMANCE EVALUATION

Extensive experiments have been undertaken to address the following inquiries: (1) the number of iterations necessary for the proposed algorithm to reach convergence towards equilibrium beaconing rate and power; (2) identifying the most efficient algorithm for rapidly converging towards equilibrium

Algorithm 2. Cooperative Bargaining Algorithm

1: Initialize c_i, a_i, C_{p_i}, C_{r_i} and Lagrange multipliers $\{\chi_i^{ite}\}_{i=1}^N$ and $\{\psi_i^{ite}\}_{i=1}^N$; set $ite = 1$;
2: Initialize $\{p_i^{ite}\}_{i=1}^N$ and $\{r_i^{ite}\}_{i=1}^N$;
3: **repeat**
4: **for** $i = 1$ to N **do**
5: (i) Update p_i^{ite} according to (26);
6: (ii) Update r_i^{ite} according to (27);
7: (iii) Update χ_i^{ite} and ψ_i^{ite} according to (28);
8: **end for**
9: (iv) Set $ite \leftarrow ite + 1$;
10: **until** Convergence or $ite = L_{max}$
11: **return** $\{p_i^{ite}\}_{i=1}^N$ and $\{r_i^{ite}\}_{i=1}^N$.

strategies; and (3) examining how system parameters influence the equilibrium beaconing rate and power. In this section, we present the outcomes of these experimental investigations, utilizing the expressions derived from the utility function discussed earlier. To illustrate, we focus on a scenario involving two vehicles.

Figures 1 and 2 provide compelling evidence of the uniqueness of joint beaconing rate and power at Nash equilibrium. The best response algorithm consistently converges towards the beaconing rate and power values corresponding to the Nash equilibrium. Similarly, Figures 3 and 4 depict the convergence of the cooperative bargaining algorithm towards the Pareto-optimal equilibrium. Furthermore, the rapid convergence of the proposed algorithms is evident from the results depicted in Figures 1, 2, 3, and 4. Specifically, the best response algorithm typically converges within five to 35 iterations, while the cooperative bargaining algorithm achieves convergence to the Pareto-optimal equilibrium after approximately 10 iterations. Consequently, the cooperative bargaining algorithm emerges as the faster-converging option, making it well-suited for real-world applications

Note that for any vehicle i, it's Nash equilibrium beaconing rate r_i and beaconing power p_i primarily depends on the parameter a_i, c_i, C_{p_i} and C_{r_i}. As such, we investigate how the Nash equilibrium points can be affected by these parameters.

Figures 5 and 6 depict variations in the beaconing rate and power of vehicles as the parameter ***a*** ranges from 1 to 20. As ***a*** increases, both the beaconing rate and power of vehicles exhibit a corresponding rise. This trend is attributed to the increase in utility as ***a*** escalates. Consequently, vehicles are more inclined to elevate their beaconing rate and power in response to the heightened utility. The escalation of parameter ***a*** prompts vehicles to utilize higher beaconing rates and power levels, aligning with the amplified utility function.

In Figures 7 and 8, we visualize the relationship between the cost ***c*** and the beaconing rate, as well as the beaconing power, for the two vehicles under consideration in this example. Notably, we observe a downward trend in both the beaconing equilibrium rate and power concerning the cost

Figure 1. Seeking the equilibrium beaconing power using the best response algorithm

