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PSK Method for Solving Mixed and Type-4 Intuitionistic Fuzzy Solid Transportation Problems

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ABSTRACT

In this article, the author categorises the solid transportation problem (STP) under uncertain environments. He formulates the mixed and fully intuitionistic fuzzy solid transportation problems (FIFSTPs) and utilizes the triangular intuitionistic fuzzy number (TIFN) to deal with uncertainty and hesitation. The PSK (P. Senthil Kumar) method for finding an intuitionistic fuzzy optimal solution for fully intuitionistic fuzzy transportation problem (FIFTP) is extended to solve the mixed and type-4 IFSTP and the optimal objective value of mixed and type-4 IFSTP is obtained in terms of triangular intuitionistic fuzzy number (TIFN). The main advantage of this method is that the optimal solution of mixed and type-4 IFSTP is obtained without using the basic feasible solution and the method of testing optimality. Moreover, the proposed method is computationally very simple and easy to understand. Finally, the procedure for the proposed method is illustrated with the help of numerical examples which is followed by graphical representation of the finding.

KEYWORDS

Intuitionistic Fuzzy Number, Intuitionistic Fuzzy Set, Mixed IFSTP, Optimal Solution, PSK Method, PSK Theorem, TFN, TIFN, TIFNs, Triangular, Type-3 IFSTP, Type-4 IFSTP

INTRODUCTION

The transportation problem is a special class of linear programming problem, widely used in the areas of inventory control, communication network, aggregate planning, employment scheduling, personal management and so on. In several real-life situations, there is a need for shipping the product from different origins (Factories) to different destinations (warehouses). The transportation problem deals with shipping commodities from different origins to various destinations. The objective of the transportation problem is to determine the optimum amount of a commodity to be transported from various supply points (origins) to different demand points (destinations) so that the total transportation cost is minimum or total transportation profit is maximum.

In the history of mathematics, Hitchcock (1941) originally developed a basic transportation problem. The transportation algorithm for solving transportation problems with equality constraints introduced by Dantzig (1963) is the simplex method specialized to the format of a table called

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transportation table. It involves two steps. First, we compute an initial basic feasible solution for the transportation problem and then, we test optimality and look at improving the basic feasible solution to the transportation problem. Swarup et al. (1997) presented tracts in operations research which deals the transportation problem when all the parameters are crisp number.

The solid transportation problem is a generalization of the classical transportation problem in which three-dimensional properties are taken into account in the objective and constraint set instead of source (origin) and destination. Shell (1955) stated an extension of well-known transportation problem is called a solid transportation problem in which bounds are given on three items, namely, supply, demand and conveyance. In many industrial problems, a homogeneous product is transported from an origin to a destination by means of different modes of transport called conveyances, such as trucks, cargo flights, goods trains, ships and so on. Haley (1962) presented the solution procedure for solving solid transportation problem, which is an extension of the modified distribution method. Patel and Tripathy (1989) proposed a computationally superior method for a solid transportation problem with mixed constraints. Basu et al. (1994) developed an algorithm for finding the optimum solution of a solid fixed charge linear transportation problem.

For finding an optimal solution, the solid transportation problem requires $m+n+l-2$ non-negative values of the decision variables to start with a basic feasible solution. Jimenez and Verdegay (1996) investigated interval multiobjective solid transportation problem via genetic algorithms. Li et al. (1997a) designed a neural network approach for a multicriteria solid transportation problem. Roy and Mahapatra (2014) gave solving solid transportation problems with multi-choice cost and stochastic supply and demand. Efficient algorithms have been developed for solving transportation problems when the coefficient of the objective function, demand, supply and conveyance values are known precisely.

Many of the distribution problems are imprecise in nature in today's world such as in corporate or in industry due to variations in the parameters. To deal quantitatively with imprecise information in making decision, Zadeh (1965) introduced the fuzzy set theory and has applied it successfully in various fields. The use of fuzzy set theory becomes very rapid in the field of optimization after the pioneering work done by Bellman and Zadeh (1970). The fuzzy set deals with the degree of membership (belongingness) of an element in the set but it does not consider the non-membership (non-belongingness) of an element in the set. In a fuzzy set the membership value (level of acceptance or level of satisfaction) lies between 0 and 1 where as in crisp set the element belongs to the set represent 1 and the element not in the set represent 0.

The unit fuzzy costs, that is, the fuzzy cost of transporting one unit from a particular supply point to a particular demand point, the fuzzy amounts available at the supply points and the fuzzy amounts required at the demand points are the parameters of the fuzzy transportation problem. Due to the lack of certainty in the parameters of a crisp transportation problem, several authors have solved transportation problems under fuzzy environment. For example, Dinagar and Palanivel (2009) investigated the transportation problem in fuzzy environment using trapezoidal fuzzy numbers. Mohideen and Kumar (2010) gave a better fuzzy optimal solution to the problems proposed by Pandian and Natarajan (2010). The transportation problem is a special kind of linear programming problem. Due to this, Nasseri et al. (2017) presented a generalized model for fuzzy linear programs with trapezoidal fuzzy numbers. Kumar (2016a, 2016b, 2017a, 2017b) formulated different types of fuzzy and intuitionistic fuzzy transportation problems and proposed a new and efficient solution method called PSK method.

Bit et al. (1993) presented a fuzzy programming approach to multiobjective solid transportation problem. Gen et al. (1995) gave a genetic algorithm for solving a bicriteria solid transportation problem with fuzzy numbers. Li et al. (1997b) discussed the genetic algorithm for solving fuzzy multiobjective solid transportation problem with fuzzy numbers. Jimenez and Verdegay (1998) proposed a solution procedure for uncertain solid transportation problem. Jimenez and Verdegay (1999) developed a parametric approach for solving fuzzy solid transportation problems by an evolutionary algorithm.

