



A Differential Epidemic Model for Information, Misinformation, and Disinformation in Online Social Networks: COVID-19 Vaccination


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ABSTRACT

These days the online social network has become a huge source of data. People are actively sharing information on these platforms. The data on online social networks can be misinformation, information, and disinformation. Because online social networks have become an important part of our lives, the information on online social networks makes a great impact on us. Here a differential epidemic model for information, misinformation, and disinformation on online social networks is proposed. The expression for basic reproduction number has been developed. Again, the stability condition for the system at both infection-free and endemic equilibriums points has been discussed. The numerical simulation has been performed to validate the theoretical results. Data available on Twitter related to COVID-19 vaccination is used to perform the experiment. Finally, the authors discuss the control strategy to minimize the misinformation and disinformation related to vaccination.

KEYWORDS

Control Strategy, COVID-19 Vaccination, Differential Epidemic Model, Numerical Simulation, Online Social Network, Stability of the System

1. INTRODUCTION

To stay connected is the basic characteristic of the human race. From thousands of years ago to till date, men have used different kinds of tools to communicate their feelings, share information and ideas with their known and new ones. In the 21st century, the most common tools that enable people

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to connect with each other are web-based social media platforms. Social media platforms have made a tremendous impact on everyone's life. The various studies and research have shown that a good number of people spend 25% of their daily life on social media (Daily time spent on social networking by internet users worldwide from 2012 to 2020, n.d.). It has played a vital role in making the global compact. The changes that are brought by the social media in our life can be felt easily. It has made communication much easier than ever before. People can share their ideas, thought, data, information, and videos through these platforms rather easily. These platforms may come in different blogs, microblogs, podcasts, weblogs etc. These days, the most commonly used social media platforms are Facebook, Whatsapp, Instagram, Twitter etc. People of different spheres and views share their opinion, facts and information through these channels.

As social media made an impact on most of the user's life, it has become a challenge for the platform providers to develop a security check system so that only reliable and useful information spread through these platforms. Social media deliver and exchange millions of information. This information can be classified into three categories i.e information, misinformation, and disinformation. Information is simply a processed data or facts. Misinformation is something false or inaccurate information that is being communicated not with a view to harmful intention. In other words, misinformation is not true but is not shared with a malicious purpose. Disinformation is also a kind of misinformation but it is communicated with a deceptive purpose. Disinformation is the most harmful form of information that is communicated and shared with a view to create panic, suspicion, and fear among the masses.

So, different types of data and information keep propagating in the sphere of these social media platforms. Some of these are useful and informative to us while some contents are being propagated with a malicious agenda. The intention behind such malicious agenda is to harm the user's reputation and to create a havoc environment over these web based platforms.

As obvious from the above, it has become a challenge for the social media service providers to provide a safeguard to their users against such malpractices that cause a form of fear and panic so that the reliability and the integrity of these service providers remain unquestioned among the population. Here a dynamic system has been proposed that will give safeguard against such malpractices to the social media users to a great extent.

1.1. Literature Review

The history of information transmission within the population is very old. Goffman and Newill considered the transmission of ideas within a population as an epidemic process. They studied the role of information retrieval in the development of such a process (Goffman & Newill, 1964). Daley and Kendall found that superficial similarities in rumor and epidemic outbreaks. They divide the whole population into three categories according to their action on a rumor, such as ignorants, spreaders, and stiflers (Daley & Kendall, 1964). Zhou, Liu and Li studied the propagation of the rumors for complex networks. They studied the rumor propagation numerically and analytically using the traditional SIR model (Zhou et al., 2007). Chen, Liu, Shen and Yuan proposed a stochastic model for rumor propagation in online social networks. They considered the infection probability as a function of ties (Cheng et al., 2013). Wen, Zhou, Zhang, Xiang, Zhou and Jia considered two factors- temporal dynamic and special dependence in their susceptible-infectious-immunized (SII) model. The previous models did not consider these two effects. They found that these factors affect the accuracy of the previous model. The SII model gives more accurate results than other traditional SIR and SIS models for rumors propagation (Wen et al., 2012).

Zhao, Xie, Gaob, Qiu, Wanga and Zhanga considered the rumor spreading model with variable forgetting rate over time. They found that a larger initial forgetting rate or faster forgetting rate decreases the rumor spreading within the network (Zhao et al., 2013). Cannarella and Spechler have given that infection, recovered SIR (irSIR) model for the online social networks. Here adoption and abandonment of network considered as infection and recovery (Cannarella & Spechler, 2014). Dhar,

Jain, Gupta proposed an SEI model for news spreading in Online social networks. They have given the criteria for rumor detection and verification. Again they proposed a revised model with media awareness as a control measure for rumors (Dhar et al., 2016). Shrivastava, Kumar, Ojha, Srivastava, Mohan and Srivastava have given a model for fake news propagation in Online social networks. They divided the population into four classes – susceptible, infectious, verified, and recovered. The model explains how the misinformation can disseminate among the groups. They used their model to eliminate the fake news related to COVID-19 which disseminated in the online social networks faster than ever (Shrivastava et al., 2020). Liang'an Huo, Sijing Chen, Xiaoxiao Xie, Jianjia He and Huiyuan Liu has been discussed in their work about the Optimal Control strategy for ISTR Rumor Propagation Model. They have considered the social reinforcement in heterogeneous network (Huo et al., 2021). Wei Zhang, Hongyong Deng, Xingmei Li and Huan Liu studied the dynamics of the rumor-spreading in complex network. They gave control mechanism for rumors in the network and also studied about effect of different parameter (Zhang et al., 2022).

From various literature reviews, it is shown that the maximum models on online social network are based on the Simple SIS, SIR and SEIS models. These models mainly explain the spread of rumor, their detection and gave control measure for it. But any news can be information, misinformation and disinformation. The existing models do not differentiate between these and could not consider the effects of these separately. This gives the idea of a differential epidemic model for rumor propagation which can be information, misinformation and disinformation. So, a differential SEIS epidemic model has been proposed for rumors propagation in online social networks. The model helps us to understand separately the propagation of misinformation, information and disinformation and also gives an idea to control them.

The rest of this paper is organized as follows: Section 2 deals with the model formulation; Section 3 explains the basic reproduction number and expression for this. Section 4 describes the infection-free equilibrium and the local and global stability of the system at infection-free equilibrium. Section 5 explains the existence of endemic equilibrium and local and global stability at endemic equilibrium. Section 6 describes the numerical simulation of the theoretical results. Finally, Section 7 contains the conclusion of the paper.