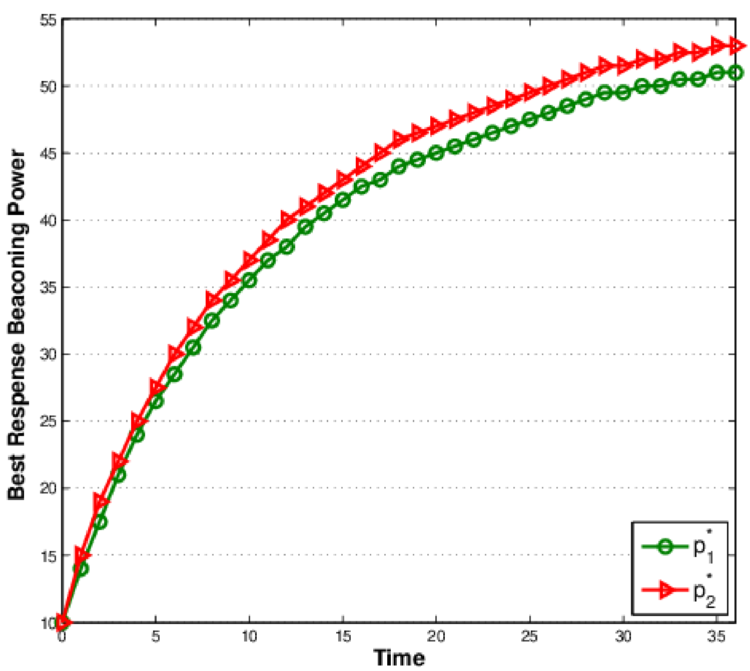


Figure 2. Seeking the beaconing equilibrium rate using the best response algorithm

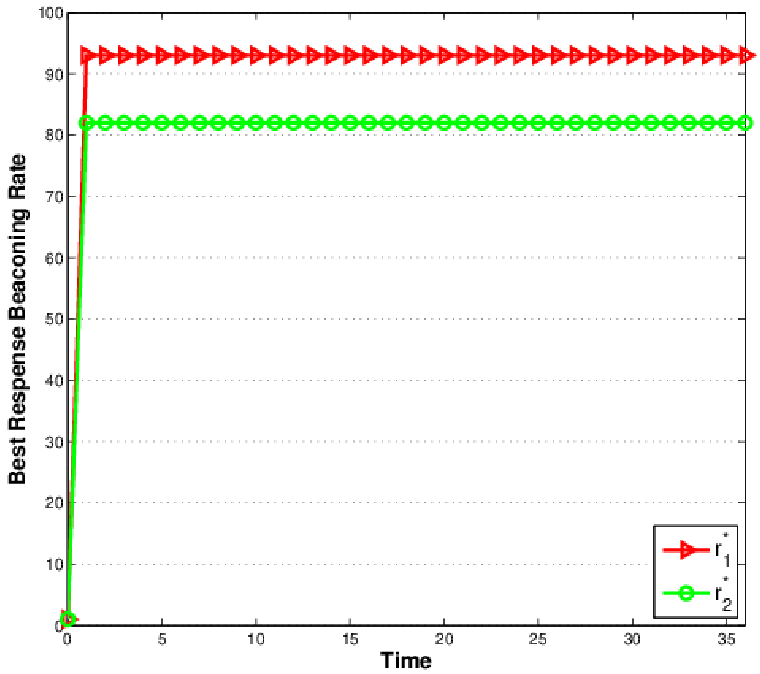


Figure 3. Seeking the equilibrium beaconing power using a cooperative bargaining algorithm