Liu (2006) presented a method to find the membership function of the fuzzy total transportation cost when the unit shipping costs, the supply and demand quantities, and the conveyance capacities are convex fuzzy numbers. Ojha et al. (2009) presented entropy based solid transportation problem for general fuzzy costs and time with fuzzy equality. Chakraborty et al. (2014) studied multi-objective multi-item solid transportation problem with fuzzy inequality constraints. Sinha et al. (2016) presented profit maximization solid transportation problem with trapezoidal interval type-2 fuzzy numbers. Thus, several researchers have solved solid transportation problems under fuzzy environment.

In conventional transportation problem supply, demand and costs are fixed crisp numbers. Therefore, in this situation the DM can predict transportation cost exactly. On the contrary in real world transportation problems, the availabilities and demands are not known exactly. These are uncertain quantities with hesitation due to various factors like lack of good communications, error in data, understanding of markets, unawareness of customers and many more. Also, the costs are in uncertain quantities with hesitation due to various factors like variation in rates of fuels, traffic jams, weather etc. In such situations, the DM cannot predict transportation cost exactly. He/She may hesitate. So, to counter these uncertainties with hesitation Atanassov (1986) proposed the intuitionistic fuzzy set (IFS) which is more reliable than the fuzzy set proposed by Zadeh (1965). The major advantage of intuitionistic fuzzy set over fuzzy set is that IFS separates the degree of membership (belongingness) and the degree of non-membership (non-belongingness) of an element in the set. With the help of IFS theory decision maker can decide about the degree of acceptance, degree of non-acceptance and degree of hesitation for some quantity. In case of transportation problem, the DM can decide about the level of acceptance and non-acceptance for the transportation cost or profit. Due to this, the application of IFS theory becomes very popular in transportation, decision making, planning, manufacturing, scheduling, etc.

Therefore, due to the applications of intuitionistic fuzzy set theory, several authors have been solved optimization problems under intuitionistic fuzzy environment. For example, Atanassov (1995) presented the ideas for intuitionistic fuzzy equations, inequalities and optimization. He formulated the optimization problems using the apparatus of the IFSs and he studied the importance of considering the concept of IFSs in optimization problems. Further, he discussed that how to use the apparatus of the IFSs in optimization problems. Ramík and Vlach (2016) studied intuitionistic fuzzy linear programming and duality: a level sets approach. Prabakaran and Ganesan (2017) presented duality theory for intuitionistic fuzzy linear programming problems. Solving intuitionistic fuzzy linear programming problem based on ranking function was proposed by Sudha and Kavithanjali (2017). Virivinti and Mitra (2018) presented handling optimization under uncertainty using intuitionistic fuzzy-logic-based expected value model. Nachammai et al. (2018) presented a comparative study of the methods of solving intuitionistic fuzzy linear programming problem. Nasseri et al. (2018) proposed an approach for solving linear programming problem with intuitionistic fuzzy objective coefficient. Thus, many authors have solved LPP under intuitionistic fuzzy environment.

Intuitionistic fuzzy solid transportation problem is a generalization of the fuzzy solid transportation problem in which input values are expressed as intuitionistic fuzzy numbers. Intuitionistic fuzzy solid transportation problem arises when the decision-maker has some vague information about the problem, that is, the data having uncertainty and hesitation in the parameters of the problem.

As there is a hesitation in the parameters of fuzzy transportation problem, several authors have been solved transportation problem under intuitionistic fuzzy environment. For example, Hussain and Kumar (2012a, 2012b, 2012c,) investigated a method for solving transportation problem in which all the parameters except transportation cost are represented by TIFN. Hussain and Kumar (2013) proposed an optimal more-for-less solution of mixed constraints intuitionistic fuzzy transportation problems. Kumar and Hussain (2014a) presented a systematic approach for solving mixed intuitionistic fuzzy transportation problems. Singh and Yadav (2014) developed efficient approach for solving type-1 intuitionistic fuzzy transportation problem where the supply, demand are TIFNs and the cost is fixed crisp number. Kumar and Hussain (2015) proposed a method for solving unbalanced intuitionistic fuzzy transportation problems. Singh and Yadav (2015) developed fuzzy programming approach for solving intuitionistic fuzzy linear fractional programming problem. Computationally simple and new method called PSK method for finding an optimal solution to fully intuitionistic fuzzy real-life transportation problems was presented by Kumar and Hussain (2016a). Recently, Kumar (2018a, 2018b, 2018c) formulated balanced and unbalanced IFTPs and solved the same by using different solution algorithms. Therefore, several authors have solved intuitionistic fuzzy transportation problems. There are several papers in the literature in which triangular intuitionistic fuzzy numbers are used for solving real life problems but to the best of our knowledge, till now no one has used triangular intuitionistic fuzzy numbers for solving the solid transportation problems.