2. MODEL FORMULATION

From above literature review shows that there are so many existing works on rumor propagation in the field of social network. But these are based on simple model. Here it is considered as the more complex model to understand the nature of rumor propagation in the field of online social network. Also it classifies the available data according as information, misinformation and disinformation for better understanding of their effect on online social network. The SEIS differential epidemic model for spreading information, misinformation and disinformation within the online social network has been considered. Here the social network users are susceptible (S) for any type of information which can be true information or rumor. In the model, the exposed users has been divided into three categories exposed-misinformation (E_m), exposed- information (E_i) and exposed-disinformation (E_d). The susceptible users go to different exposed classes according to their status of the account that contains misinformation or information or disinformation. Further, considered exposed-misinformation users can spread misinformation as well as information and disinformation. In Similar manner other two types of exposed classes can behave. Again, there are three infectious classes like exposed classes. These are misinformation (I_m), information (I_i) and disinformation (I_d). If exposed users start spreading the information, misinformation and disinformation then they go to respective infectious classes according to the category of information. Also after applying some recovery mechanism misinformation and disinformation can remove and users again become susceptible. Here consider N as total number of users of online social network. The main aim of the model is to reduce the misinformation and disinformation within the network for a healthy social network environment.

$$\frac{dS}{dt} = \lambda - (\mu + \beta_m I_m + \beta_i I_i + \beta_d I_d)S + (\gamma_m I_m + \gamma_i I_i + \gamma_d I_d) \quad (1a)$$

$$\frac{dE_m}{dt} = P_m (\beta_m SI_m + \beta_i SI_i + \beta_d SI_d) - (\mu + \varepsilon_{mm} + \varepsilon_{mi} + \varepsilon_{md})E_m \quad (1b)$$

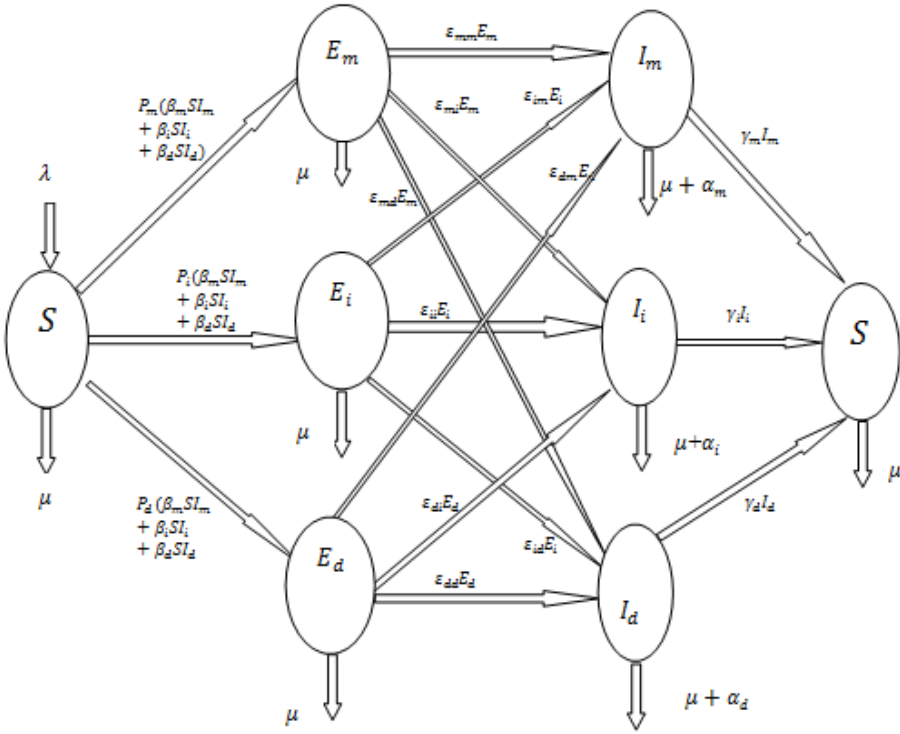
$$\frac{dE_i}{dt} = P_i (\beta_i SI_i + \beta_d SI_d + \beta_m SI_m) - (\mu + \varepsilon_{ii} + \varepsilon_{id} + \varepsilon_{im})E_i \quad (1c)$$

$$\frac{dE_d}{dt} = P_d (\beta_d SI_d + \beta_i SI_i + \beta_m SI_m) - (\mu + \varepsilon_{dd} + \varepsilon_{di} + \varepsilon_{dm})E_d \quad (1d)$$

Table 1. Description of parameters used

| Symbols | Descriptions |
|--|---|
| λ | The rate of the new population joining the network |
| P_m or P_1 | The probability of susceptible users at which it goes to misinformation exposed classes |
| P_i or P_2 | The probability of susceptible users at which it goes to information exposed classes |
| P_d or P_3 | The probability of susceptible users at which it goes to disinformation exposed classes |
| ε_{mm} or ε_{11} | The rate of exposed misinformation users become infectious misinformation |
| ε_{mi} or ε_{12} | The rate of exposed misinformation users become infectious information |
| ε_{md} or ε_{13} | The rate of exposed misinformation users become infectious disinformation |
| ε_{im} or ε_{21} | The rate of exposed information users become infectious misinformation |
| ε_{ii} or ε_{22} | The rate of exposed information users become infectious information |
| ε_{id} or ε_{23} | The rate of exposed information users become infectious disinformation |
| ε_{dm} or ε_{31} | The rate of exposed disinformation users become infectious misinformation |
| ε_{di} or ε_{32} | The rate of exposed disinformation users become infectious information |
| ε_{dd} or ε_{33} | The rate of exposed disinformation users become infectious disinformation |
| γ_m or γ_1 | The recovery rate of misinformation class |
| γ_i or γ_2 | The recovery rate of information class |
| γ_d or γ_3 | The recovery rate of disinformation class |
| α_m or α_1 | The rate at which misinformation infectious class users blocked from the network due to their malicious behavior. |
| α_i or α_2 | The rate at which information infectious class users blocked from the network due to their malicious behavior |
| α_d or α_3 | The rate at which disinformation infectious class users blocked from the network due to their malicious behavior |

Figure 1. Differential SEIS Epidemic Model for Misinformation, Information and Disinformation in Online social network



$$\frac{dI_m}{dt} = (\varepsilon_{mm} E_m + \varepsilon_{im} E_i + \varepsilon_{dm} E_d) - (\mu + \alpha_m + \gamma_m) I_m \quad (1e)$$

$$\frac{dI_i}{dt} = (\varepsilon_{ii} E_i + \varepsilon_{mi} E_m + \varepsilon_{di} E_d) - (\mu + \alpha_i + \gamma_i) I_i \quad (1f)$$

$$\frac{dI_d}{dt} = (\varepsilon_{dd} E_d + \varepsilon_{md} E_m + \varepsilon_{id} E_i) - (\mu + \alpha_d + \gamma_d) I_d \quad (1g)$$

For notational convenience, consider here 1 for misinformation (m), 2 for information (i) and 3 for disinformation (d) users in the subsequent discussion. This makes system in (1a-1g) to appear as follows

$$\frac{dS}{dt} = \lambda - \sum_{j=1}^3 (\mu + \beta_j I_j) S + \sum_{k=1}^3 \gamma_k I_k \quad (1h)$$

$$\frac{dE_n}{dt} = P_n \sum_{j=1}^3 \beta_j S I_j - \left(\mu + \sum_{j=1}^3 \varepsilon_{nj} \right) E_n \quad (1i)$$

$$\frac{dI_k}{dt} = \sum_{n=1}^3 \varepsilon_{nk} E_n - (\mu + \alpha_k + \gamma_k) I_k \quad (1j)$$