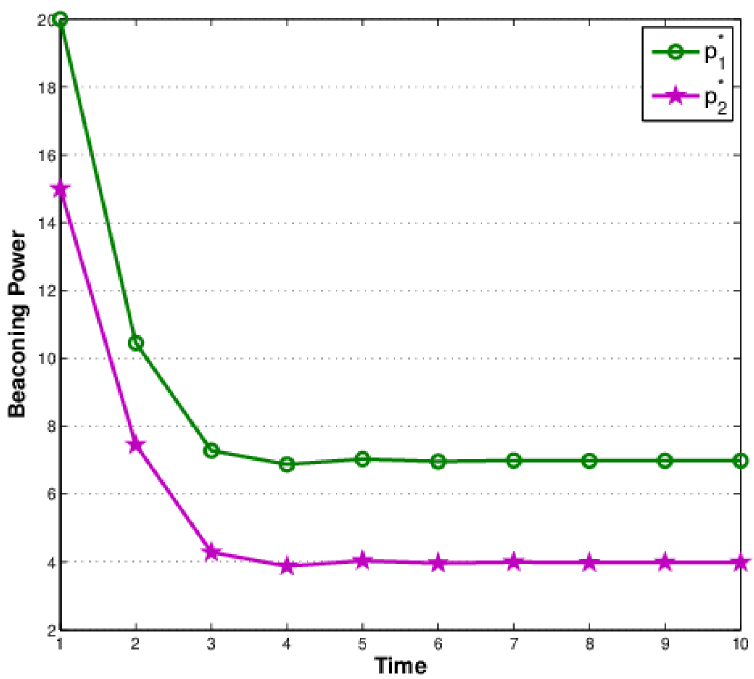


Figure 4. Seeking the beaconing equilibrium rate using a cooperative bargaining algorithm