The allocation problem is one of the most important problems of management science. In general, both the transportation problems and the assignment problems are called allocation problems or optimization problems. The transportation problem deals with assigning sources to destinations and the assignment problem deals with assigning jobs to machines. An assignment problem is a particular case of transportation problem where the sources are assignees and the destinations are tasks. Furthermore, every source has a supply of 1 (since each assignee is to be assigned to exactly one task) and every destination has a demand of 1 (since each task is to be performed by exactly one assignee). Also, the objective is to minimize the total cost or to maximize the total profit of allocation. Hence, every intuitionistic fuzzy assignment problem can be represented by intuitionistic fuzzy transportation problem if their supply of sources and demand of destinations should be exactly one. In general, the objective of the allocation problem is to assign the available resources in an economic way. When the resources to be allocated are scarce, a well-planned action is necessary for a decision-maker to attain the optimal utility. If the supplying sources and the receiving agents are limited, the best pattern of the allocation to get the maximum return or the best plan with the least cost, whichever may be applicable to the problem, is to be found out. In literature, Kumar and Hussain (2014b, 2014c, 2014d) proposed different methods to solve the different kinds of intuitionistic fuzzy assignment problem. Kumar and Hussain (2016b, 2016c) presented the solution methods for solving fully intuitionistic fuzzy real-life assignment problem and unbalanced intuitionistic fuzzy assignment problem. Kumar (2018d) developed a simple and efficient algorithm for solving type-1 intuitionistic fuzzy solid transportation problems. Kumar (2018e) presented the PSK method for solving intuitionistic fuzzy solid transportation problems. Recently, various kinds of optimization problems under fuzzy and intuitionistic fuzzy environment were presented by Kumar (2018f, 2018g, 2018h, 2019a, 2020a, 2020b).

Ranking of alternatives in intuitionistic fuzzy environment plays a major role in decision making. Burillo et al. (1994) proposed definition of intuitionistic fuzzy number and studied its properties. A number of researchers like Grzegorzewski (2003), Nehi et al. (2005), Nayagam et al. (2008), Guha and Chakraborty (2010), Deng Feng Li et al. (2010), Nehi (2010), Das and Guha (2013), Shabani and Jamkhaneh (2014) studied IFNs and analyzed its properties. Corresponding to every intuitionistic fuzzy number, Varghese and Kuriakose (2012) have proposed its crisp equivalent using its non-membership and membership function. Mahapatra and Roy (2009), Shaw and Roy (2012), Mahapatra and Roy (2013), Velu et al. (2017), Kumar et al. (2017) have proposed ranking methods and some arithmetic operations on triangular/trapezoidal intuitionistic fuzzy numbers.

In this article, PSK method for finding the intuitionistic fuzzy optimal solution for fully intuitionistic fuzzy transportation problem is extended to solve the mixed and type-4 IFSTPs in single stage. The optimal object value of mixed and type-4 IFSTP is obtained in terms of TIFN. The existing ordering procedure of Varghese and Kuriakose is used to transform the mixed and type-4 IFSTP into a crisp one so that the conventional method may be applied to solve the STP. The occupied cells of crisp STP that we obtained are as same as the occupied cells of mixed and type-4 IFSTP, but the value of occupied cells of mixed and type-4 IFSTP is the maximum possible value of crisp supply (or fuzzy/intuitionistic fuzzy supply), crisp demand (or fuzzy/ intuitionistic fuzzy demand) and crisp conveyance capacities (or fuzzy/intuitionistic fuzzy conveyance capacities). On the basis of this idea the solution procedure is differs from STP to mixed and type-4 IFSTP in allocation step only. Therefore, the PSK method for solving FIFTP is extended to solve the mixed and type-4 IFSTP. Moreover, the author proved a theorem, which states that every solution obtained by PSK method to fully intuitionistic fuzzy solid transportation problem with equality constraints is a fully intuitionistic fuzzy optimal.

The article is organized in the following manner. Some preliminary definitions and the Kumar and Hussain's (2016a) multiplication operation for TIFN will be explained in Section 2. Section 3 presents the ranking procedure and ordering principles of TIFN. Section 4 describes the definition of fully intuitionistic fuzzy solid transportation problem (FIFSTP) and its mathematical formulation. Section 5 consists of the PSK Method with new theorems and remarks. Section 6 provides the numerical example, results and discussion. The last section draws some conclusions.

PRELIMINARIES

In this section, some basic definitions and Kumar and Hussain's (2016a) multiplication operation is given.

Definition: Let X be a finite universal set. An intuitionistic fuzzy set A in X is an object having the form $A=\left\{\left\langle x,\mu_A\left(x\right),\vartheta_A\left(x\right)\right\rangle : x\in X\right\}$, where the functions $\mu_A\left(x\right),\vartheta_A\left(x\right): X\to\left[0,1\right]$ define respectively, the degree of membership and degree of non – membership of the element $x\in X$ to the set A, which is a subset of X, and for every element $x\in X$, $0\le\mu_A\left(x\right)+\vartheta_A\left(x\right)\le 1$. Furthermore, we have $\pi_A\left(x\right)=1-\mu_A\left(x\right)-\vartheta_A\left(x\right)$ called the intuitionistic fuzzy set index or hesitation margin of x in A. $\pi_A\left(x\right)$ is the degree of indeterminacy of $x\in X$ to the IFS A and $\pi_A\left(x\right)\in\left[0,1\right]$ i.e., $\pi_A\left(x\right): X\to\left[0,1\right]$ and $0\le\pi_A\left(x\right)\le 1$ for every $x\in X$. $\pi_A\left(x\right)$ expresses the lack of knowledge of whether x belongs to IFS A or not.

For example, let A be an intuitionistic fuzzy set with $\mu_A\left(x\right)=0.5$ and $\vartheta_A\left(x\right)=0.4\Rightarrow\pi_A\left(x\right)=1-\left(0.5+0.4\right)=0.1$. It can be interpreted as "the degree that the object x belongs to IFS A is 0.5, the degree that the object x does not belongs to IFS A is 0.4 and the degree of hesitancy is 0.1".