3. BASIC REPRODUCTION NUMBER (R_0)

Basic Reproduction Number is defined as number of secondary users infected due to one primary infected user during his life period. Here the primary users are responsible for spreading misinformation, information and disinformation. It is an important parameter for analyzing the spreading of rumors within the network. The expression for Basic Reproduction Number has been calculated using the next generation technique (Van den Driessche & Watmough, 2002). For this two matrices F and V are considered. Here F stands for new rumor in the network and V is the rate at which rumors transmit within the network.

$$F = \begin{bmatrix} P_n \sum_{n=1}^3 \beta_j S I_j \\ 0 \\ 0 \end{bmatrix}; \quad V = \begin{bmatrix} (\mu + \sum_{j=1}^3 \varepsilon_{nj}) E_n \\ (\mu + \alpha_k + \gamma_k) I_k - \sum_{k=1}^3 \varepsilon_{nk} E_n \\ \sum_{n=1}^3 (\mu + \beta_j I_j) S - \sum_{j=1}^3 \gamma_j I_j - \lambda \end{bmatrix}$$

$$\bar{F} = \begin{bmatrix} 0 & P_n \sum_{n=1}^3 \beta_j \\ 0 & 0 \end{bmatrix}; \quad \bar{V} = \begin{bmatrix} \mu + \sum_{j=1}^3 \varepsilon_{nj} & 0 \\ -\sum_{j=1}^3 \varepsilon_{nk} & (\mu + \alpha_k + \gamma_k) \end{bmatrix}$$

$$\bar{V}^{-1} = \frac{1}{\left(\mu + \sum_{n=1}^3 \varepsilon_{nj} \right) (\mu + \alpha_k + \gamma_k)} \begin{bmatrix} (\mu + \alpha_k + \gamma_k) & 0 \\ \sum_{j=1}^3 \varepsilon_{nk} & \mu + \sum_{j=1}^3 \varepsilon_{nj} \end{bmatrix}$$

$$R_0 = \rho(\bar{F}\bar{V}^{-1}) = \frac{\sum_{j=1}^3 P_n \sum_{n=1}^3 \varepsilon_{nj} \beta_j}{\left(\mu + \sum_{j=1}^3 \varepsilon_{nj} \right) (\mu + \alpha_k + \gamma_k)}$$

4. INFECTION FREE EQUILIBRIUM (IFE)

The infection-free equilibrium is defined as the state in which the system has no rumor. Let infection-free equilibrium of the system (1h-1j) is $P(S, 0, 0)$.

Theorem 1: The system (1h-1j) is local asymptotical stable at Infection free equilibrium $P(S, 0, 0)$ when $R_0 < 1$ and unstable when $R_0 > 1$.

Proof: The Jacobian at IFE $P(S, 0, 0)$ of equation (1g-1i) is

$$J = \begin{bmatrix} -\mu & 0 & \sum_{k=1}^3 \gamma_k - \sum_{j=1}^3 \beta_j \\ 0 & -\left(\mu + \sum_{j=1}^3 \varepsilon_{nj}\right) & P_n \sum_{j=1}^3 \beta_j \\ 0 & \sum_{j=1}^3 \varepsilon_{nj} & -(\mu + \alpha_k + \gamma_k) \end{bmatrix}$$

Here First eigen values of Jacobian Matrix J is $\omega_1 = -\mu$ and other two can be calculated from equation:

$\omega^2 + \left\{ \left(\mu + \sum_{j=1}^3 \varepsilon_{nj} \right) + (\mu + \alpha_k + \gamma_k) \right\} \omega + \left(\mu + \sum_{j=1}^3 \varepsilon_{nj} \right) (\mu + \alpha_k + \gamma_k) (1 - R_0) = 0$. Clearly, sum of eigen values of above equation is negative and product of eigen values is positive when $R_0 < 1$.

Thus all eigen values of above Jacobian matrix are negative when $R_0 < 1$.

Hence the system (1h-1j) is local asymptotical stable at IFE $(S, 0, 0)$ when $R_0 < 1$ and unstable when $R_0 > 1$.

Theorem 2: The system (1h-1j) is global asymptotical stable at infection free equilibrium $P(S, 0, 0)$ when $R_0 < 1$ and unstable when $R_0 > 1$.

Proof: For global stability of the system (1h-1j), define the Lapunov's Function L as

$$L = \sum_{j=1}^3 \varepsilon_{nj} E_n + \left(\mu + \sum_{j=1}^3 \varepsilon_{nj} \right) I_k$$

$$\frac{dL}{dt} = \sum_{j=1}^3 \varepsilon_{nj} \frac{dE_n}{dt} + \left(\mu + \sum_{j=1}^3 \varepsilon_{nj} \right) \frac{dI_k}{dt}$$

$$\frac{dL}{dt} = \left(\mu + \sum_{j=1}^3 \varepsilon_{nj} \right) (\mu + \alpha_k + \gamma_k) \left(\frac{\sum_{j=1}^3 P_n \sum_{n=1}^3 \varepsilon_{nj} \beta_j}{\left(\mu + \sum_{j=1}^3 \varepsilon_{nj} \right) (\mu + \alpha_k + \gamma_k)} - 1 \right) I_k$$

$$\frac{dL}{dt} = (\mu + \alpha_n + \gamma_n) \left(\mu + \sum_{n=1}^3 \varepsilon_{nj} \right) (R_0 - 1)$$

From above it can be seen that when $R_0 < 1$ then $\frac{dL}{dt} < 0$ and $\frac{dL}{dt} = 0$ when $E_n = 0$ and $I_k = 0$.

Clearly most extensive invariant set in $\left\{ (S, E_n, I_k) \in \Gamma : \frac{dL}{dt} < 0 \right\}$ is singleton set $\{P\}$. Hence from Lasalle's principle system (1h-1j) is globally stable at IFE when $R_0 < 1$ (LaSalle & Lefschetz, 1976).

5. ENDEMIC EQUILIBRIUM AND EXISTENCE

Endemic equilibrium is defined as the state when system persist rumor for long time. Let (S^*, E_n^*, I_k^*) be endemic equilibrium for system (1h-1j). On solving (1h-1j)

$$S^* = \frac{\sum_{j=1}^3 P_n \sum_{n=1}^3 \varepsilon_{nj} \beta_j}{\left(\mu + \sum_{j=1}^3 \varepsilon_{nj} \right) (\mu + \alpha_k + \gamma_k)} = \frac{1}{R_0}$$

$$I_n^* = \frac{\lambda - \mu S^*}{\sum_{j=1}^3 \beta_j S^* - \sum_{j=1}^3 \gamma_k}$$

$$E_n^* = \frac{P_n \sum_{j=1}^3 \beta_j S^* - \sum_{j=1}^3 \gamma_k}{\left(\mu + \sum_{j=1}^3 \varepsilon_{nj} \right)}$$

Clearly the endemic equilibrium (S^*, E_n^*, I_k^*) exists only when $R_0 > 1$.