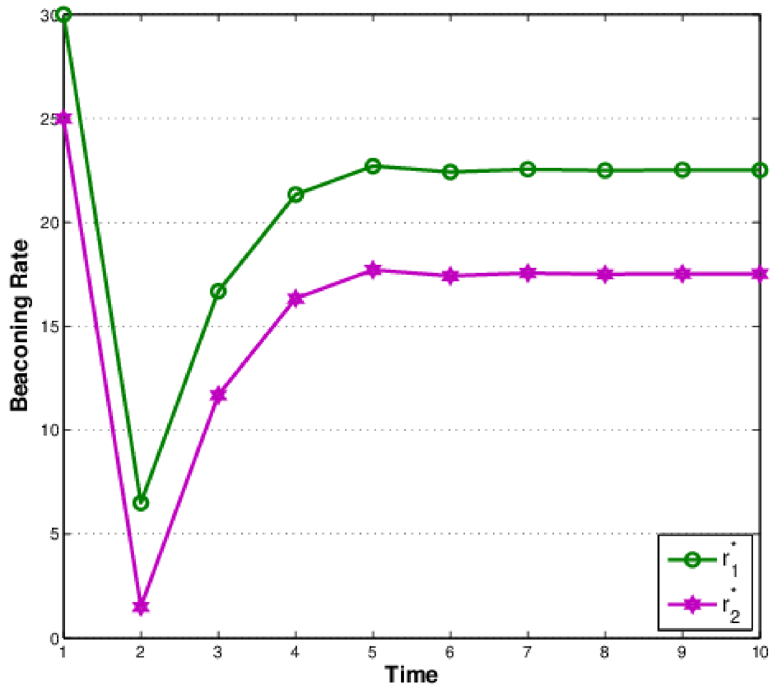


Figure 5. Beaconing power with respect to a

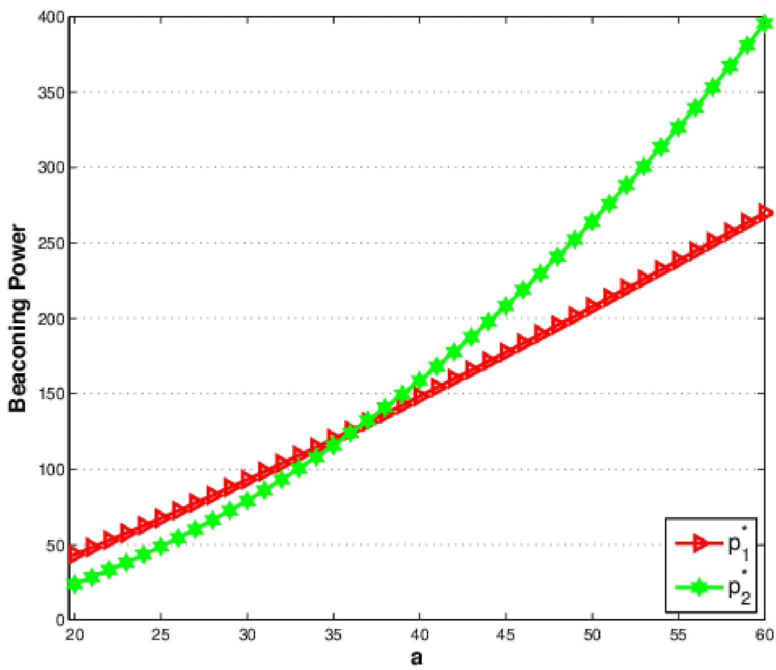


Figure 6. Beaconing rate with respect to a

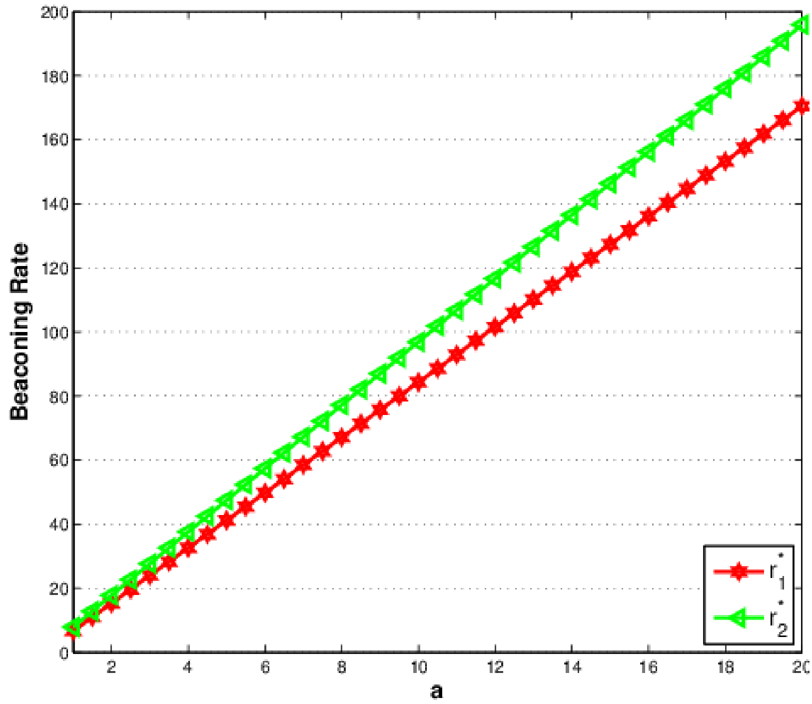


Figure 7. Beaconing power with respect to c

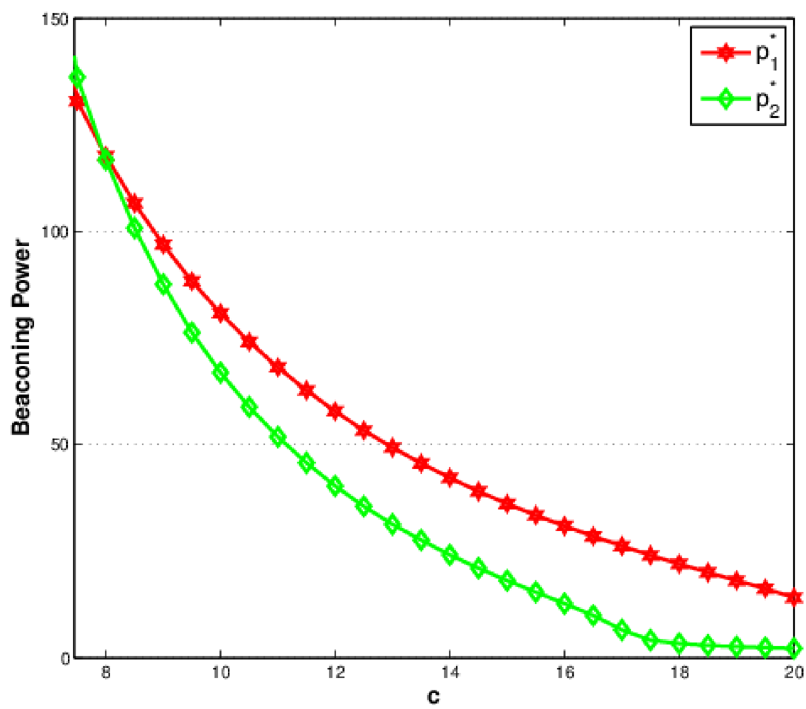
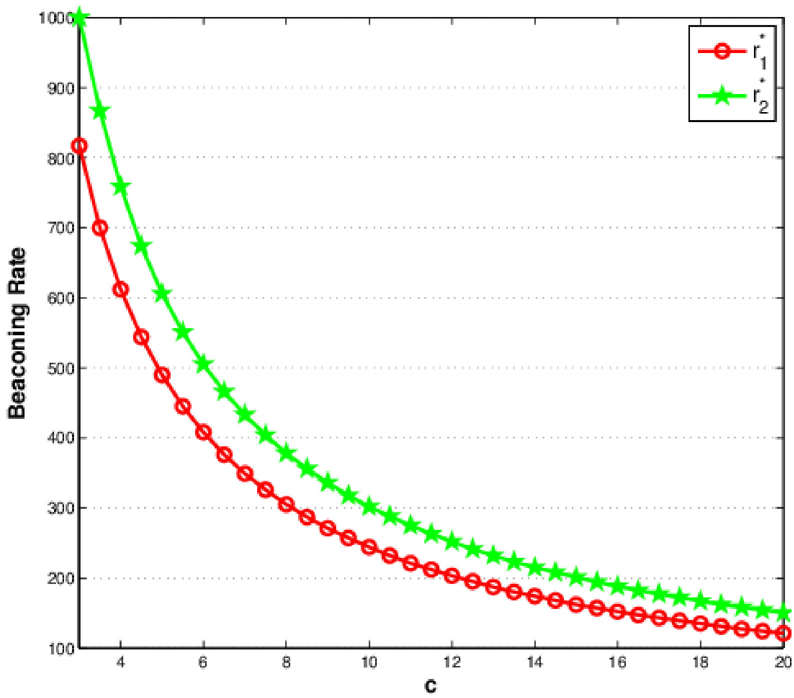


Figure 8. Beaconing rate with respect to c



c. As the cost *c* escalates, vehicles incur higher expenses during periods of heightened congestion, resulting in diminished payoffs. Consequently, vehicles are motivated to reduce their beaconing rate and power to alleviate congestion costs.

Figures 9 and 10 show both the beaconing power and the beaconing for the non-cooperative games and the cooperative strategic beaconing obtained using the Nash bargaining solution. When energy cost (C_r and C_p) rise, there is a noticeable decrease in both beaconing power and beaconing rate. Notably, the strategic beaconing strategy derived from the Nash bargaining solution consistently displays lower levels as energy costs elevate in comparison to the non-cooperative beaconing strategy. A notable distinction is that the Nash bargaining solution-based strategic beaconing scheme demonstrates superior energy efficiency across all energy cost values when contrasted with the non-cooperative strategy. Consequently, the Nash bargaining solution scheme ensures an extended network lifetime compared to its non-cooperative counterpart.

6. CONCLUSION

This paper tackles the challenge of jointly controlling beaconing rate and power in VANETs using S-modular theory. We model the competition among vehicles in VNETs as both a non-cooperative and cooperative game, wherein each vehicle selects its beaconing rate and power. Equilibrium analysis is conducted, and we propose three distributed algorithms for computing the equilibrium point. Through simulations, we demonstrate how system parameters impact joint beaconing rate and power, while also revealing the iteration counts required by each algorithm to achieve convergence. Our analysis and simulation outcomes offer valuable insights into the intricate dynamics among vehicles, whether in competitive or cooperative scenarios, thereby facilitating the optimization of vehicle strategies.