Definition (*Mahapatra, B. S., & Mahapatra, G. S. (2010)*): An Intuitionistic Fuzzy Number (IFN) $\tilde{A}^I$ is:

1. An intuitionistic fuzzy subset of the real line R,
2. Normal, that is, there is some $x_0\in R$ such that $\mu_{\tilde{A}^I}\left(x_0\right)=1,\ \vartheta_{\tilde{A}^I}\left(x_0\right)=0$,
3. Convex for the membership function $\mu_{\tilde{A}^I}\left(x\right)$, that is, $\mu_{\tilde{A}^I}\left(\lambda x_1+\left(1-\lambda\right)x_2\right)\ge\min\left(\mu_{\tilde{A}^I}\left(x_1\right),\mu_{\tilde{A}^I}\left(x_2\right)\right)$, for every $x_1,x_2\in R,\ \lambda\in\left[0,1\right]$,
4. Concave for the non – membership function $\vartheta_{\tilde{A}^I}\left(x\right)$, i.e., $\vartheta_{\tilde{A}^I}\left(\lambda x_1+\left(1-\lambda\right)x_2\right)\le\max\left(\vartheta_{\tilde{A}^I}\left(x_1\right),\vartheta_{\tilde{A}^I}\left(x_2\right)\right)$, for every $x_1,x_2\in R$, $\lambda\in\left[0,1\right]$.

Definition: A fuzzy number A is defined to be a triangular fuzzy number (TFN) if its membership functions $\mu_A:\mathbb{R}\to[0, 1]$ is equal to:

$$\mu_A(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1} & if\, x\in[a_1,a_2]\\ \dfrac{a_3-x}{a_3-a_2} & if\, x\in[a_2,a_3]\\ 0 & \text{Otherwise}\end{cases}$$

Definition: A Triangular Intuitionistic Fuzzy Number ($\tilde{A}^I$ is an intuitionistic fuzzy set in R with the following membership function $\mu_A(x)$ and non-membership function $\vartheta_A(x)$:) :

$$\mu_A(x)=\begin{cases}0, & \text{for}\, x<a_1\\ \dfrac{x-a_1}{a_2-a_1}, & \text{for}\, a_1\le x\le a_2\\ 1, & \text{for}\, x=a_2\\ \dfrac{a_3-x}{a_3-a_2}, & \text{for}\, a_2\le x\le a_3\\ 0, & \text{for}\, x>a_3\end{cases} \quad \text{and} \quad \vartheta_A(x)=\begin{cases}1, & \text{for}\, x<a_1'\\ \dfrac{a_2-x}{a_2-a_1'}, & \text{for}\; a_1'\le x\le a_2\\ 0, & \text{for}\; x=a_2\\ \dfrac{x-a_2}{a_3'-a_2}, & \text{for}\, a_2\le x\le a_3'\\ 1, & \text{for}\; x>a_3'\end{cases}$$

where $a_1' \le a_1 \le a_2 \le a_3 \le a_3'$ and $\mu_A(x), \vartheta_A(x) \le 0.5$ for $\mu_A(x) = \vartheta_A(x)$, $\forall x \in R.$ This TIFN is denoted by $\tilde{A}^I = (a_1, a_2, a_3)(a_1', a_2, a_3')$.

Particular Cases

Let $\tilde{A}^I = (a_1, a_2, a_3)(a_1', a_2, a_3')$ be a TIFN. Then the following cases arise.

Case 1: If $a_1' = a_1$, $a_3' = a_3$ then $\tilde{A}^I$ represent Triangular Fuzzy Number (TFN). It is denoted by $\tilde{A} = (a_1, a_2, a_3)$.

Case 2: If $a_1' = a_1 = a_2 = a_3 = a_3' = m$ then $\tilde{A}^I$ represent a real number m.

Definition: Let $\tilde{A}^I = (a_1, a_2, a_3)(a_1', a_2, a_3')$ and $\tilde{B}^I = (b_1, b_2, b_3)(b_1', b_2, b_3')$ be any two TIFNs then the arithmetic operations as follows:

- **Addition:** $\tilde{A}^I \oplus \tilde{B}^I = (a_1+b_1, a_2+b_2, a_3+b_3)(a_1'+b_1', a_2+b_2, a_3'+b_3')$
- **Subtraction:** $\tilde{A}^I \ominus B^I = (a_1-b_3, a_2-b_2, a_3-b_1)(a_1'-b_3', a_2-b_2, a_3'-b_1')$
- **Multiplication:** Kumar and Hussain's (2016a) multiplication operation:

$$\tilde{A}^I \otimes \tilde{B}^I = \left(a_1\Re(\tilde{B}^I), a_2\Re(\tilde{B}^I), a_3\Re(\tilde{B}^I)\right)\left(a_1'\Re(\tilde{B}^I), a_2\Re(\tilde{B}^I), a_3'\Re(\tilde{B}^I)\right) \text{if } \Re(\tilde{A}^I), \Re(\tilde{B}^I) \ge 0$$

Remark: All the parameters of the conventional STP such as supply, demand, cost and conveyance capacity are in positive. Since in transportation problems, negative parameters have no physical meaning. Hence, in the proposed method all the parameters may be assumed as non-negative triangular intuitionistic fuzzy number. On the basis of this idea we need not further investigate the multiplication operation under the condition that $\Re\left(\tilde{A}^I\right), \Re\left(\tilde{B}^I\right) < 0$.