Theorem 3: The system (1h-1j) is locally stable at endemic equilibrium (S^*, E_n^*, I_k^*) .

Proof: For global stability of the system (1h-1j) consider the Jacobian matrix of the system at endemic equilibrium (S^*, E_n^*, I_k^*) .

$$J = \begin{bmatrix} -\sum_{j,k=1}^3 (\mu + \beta_j I_k^*) & 0 & \sum_{j,k=1}^3 (\gamma_k - \beta_j S^*) \\ P_n \sum_{j,k=1}^3 \beta_j I_k^* & -\left(\mu + \sum_{j=1}^3 \varepsilon_{nj} \right) & P_n \sum_{j=1}^3 \beta_j S^* \\ 0 & \sum_{j=1}^3 \varepsilon_{nj} & -(\mu + \alpha_k + \gamma_k) \end{bmatrix}$$

$$J = -\sum_{j,k=1}^3 (\mu + \beta_j I_k^*) \left\{ \left(\mu + \sum_{j=1}^3 \varepsilon_{nj} \right) (\mu + \alpha_k + \gamma_k) - \sum_{j=1}^3 \varepsilon_{nj} P_n \sum_{j=1}^3 \beta_j S^* \right\} - P_n \sum_{n=1}^3 \beta_j I_k^* (\beta_j S^* - \gamma_k) \varepsilon_{nj}$$

$$J = 0 - P_n \sum_{j,k=1}^3 \beta_j I_k^* (\beta_j S^* - \gamma_k) \varepsilon_{nj}$$

$$J = -P_n \sum_{n=1}^3 \beta_j I_k^* (\beta_j S^* - \gamma_k) \varepsilon_{nj}$$

Here, $\det(J) < 0, \text{tr}(J) < .$ Hence the system (1h-1j) will be locally stable when $R_0 > 1$ at endemic equilibrium (S^*, E_n^*, I_k^*) .

Theorem 4: The system (1h-1j) is globally stable at endemic equilibrium (S^*, E_n^*, I_k^*) .

Proof: Consider Lapunov's Function L (McCluskey, 2006) at endemic equilibrium (S^*, E_n^*, I_k^*) .

$$L = (S - S^* \log S) + A(E_n - E_n^* \log E_n) + B(I_k - I_k^* \log I_k)$$

$$\frac{dL}{dt} = \left(1 - \frac{S^*}{S}\right) S' + A \left(1 - \frac{E_n^*}{E_n}\right) E_n' + B \left(1 - \frac{I_k^*}{I_k}\right) I_k'$$

Putting value of S', E_n', I_k' from (1h-1j)

$$\begin{aligned} \frac{dL}{dt} = & \left(1 - \frac{S^*}{S}\right) \left(\lambda - \sum_{j=1}^3 (\mu + \beta_j I_j) S + \sum_{k=1}^3 \gamma_k I_k \right) + \\ & A \left(1 - \frac{E_n^*}{E_n}\right) \left(P_n \sum_{j=1}^3 \beta_j S I_j - \left(\mu + \sum_{j=1}^3 \varepsilon_{nj} \right) E_n \right) + \\ & B \left(1 - \frac{I_k^*}{I_k}\right) \left(\sum_{k=1}^3 \varepsilon_{nk} E_n - (\mu + \alpha_k + \gamma_k) I_k \right) \end{aligned}$$

Again from (1h-1j)

$$\lambda = \sum_{j=1}^3 (\mu + \beta_j I_j) S - \sum_{k=1}^3 \gamma_k I_k \quad ; \quad \beta_n = \frac{\left(\mu + \sum_{j=1}^3 \varepsilon_{nj} \right) E_n^*}{P_n S^* I_k^*} ; (\mu + \alpha_k + \gamma_k) = \sum_{k=1}^3 \varepsilon_{nk} E_n$$

On putting these values

$$\begin{aligned} \frac{dL}{dt} = & \left(1 - \frac{S^*}{S}\right) \left\{ \beta_n S I_k^* - \beta_n S I_k - \mu (S - S^*) + \sum_{n=1}^3 \gamma_n (I_k - I_k^*) \right\} + \\ & A \left(1 - \frac{E_n^*}{E_n}\right) \left\{ \left(\mu + \sum_{j=1}^3 \varepsilon_{nj} \right) E_n^* \frac{S I_n}{S^* I_n^*} - \left(\mu + \sum_{k=1}^3 \varepsilon_k \right) E_n \right\} + \\ & B \left(1 - \frac{I_k^*}{I_k}\right) \left(\sum_{n=1}^3 \varepsilon_{nj} E_n - \sum_{j=1}^3 \varepsilon_{nj} \frac{E_n^* I_n}{I_n^*} \right) \\ \frac{dL}{dt} = & \frac{-\mu (s - s^*)}{s} + \frac{(\mu + \sum_{j=1}^8 \varepsilon_{nj}) E_n^*}{P_n} \left(1 - \frac{S I_k}{S^* I_k^*}\right) \left(1 - \frac{S^*}{S}\right) + \\ & A E^* (\mu + \sum_{j=1}^3 \varepsilon_{nj}) \left(1 - \frac{E_n^*}{E_n}\right) \left(\frac{S I_k}{S^* I_k^*} - \frac{E_n}{E_n^*} \right) + \\ & B E^* \left(1 - \frac{I_k^*}{I_k}\right) (\mu + \sum_{j=1}^3 \varepsilon_{nj}) \left(\frac{\varepsilon_{nj}}{(\mu + \sum_{j=1}^3 \varepsilon_{nj}) E_n^*} \frac{E_n}{I_n^*} - \frac{\varepsilon_{nj}}{(\mu + \sum_{j=1}^8 \varepsilon_{nj}) I_k^*} \right) \end{aligned}$$

$$\frac{dL}{dt} = \frac{-\mu(S - S^*)}{S} + \frac{\left(\mu + \sum_{j=1}^3 \varepsilon_{nj}\right) E_n^*}{P_n} f(x, y, z; P)$$

Let Here $(x, y, z) = \left(\frac{S}{S^*}, \frac{E_n}{E_n^*}, \frac{I_k}{I_k^*}\right)$

$$\begin{aligned} f(x, y, z; P) &= \left(1 - xz\right) \left(1 - \frac{1}{x}\right) + PA \left(1 - \frac{1}{y}\right) (xz - y) + BP \left(1 - \frac{1}{z}\right) \left(y - z\right) \frac{\varepsilon_{nj}}{\left(\mu + \sum_{j=1}^3 \varepsilon_{nj}\right)} \\ &= \left(1 - \frac{1}{x} - xz + z\right) + PA \left(xz - y - \frac{xz}{y} + 1\right) + BP \frac{\varepsilon_{nj}}{\left(\mu + \sum_{j=1}^3 \varepsilon_{nj}\right)} \left(y - z - \frac{y}{z} + 1\right) \end{aligned}$$

On equating co-efficient of z, y and xz

$$B = \frac{\left(\mu + \sum_{j=1}^3 \varepsilon_{nj}\right)}{P \varepsilon_{nj}} \text{ and } A = \frac{1}{P}$$

On putting these values

$$f(x, y, z; P) = \left(3 - \frac{1}{x} - \frac{xz}{y} - \frac{y}{z}\right) \leq 0;$$

Since A.M \geq G.M and also equality holds to zero when x=1 and y=z. Thus $\frac{dL}{dt} \leq 0$ iff

$S = S^*$. Hence (S^*, E_n^*, I_k^*) is globally asymptotically stable.