Figure 9. Beaconing power with respect to C_p

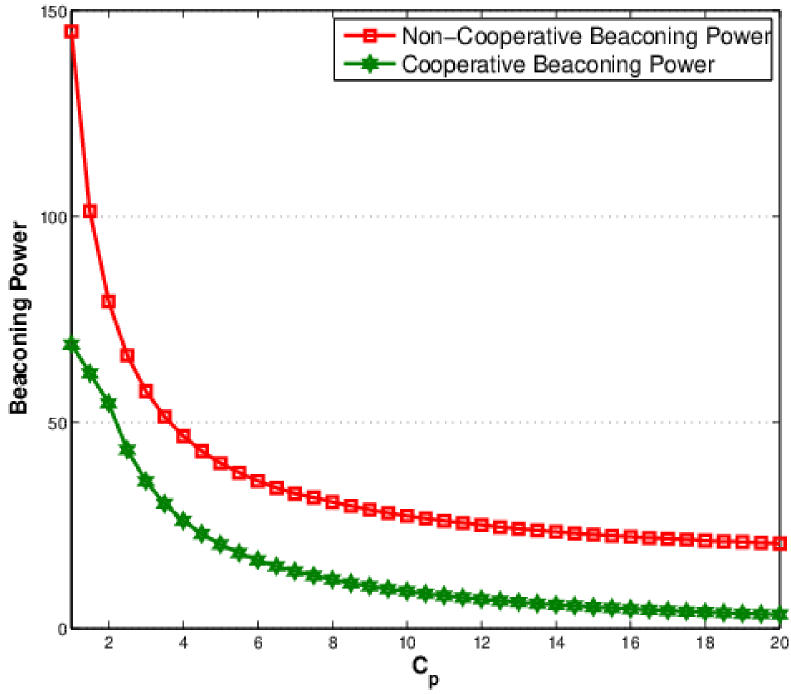
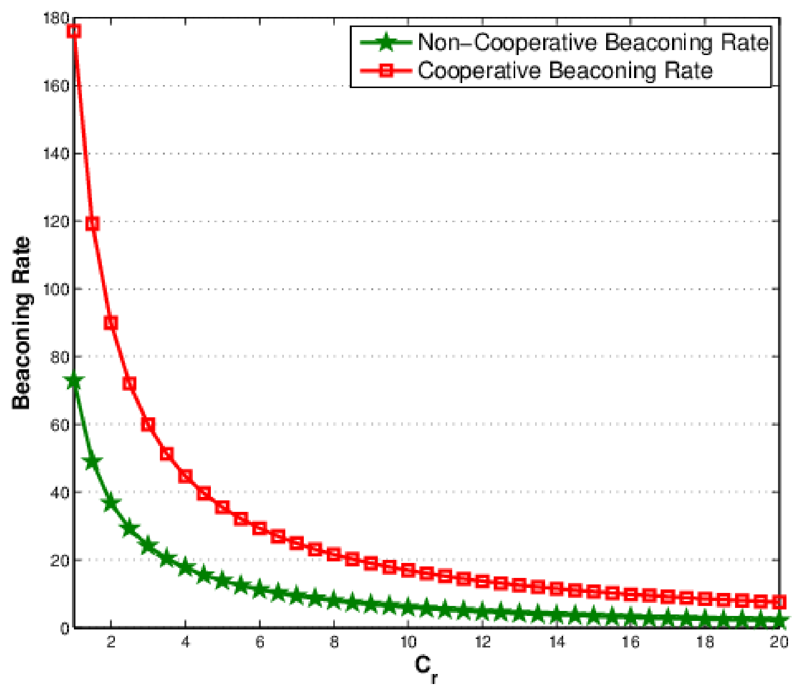


Figure 10. Beaconing rate with respect to C_r



CONFLICTS OF INTEREST

We wish to confirm that there are no known conflicts of interest associated with this publication and there has been no significant financial support for this work that could have influenced its outcome.

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