Scalar multiplication:

1. $\mathrm{k\tilde{A}^I} = \left(\mathrm{ka_1, ka_2, ka_3}\right)\left(\mathrm{ka_1', ka_2, ka_3'}\right)$, for $k \geq 0$
2. $k\tilde{A}^I = \left(ka_3, ka_2, ka_1\right)\left(ka_3', ka_2, ka_1'\right)$, for $k < 0$

COMPARISON OF TIFN

Definition: Let $\tilde{A}^I = \left(a_1, a_2, a_3\right)\left(a_1', a_2, a_3'\right)$ and $\tilde{B}^I = \left(b_1, b_2, b_3\right)\left(b_1', b_2, b_3'\right)$ be two TIFNs. Then the set of TIFNs is defined as follows:

1. $\Re\left(\tilde{A}^I\right) > \Re\left(\tilde{B}^I\right)$ if and only if $\tilde{A}^I \succ \tilde{B}^I$;
2. $\Re\left(\tilde{A}^I\right) < \Re\left(\tilde{B}^I\right)$ if and only if $\tilde{A}^I \prec \tilde{B}^I$;
3. $\Re\left(\tilde{A}^I\right) = \Re\left(\tilde{B}^I\right)$ if and only if $\tilde{A}^I \approx \tilde{B}^I$, where:

$$\Re\left(\tilde{A}^I\right) = \frac{1}{3}\left[\frac{\left(a_3' - a_1'\right)\left(a_2 - 2a_3' - 2a_1'\right) + \left(a_3 - a_1\right)\left(a_1 + a_2 + a_3\right) + 3\left(a_3'^2 - a_1'^2\right)}{a_3' - a_1' + a_3 - a_1}\right]$$

$$\Re\left(\tilde{B}^I\right) = \frac{1}{3}\left[\frac{(b_3' - b_1')\left(b_2 - 2b_3' - 2b_1'\right) + \left(b_3 - b_1\right)\left(b_1 + b_2 + b_3\right) + 3\left(b_3'^2 - b_1'^2\right)}{b_3' - b_1' + b_3 - b_1}\right]$$

Whenever the above formula doesn't provide finite value then we can make use of the following formula. The score function for the membership function $\mu_A\left(x\right)$ is denoted by $S\left(\mu_A\left(x\right)\right)$ and is defined by $S\left(\mu_A\left(x\right)\right) = \frac{a_1 + 2a_2 + a_3}{4}$.

The score function for the non-membership function $\vartheta_A\left(x\right)$ is denoted by $S\left(\vartheta_A\left(x\right)\right)$ and is defined by $S\left(\vartheta_A\left(x\right)\right) = \frac{a_1' + 2a_2 + a_3'}{4}$.

The accuracy function of $\tilde{A}^I$ is denoted by $f\left(\tilde{A}^I\right)$ and is defined by:

$$f\left(\tilde{A}^I\right) = \frac{S\left(\mu_A\left(x\right)\right) + S\left(\vartheta_A\left(x\right)\right)}{2} = \frac{\left(a_1 + 2a_2 + a_3\right) + (a_1' + 2a_2 + a_3')}{8}$$

From the accuracy function, we have:

1. $f\left(\tilde{A}^I\right) > f\left(\tilde{B}^I\right)$ if and only if $\tilde{A}^I \succ \tilde{B}^I$;
2. $f\left(\tilde{A}^I\right) < f\left(\tilde{B}^I\right)$ if and only if $\tilde{A}^I \prec \tilde{B}^I$;

3. $f\left(\tilde{A}^I\right) = f\left(\tilde{B}^I\right)$ if and only if $\tilde{A}^I \approx \tilde{B}^I$.

Definition: The ordering $\succeq$ and $\preccurlyeq$ between any two TIFNs $\tilde{A}^I$ and $\tilde{B}^I$ are defined as follows:

1. $\tilde{A}^I \succcurlyeq \tilde{B}^I$ iff $\tilde{A}^I \succ \tilde{B}^I$ or $\tilde{A}^I \approx \tilde{B}^I$;
2. $\tilde{A}^I \preccurlyeq \tilde{B}^I$ iff $\tilde{A}^I \prec \tilde{B}^I$ or $\tilde{A}^I \approx \tilde{B}^I$

Definition: Let $\left\{\tilde{\omega}_r^I, r = 1,2,\ldots,m\right\}$ be a set of TIFNs. If $\Re\left(\tilde{\omega}_p^I\right) \le \Re\left(\tilde{\omega}_r^I\right)$ for all *r*, then the TIFN $\tilde{\omega}_p^I$ is the minimum of $\left\{\tilde{\omega}_r^I, r = 1,2,\ldots,m\right\}$.

Definition: Let $\left\{\tilde{\omega}_r^I, r = 1,2,\ldots,m\right\}$ be a set of TIFNs. If $\Re\left(\tilde{\omega}_s^I\right) \ge \Re\left(\tilde{\omega}_r^I\right)$ for all *r*, then the TIFN $\tilde{\omega}_s^I$ is the maximum of $\left\{\tilde{\omega}_r^I, r = 1,2,\ldots,m\right\}$.

FULLY INTUITIONISTIC FUZZY SOLID TRANSPORTATION PROBLEM AND ITS MATHEMATICAL FORMULATION

The following basic terminologies used in this article are defined in this section.

Definition: If the solid transportation problem has at least one of the parameters (cost) or three of the parameters (supply, demand and conveyance capacity) or all of the parameters (supply, demand, conveyance capacity and cost) in intuitionistic fuzzy numbers then the problem is called IFSTP. Otherwise it is not an IFSTP.

Further, solid intuitionistic fuzzy transportation problem can be classified into four categories. They are:

1. Type-1 intuitionistic fuzzy solid transportation problem (type-1 IFSTP);
2. Type-2 intuitionistic fuzzy solid transportation problem (type-2 IFSTP);
3. Type-3 intuitionistic fuzzy solid transportation problem (type-3 IFSTP or Mixed Intuitionistic Fuzzy solid Transportation Problem (MIFSTP));
4. Type-4 intuitionistic fuzzy solid transportation problem (type-4 IFSTP or Fully Intuitionistic Fuzzy solid Transportation Problem (FIFSTP)).