From LaSalle's Extension the solutions which intersect the interior of $R_3 > 0$ limit to an invariant set contained in $\{(S, E_n, I_k) \in R_3 > 0 : S = S^*\}$.

The only invariant set in $\{(S, E_n, I_k) \in R_3 > 0 : S = S^*\}$ is the set consisting of the endemic equilibrium (S^*, E_n^*, I_k^*) . Thus, all solutions of the system (1h-1j) which intersect the interior of $R_3 > 0$ limit to (S^*, E_n^*, I_k^*) . In fact, (S^*, E_n^*, I_k^*) is globally asymptotically stable for all non-negative initial conditions for which $E_n + I_k > 0$.

6. NUMERICAL SIMULATION

Example 1: In this example, consider the case, $R_0 < 1$. It is clear from Figure 2 that all the infected and exposed classes become nearly equal to zero with time. This means that no further spreading of unwanted messages in the network. Again, the dynamic of different infectious and exposed classes with time has been given in Figure 3 for more clear observation of the spreading of rumors in the

online social network. It is found that all infectious and exposed classes become nearly equal to zero after some time. This means system acquired stability under a given condition. Thus $R_0 < 1$ is an ideal condition for rumor free environment within the network. Hence the numerical simulation also satisfies the situation considered in Theorem 1 and Theorem 2. The set of the parameter used in Figures 2 and Figures 3 are given in the Table 2.

Example 2: In this example consider, $R_0 = 1.62 > 1$ which is opposite situation of above example. It is observed in Figure 4 that the system persist unwanted message for a long time. After some time all the exposed and infectious classes become stable. Thus the system is asymptotically stable at endemic equilibrium when $R_0 > 1$. This satisfies our consideration in the Theorem 3 and Theorem 4 numerically. Again, the dynamics of different exposed and infectious classes with time

Table 2. List of parameter used for Figure 2 and Figure 3

| Parameters | λ | P_1 | P_2 | P_3 | 2_1 | 2_2 | 2_3 | μ_{11} | μ_{12} | μ_{13} | μ_{21} | μ_{22} |
|------------|------------|------------|------------|------------|--------|--------|--------|------------|------------|------------|------------|------------|
| Values | 0.8 | 0.3 | 0.3 | 0.4 | 0.3 | 0.3 | 0.3 | 0.6 | 0.6 | 0.6 | 0.6 | 0.6 |
| Parameters | μ_{23} | μ_{31} | μ_{32} | μ_{33} | 3_1 | 3_2 | 3_3 | \pm_1 | \pm_2 | \pm_3 | μ | |
| Values | 0.6 | 0.6 | 0.6 | 0.6 | 0.7 | 0.7 | 0.7 | 0.5 | 0.5 | 0.5 | 0.3 | |

Figure 2. Differential SEIS Model for $R_0 < 1$

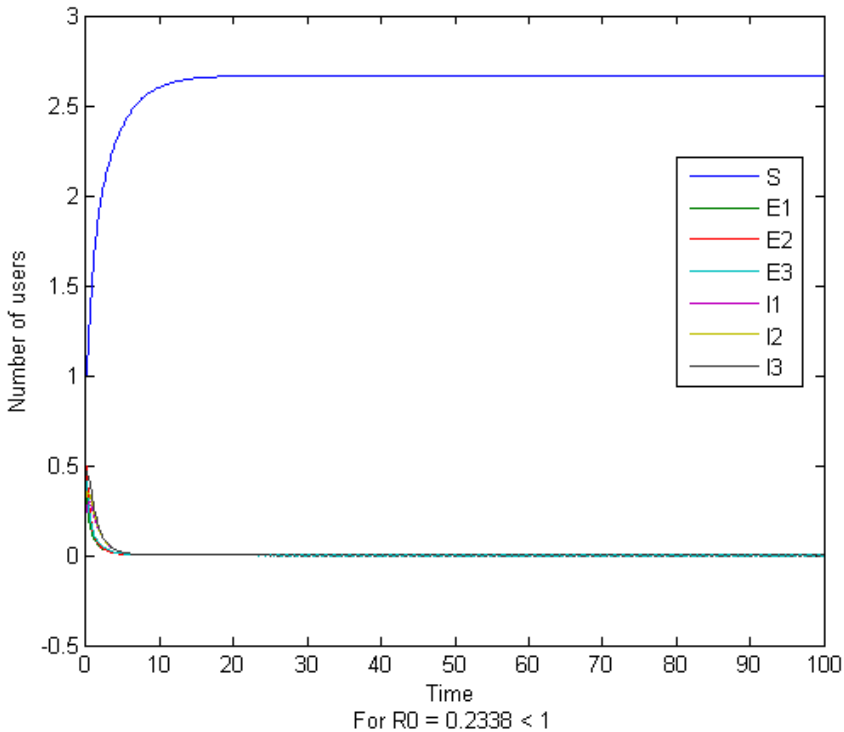
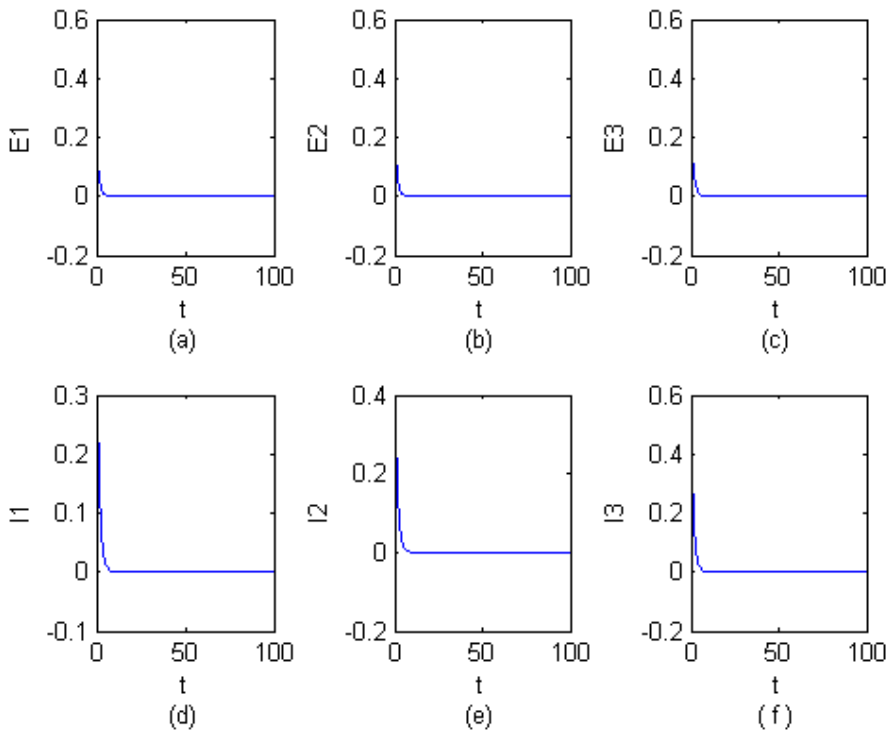


Figure 3. Variation of different Exposed and Infectious classes with time for $R_0 < 1$.



has been studied in Figure 5. The numerical result in the Figure 5 shows that the unwanted message spread for a long time within the network and finally becomes constant. Parameters used in Figure 4 and Figure 5 are given in Table 3.

7. DATA SET

Here the real-time data from Twitter has been used for our work. The data set is obtained by using the Twitter API. Here Martin Hawskey spreadsheet tool TAGS v6.1 used for collection of tweets (Hawksey, 2013). Here the data of the **# Covid 19 vaccination** for the period 13/02/21 to 20/02/21 has been collected. For **# Covid 19 vaccination** approx 3000 tweets are collected for the said time

Table 3. Parameters used in Figure 4 and Figure 5

| Parameters | λ | P_1 | P_2 | P_3 | z_1 | z_2 | z_3 | μ_{11} | μ_{12} | μ_{13} | μ_{21} | μ_{22} |
|------------|------------|------------|------------|------------|-------|-------|-------|------------|------------|------------|------------|------------|
| Values | 0.9 | 0.3 | 0.3 | 0.4 | 0.9 | 0.9 | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 |
| Parameters | μ_{23} | μ_{31} | μ_{32} | μ_{33} | s_1 | s_2 | s_3 | \pm_1 | \pm_2 | \pm_3 | μ | |
| Values | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 | 0.1 | 0.1 | 0.1 | 0.1 | |

Figure 4. Differential SEIS Model for $R_0 > 1$

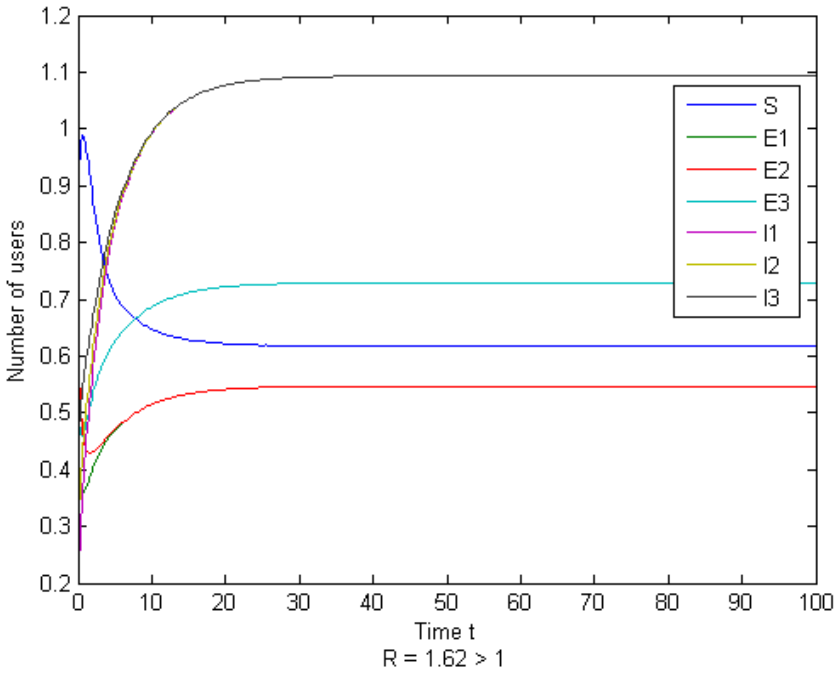
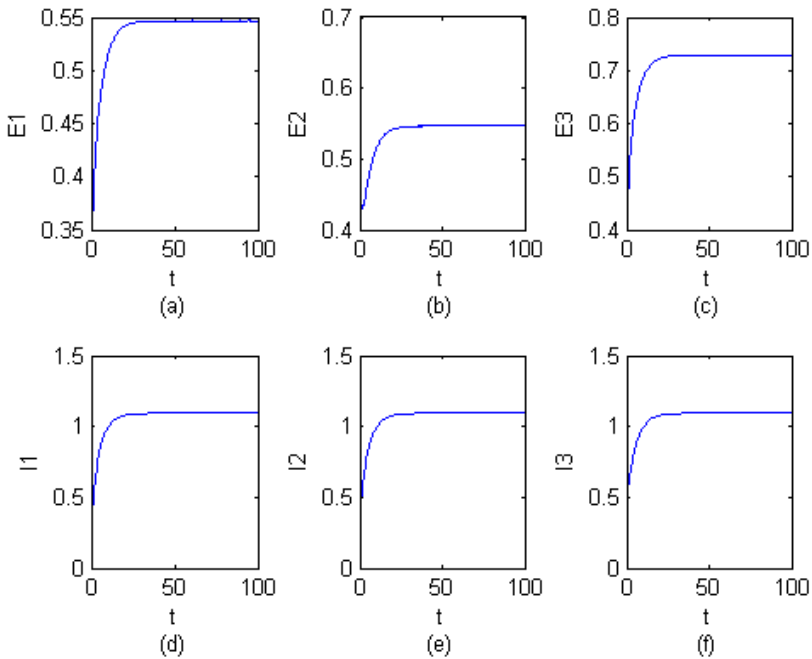


Figure 5. Variation of different infectious classes with time for $R_0 > 1$.