Definition: A solid transportation problem having intuitionistic fuzzy availabilities, intuitionistic fuzzy demands and intuitionistic fuzzy conveyance capacity but crisp costs is termed as intuitionistic fuzzy solid transportation problem of type-1.

Definition: A solid transportation problem having crisp availabilities crisp demands and crisp conveyance capacity but intuitionistic fuzzy costs is termed as intuitionistic fuzzy solid transportation problem of type-2.

Definition: The solid transportation problem is said to be the type-3 intuitionistic fuzzy solid transportation problem or mixed intuitionistic fuzzy solid transportation problem if all the parameters of the solid transportation problem (such as supplies, demands, conveyance capacities and costs) must be in the mixture of crisp numbers, triangular fuzzy numbers and triangular intuitionistic fuzzy numbers.

Definition: The solid transportation problem is said to be the type-4 intuitionistic fuzzy solid transportation problem or fully intuitionistic fuzzy solid transportation problem if all the parameters of the solid transportation problem (such as supplies, demands, conveyance capacities and costs) must be in intuitionistic fuzzy numbers.

Definition: The intuitionistic fuzzy solid transportation problem is said to be balanced intuitionistic fuzzy solid transportation problem if total intuitionistic fuzzy supply and total intuitionistic fuzzy demand is equal to total intuitionistic fuzzy conveyance capacity.

That is:

$$\sum_{i=1}^{m}\tilde{a}_i^I = \sum_{j=1}^{n}\tilde{b}_j^I = \sum_{k=1}^{l}\tilde{e}_k^I$$

Definition: The intuitionistic fuzzy solid transportation problem is said to be an unbalanced intuitionistic fuzzy solid transportation problem if total intuitionistic fuzzy supply and total intuitionistic fuzzy demand is not equal to total intuitionistic fuzzy conveyance capacity.

That is:

$$\sum_{i=1}^{m}\tilde{a}_i^I \neq \sum_{j=1}^{n}\tilde{b}_j^I \neq \sum_{k=1}^{l}\tilde{e}_k^I$$

Definition: A set of intuitionistic fuzzy non-negative allocations $\tilde{x}_{ijk}^I > \tilde{0}^I$ satisfies the supply, demand and conveyance restriction (i.e., which satisfies the Equations (1), (2) and (3)) is known as intuitionistic fuzzy feasible solution.

Definition: Any feasible solution is an intuitionistic fuzzy basic feasible solution if the number of non-negative allocations is at most $(m+n+l-2)$ where *m* is the number of origins and *n* is the number of destinations and *l* is the number of conveyances in the $m \times n \times l$ solid transportation table.

Definition: If the intuitionistic fuzzy basic feasible solution contains less than $(m+n+l-2)$ non-negative allocations in $m \times n \times l$ solid transportation table, it is said to be degenerate.

Definition: Any intuitionist fuzzy feasible solution to a solid transportation problem containing *m* origins and *n* destinations and *l* conveyances is said to be intuitionist fuzzy non-degenerate, if it contains exactly $(m+n+l-2)$ occupied cells.

Definition: The intuitionistic fuzzy basic feasible solution is said to be intuitionistic fuzzy optimal solution if it minimizes the total intuitionistic fuzzy transportation cost, that is, minimize $\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\tilde{c}_{ijk}^I \otimes \tilde{x}_{ijk}^I$ subject to the constraints (or) it maximizes the total intuitionistic fuzzy transportation profit.

Definition (Mathematical Formulation of FIFSTP): Consider transportation with *m* origins, *n* destinations and *l* conveyances. Let $\tilde{c}_{ijk}^I = \left(c_{ijk}^1, c_{ijk}^2, c_{ijk}^3\right)\left(c_{ijk}^{1'}, c_{ijk}^2, c_{ijk}^{3'}\right)$ be the unit cost of transporting one unit of the product from i[th] origin to j[th] destination by means of the k[th] conveyance. Let $\tilde{a}_i^I = \left(a_i^1, a_i^2, a_i^3\right)\left(a_i^{1'}, a_i^2, a_i^{3'}\right)$ be the quantity of commodity available at origin *i*. Let $\tilde{b}_j^I = \left(b_j^1, b_j^2, b_j^3\right)\left(b_j^{1'}, b_j^2, b_j^{3'}\right)$ be the amount of the quantity of commodity needed at destination *j*. Let $\tilde{e}_k^I$ be the amount of the material transported by k[th] conveyance. Let $\tilde{x}_{ijk}^I = \left(x_{ijk}^1, x_{ijk}^2, x_{ijk}^3\right)\left(x_{ijk}^{1'}, x_{ijk}^2, x_{ijk}^{3'}\right)$ be the number of units of quantity transported from i[th] origin to j[th] destination by means of the k[th] conveyance. Our objective is to minimize the total intuitionistic fuzzy transportation cost satisfying intuitionistic fuzzy supply, intuitionistic fuzzy demand and intuitionistic fuzzy conveyance constraints.