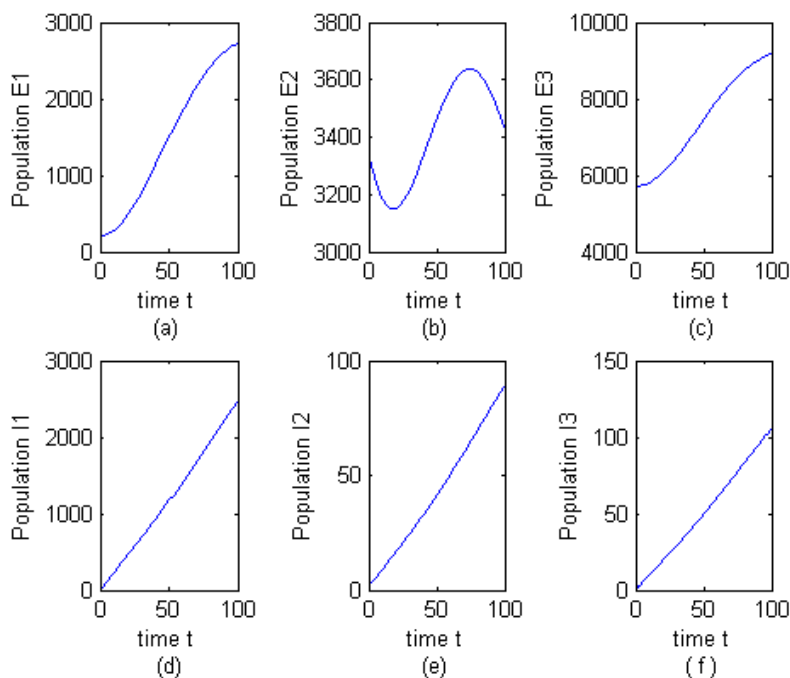


period. It seems that Twitter has about 330 million users in one month out of which 32% of users use English language (Research on 100 Million Tweets, n.d.; Wikipedia, n.d.). 15% of the users are blocked due to their malicious behavior on the network (An in depth analysis of abuse on Twitter, n.d.). The total number of English language users considered here under the susceptible class. The tweets with the most number of re-tweet extracted from these tweets. Among these tweets, that one which contains disinformation related to # **Covid-19 vaccinations** has been extracted. The number of re-tweets on this disinformation is considered as misinformation. Other than these tweets, all other tweets are considered as information. The tweets and re-tweets carry information, misinformation, and disinformation are considered under corresponding infection classes. Again the number of followers related to accounts that contains information, misinformation and disinformation has been counted and considered it under corresponding exposed classes.

8. EXPERIMENTAL RESULT

In this section, the above real data set has been used. The simulation of the system of differential equations using the above data set has been done with the help of Matlab. The R-K-Method of 4th order is used for the experimental purpose. The different exposed and infectious classes against time have been plotted in Figure 6 (a)-6 (f). It is found that exposed misinformation and disinformation increase gradually but the rate of disinformation is more than misinformation (Figure 6(a) & 6(c)). The rate is higher because malicious users spread the disinformation intentionally in the network. Again, exposed information is not showing constant behavior with respect to time, because it is not doing in a malicious manner in the network (Figure 6(b)). Further, the rate of different infectious classes with respect to time is not the same (Figure 6(d)-6(f)). The rate of misinformation is found

Figure 6. Variation in different exposed and infectious classes against time for # Covid 19 Vaccination



very high with respect to the other two classes, which is natural in the social networks. Thus our experimental results justified the real phenomenon of the social networks.

9. CONTROL STRATEGY

In this section, the numerical simulation has been used for giving an idea about how to control rumor propagation in online social networks. The conditions that support the spread of misinformation and disinformation within the network have been considered here. The data set which supports the propagation of rumors is given below in the table:

It is obvious from the Figure 7 and Figure 8 that the rate of misinformation and disinformation in the network is very high. It is because the data set given in table 4 supports the condition. Firstly the probability of spreading of misinformation and disinformation is higher than spreading of true information. Again, the contact rate is taken very high for users of infected misinformation and disinformation classes. The rates of users of various exposed classes have been taken in such manner that most of users go to infected misinformation and disinformation classes. The recovery rate of infected misinformation and disinformation classes is less than the information class. Also the rate of blocking of malicious users is very low. Here the rate of blocking of the users due to their malicious behavior within the network is taken zero for infectious-information class. All the conditions taken above support the spreading of misinformation and disinformation. These conditions are practically true and can be seen in the case of various online social networks.

Now consider the various parameters in Table 4 in such a manner that the rate of exposed and infectious information is higher than others (Figure 9 and Figure 10). This means that the only true information spreads throughout the network. The parameters used in Table 5 give an idea to minimize the false news and maximize the true information in the social network. The social network service provider should develop a mechanism regarding authentication of any news so that the probability of spreading of misinformation and disinformation can be reduced. Again contact rate with respect to the malicious users should be less in comparison to authenticate users in the network. Also, there should

Table 4. Parameters used for Figure 7 and Figure 8

| Parameters | λ | P_1 | P_2 | P_3 | 2_1 | 2_2 | 2_3 | μ_{11} | μ_{12} | μ_{13} | μ_{21} | μ_{22} |
|------------|------------|------------|------------|------------|--------|--------|--------|------------|------------|------------|------------|------------|
| Values | 0.9 | 0.5 | 0.1 | 0.4 | 0.9 | 0.3 | 0.7 | 0.6 | 0.2 | 0.5 | 0.6 | 0.2 |
| Parameters | μ_{23} | μ_{31} | μ_{32} | μ_{33} | 3_1 | 3_2 | 3_3 | \pm_1 | \pm_2 | \pm_3 | μ | R_0 |
| Values | 0.5 | 0.6 | 0.2 | 0.5 | 0.2 | 0.6 | 0.2 | 0.1 | 0.0 | 0.1 | 0.1 | 2.215 |

Table 5. Parameters used for Figure 9 and Figure 10

| Parameters | λ | P_1 | P_2 | P_3 | 2_1 | 2_2 | 2_3 | μ_{11} | μ_{12} | μ_{13} | μ_{21} | μ_{22} |
|------------|------------|------------|------------|------------|--------|--------|--------|------------|------------|------------|------------|------------|
| Values | 0.9 | 0.3 | 0.5 | 0.2 | 0.3 | 0.8 | 0.2 | 0.1 | 0.6 | 0.2 | 0.1 | 0.6 |
| Parameters | μ_{23} | μ_{31} | μ_{32} | μ_{33} | 3_1 | 3_2 | 3_3 | \pm_1 | \pm_2 | \pm_3 | M | R_0 |
| Values | 0.2 | 0.1 | 0.6 | 0.2 | 0.6 | 0.4 | 0.8 | 0.5 | 0.0 | 0.5 | 0.001 | 1.667 |

be such a condition in the network so that the rate of propagation of data from the exposed information, disinformation, and misinformation classes to infectious misinformation and disinformation should be less than the propagation of data from the exposed information, disinformation, and misinformation classes to infectious information class. Finally, recovery rate and removing rate of malicious users must be higher in the network. All these conditions together make the environment of any social network site rumor-free and healthy for the users.

10. CONCLUSION

Here a differential epidemic model for online social networks has been proposed. There are so many existing models in the field of rumors propagation but they do not differentiate among information, misinformation and disinformation. Here the differentiation among information, misinformation and disinformation has been considered and their propagation nature is observed separately. It also gives an idea to reduce misinformation and disinformation. This makes the paper unique from the existing works. Again, the expression for Basic reproduction number R_0 has been discussed. It is found that the system is locally and globally stable at infection-free equilibrium when $R_0 < 1$. The System is locally and globally stable at endemic equilibrium when $R_0 > 1$. The Numerical simulation validates our results. The experimental results show the nature of the spreading of data related to COVID-19 vaccination on Twitter. The experimental results matched with the actual conditions related to rumors about COVID-19 vaccination. The proposed control strategy can play an important role in order to minimize the propagation of rumors on the online social network.