A STP having uncertainty and hesitation in transportation costs, supply, demand and conveyance (i.e., capacity of different modes of transport) can be formulated as follows:

(FIFSTP) (P) Minimize $\tilde{Z}^I = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\tilde{c}_{ijk}^I \otimes \tilde{x}_{ijk}^I$

subject to:

$$\sum_{i=1}^{m}\sum_{k=1}^{l}\left(x_{ijk}^1, x_{ijk}^2, x_{ijk}^3\right)\left(x_{ijk}^{1'}, x_{ijk}^2, x_{ijk}^{3'}\right) \approx \left(a_i^1, a_i^2, a_i^3\right)\left(a_j^{1'}, a_j^2, a_j^{3'}\right), \text{for } i = 1,2,\ldots,m \quad (1)$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l}\left(x_{ijk}^1, x_{ijk}^2, x_{ijk}^3\right)\left(x_{ijk}^{1'}, x_{ijk}^2, x_{ijk}^{3'}\right) \approx \left(b_j^1, b_j^2, b_j^3\right)\left(b_j^{1'}, b_j^2, b_j^{3'}\right), \text{for } j = 1,2,\ldots,n \quad (2)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left(x_{ijk}^1, x_{ijk}^2, x_{ijk}^3\right)\left(x_{ijk}^{1'}, x_{ijk}^2, x_{ijk}^{3'}\right) \approx \left(e_k^1, e_k^2, e_k^3\right)\left(e_k^{1'}, e_k^2, e_k^{3'}\right), \text{for } k = 1,2,\ldots,l \quad (3)$$

$$\left(x_{ijk}^1, x_{ijk}^2, x_{ijk}^3\right)\left(x_{ijk}^{1'}, x_{ijk}^2, x_{ijk}^{3'}\right) \succcurlyeq \tilde{0}^I, \text{for } i = 1,2,\ldots,m; \; j = 1,2,\ldots,n \text{ and } k = 1,2,\ldots,l \quad (4)$$

where:

m = the number of supply points
n = the number of demand points
l = the number of conveyances

When the supplies, demands and costs are intuitionistic fuzzy numbers, then the minimum total cost becomes an intuitionistic fuzzy number. Symbolically it can be noted that $\tilde{Z}^I$ where:

$$\tilde{Z}^I = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\tilde{c}_{ijk}^I \otimes \tilde{x}_{ijk}^I$$

Hence it cannot be minimized directly. For solving the problem we convert the intuitionistic fuzzy supplies $\left(\tilde{a}_i^I\right)$, intuitionistic fuzzy demands $\left(\tilde{b}_j^I\right)$, intuitionistic fuzzy conveyance capacities $\left(\tilde{e}_k^I\right)$ and the intuitionistic fuzzy costs $\left(\tilde{c}_{ijk}^I\right)$ into crisp ones by an intuitionistic fuzzy number ranking method of Varghese and Kuriakose.

Consider transportation with *m* origins, *n* destinations and *l* conveyances. Let c_{ijk} be the unit cost of transporting one unit of the product from i^th^ origin to j^th^ destination by means of the k^th^ conveyance. Let a_i be the quantity of commodity available at origin *i*. Let b_j be the amount of the quantity of commodity needed at destination *j*. Let e_k be the amount of the material transported by k^th^ conveyance. Let x_{ijk} be the number of units of quantity transported from i^th^ origin to j^th^ destination by means of the k^th^ conveyance. Our aim is to determine transportation schedule to minimize the transportation cost satisfying supply, demand and conveyance constraints:

(P*) Minimize $\tilde{Z}^I = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\Re\left(\tilde{c}_{ijk}^I\right) \otimes \Re\left(\tilde{x}_{ijk}^I\right)$

subject to:

$$\sum_{j=1}^{n}\sum_{k=1}^{l}\Re\left(\tilde{x}_{ijk}^I\right) \approx \Re\left(\tilde{a}_j^I\right), \text{for } j = 1,2,\ldots,m \tag{5}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l}\Re\left(\tilde{x}_{ijk}^I\right) \approx \Re\left(\tilde{b}_j^I\right), \text{for } j = 1,2,\ldots,n \tag{6}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\Re\left(\tilde{x}_{ijk}^I\right) \approx \Re\left(\tilde{e}_k^I\right), \ \text{for } k = 1,2,\ldots,l \tag{7}$$

$$\begin{aligned}\Re\left(\tilde{x}_{ijk}^I\right) \succcurlyeq \tilde{0}^I, &\text{for } \ i = 1,2,\ldots,m \\ &\text{for } j = 1,2,\ldots,n \text{ and} \\ &k = 1,2,\ldots,l\end{aligned} \tag{8}$$

Since $\mathbf{\Re}\left(\tilde{c}_{ijk}^I\right), \ \Re\left(\tilde{a}_i^I\right), \Re\left(\tilde{b}_j^I\right), \Re\left(\tilde{e}_k^I\right),$ all are crisp values, this problem (P*) is obviously the crisp solid transportation problem of the form (P) which can be solved by the conventional method namely the Modified Distribution Method or Min Zero- Min Cost method. Once the optimal solution x^* of Model (P*) is found, the optimal intuitionistic fuzzy objective value $\tilde{Z}^{I^*}$ of the original problem can be calculated as:

$$\tilde{Z}^{I^*} = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\tilde{c}_{ijk}^I \otimes \tilde{x}_{ijk}^{I^*}$$

where, $\tilde{c}_{ijk}^I = \left(c_{ijk}^1, c_{ijk}^2, c_{ijk}^3\right)\left(c_{ijk}^{1'}, c_{ijk}^2, c_{ijk}^{3'}\right)$, $\tilde{x}_{ijk}^I = \left(x_{ijk}^1, x_{ijk}^2, x_{ijk}^3\right)\left(x_{ijk}^{1'}, x_{ijk}^2, x_{ijk}^{3'}\right)$. If *m*=3, *n*=3 and *l*=3 then the FIFSTP (see Table 1) and its equivalent crisp STP (refer to Table 2) can be stated in the tabular form shown.