Figure 7. Variation of Exposed Classes with respect to time for condition supporting the spreading of misinformation and disinformation in the network

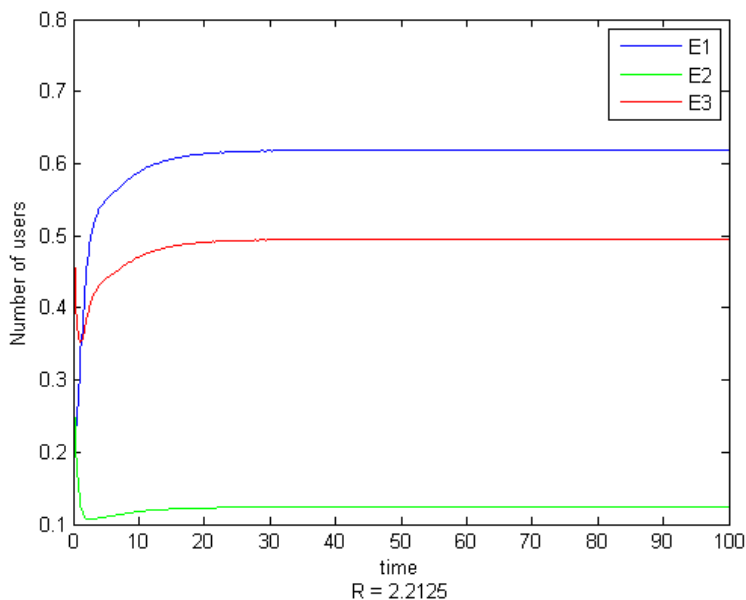


Figure 8. Variation of Infectious Classes with respect to time for condition supporting the spreading of misinformation and disinformation in the network

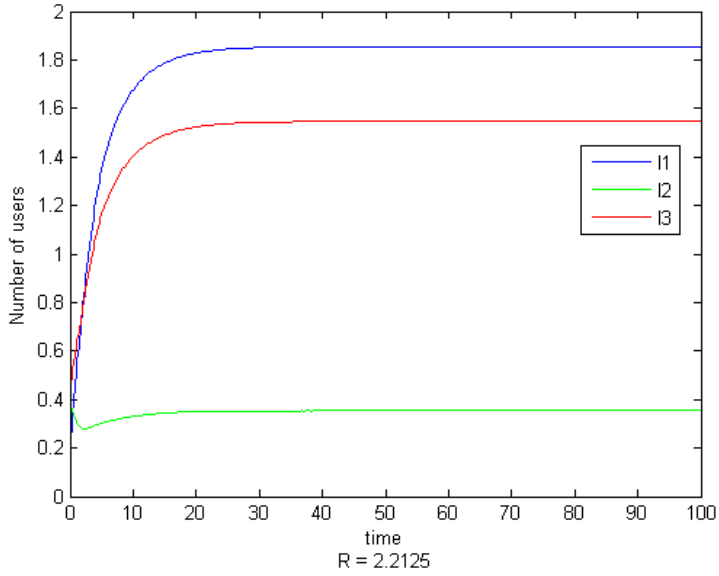


Figure 9. Variation of Exposed Classes with respect to time for condition control the spreading of misinformation and disinformation in the network

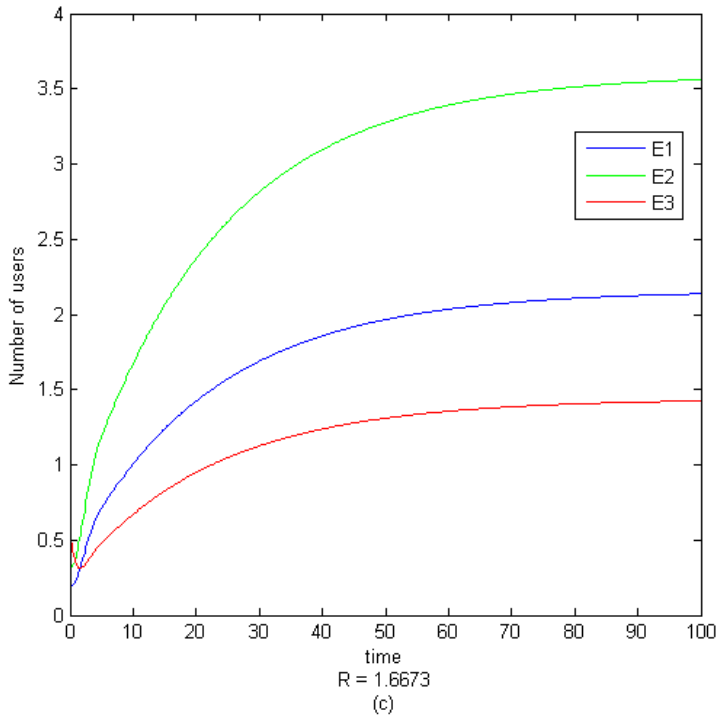
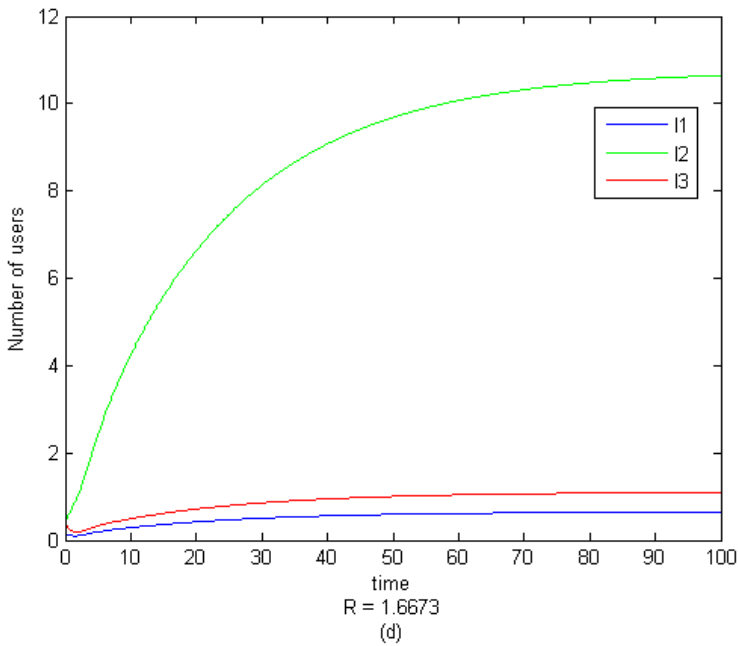


Figure 10. Variation of Exposed Classes with respect to time for condition control the spreading of misinformation and disinformation in the network



11. FUTURE SCOPE OF WORK AND RESEARCH CHALLENGES

More relevant data related to the various aspects of the work need to be collected and the required information has to be extracted with the help of MATLAB or other appropriate techniques. The challenges in this type of work is to collect the data for online social network and also find the tools for defense architecture which will both cost efficient as well as more effective from the existing techniques.

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