Table 1. Tabular representation of 3 × 3 × 3 FIFSTP

										Capacity $\tilde{e}^I_k$
	E_1			E_1			E_1			$\tilde{e}^I_1$
		E_2			E_2			E_2		$\tilde{e}^I_2$
			E_3			E_3			E_3	$\tilde{e}^I_3$
	D_1			D_2			D_3			**Supply** $\tilde{a}^I_i$
O_1	$\tilde{c}^I_{111}$	$\tilde{c}^I_{112}$	$\tilde{c}^I_{113}$	$\tilde{c}^I_{121}$	$\tilde{c}^I_{122}$	$\tilde{c}^I_{123}$	$\tilde{c}^I_{131}$	$\tilde{c}^I_{132}$	$\tilde{c}^I_{133}$	$\tilde{a}^I_1$
O_2	$\tilde{c}^I_{211}$	$\tilde{c}^I_{212}$	$\tilde{c}^I_{213}$	$\tilde{c}^I_{221}$	$\tilde{c}^I_{222}$	$\tilde{c}^I_{223}$	$\tilde{c}^I_{231}$	$\tilde{c}^I_{232}$	$\tilde{c}^I_{233}$	$\tilde{a}^I_2$
O_3	$\tilde{c}^I_{311}$	$\tilde{c}^I_{312}$	$\tilde{c}^I_{313}$	$\tilde{c}^I_{321}$	$\tilde{c}^I_{322}$	$\tilde{c}^I_{323}$	$\tilde{c}^I_{331}$	$\tilde{c}^I_{332}$	$\tilde{c}^I_{333}$	$\tilde{a}^I_3$
Demand $\tilde{b}^I_j$	$\tilde{b}^I_1$			$\tilde{b}^I_2$			$\tilde{b}^I_3$			

Result 1: The balanced condition is the necessary and sufficient condition for the existence of a feasible solution to the SIFTP.

Result 2: If $l = 1$, the number of conveyances is only one, the above problem reduces to an IFTP.

Now a new method is proposed, namely, PSK method for finding an optimal solution to the mixed and fully intuitionistic fuzzy solid transportation problems.

PSK METHOD

The PSK method proceeds as follows:

Step 1: Formulate a TP with different origins, numerous destinations and various types of conveyances. This type of problem is called STP or three-dimensional transportation problems. Consider the STP having all the parameters such as supply, demand, unit transportation cost and conveyance capacities must be a mixture of crisp numbers, triangular fuzzy numbers and triangular intuitionistic fuzzy numbers (This situation is known as MIFSTP).

Step 2: Examine whether the total intuitionistic fuzzy supply, total intuitionistic fuzzy demand and total intuitionistic fuzzy conveyance capacities are all equal (or its all ranking index must be equal). If not, change it. This step gives the balanced mixed intuitionistic fuzzy solid transportation problem (BMIFSTP).

Step 3: After using step 2, convert BMIFSTP into balanced fully intuitionistic fuzzy solid transportation problem (BFIFSTP) using the following steps:

Table 2. Tabular representation of crisp 3 × 3 × 3 STP

										Capacity $\tilde{e}_k^I$
	E_1			E_1			E_1			$\Re(\tilde{e}_1^I)$
		E_2			E_2			E_2		$\Re(\tilde{e}_2^I)$
			E_3			E_3			E_3	$\Re(\tilde{e}_3^I)$
	D_1			D_2			D_3			**Supply** $\tilde{a}_i^I$
O_1	$\Re(\tilde{c}_{111}^I)$	$\Re(\tilde{c}_{112}^I)$	$\Re(\tilde{c}_{113}^I)$	$\Re(\tilde{c}_{121}^I)$	$\Re(\tilde{c}_{122}^I)$	$\Re(\tilde{c}_{123}^I)$	$\Re(\tilde{c}_{131}^I)$	$\Re(\tilde{c}_{132}^I)$	$\Re(\tilde{c}_{133}^I)$	$\Re(\tilde{a}_1^I)$
O_2	$\Re(\tilde{c}_{211}^I)$	$\Re(\tilde{c}_{212}^I)$	$\Re(\tilde{c}_{213}^I)$	$\Re(\tilde{c}_{221}^I)$	$\Re(\tilde{c}_{222}^I)$	$\Re(\tilde{c}_{223}^I)$	$\Re(\tilde{c}_{231}^I)$	$\Re(\tilde{c}_{232}^I)$	$\Re(\tilde{c}_{233}^I)$	$\Re(\tilde{a}_2^I)$
O_3	$\Re(\tilde{c}_{311}^I)$	$\Re(\tilde{c}_{312}^I)$	$\Re(\tilde{c}_{313}^I)$	$\Re(\tilde{c}_{321}^I)$	$\Re(\tilde{c}_{322}^I)$	$\Re(\tilde{c}_{323}^I)$	$\Re(\tilde{c}_{331}^I)$	$\Re(\tilde{c}_{332}^I)$	$\Re(\tilde{c}_{333}^I)$	$\Re(\tilde{a}_3^I)$
Demand $\tilde{b}_j^I$	$\Re(\tilde{b}_1^I)$			$\Re(\tilde{b}_2^I)$			$\Re(\tilde{b}_3^I)$			

1. If any one or more in the supplies/demands/conveyance capacities/costs of a transportation problem having a real number say (a_1) that can be expanded as a TIFN $a_1 = (a_1, a_1, a_1)(a_1, a_1, a_1)$;
2. If any one or more in the supplies/demands/conveyance capacities/costs of a transportation problem having a triangular fuzzy number say (a_1, a_2, a_3) that can be expanded as a TIFN $(a_1, a_2, a_3) = (a_1, a_2, a_3)(a_1, a_2, a_3)$;
3. If any one or more in the supplies/demands/conveyance capacities/costs of a transportation problem having a TIFN say $(a_1, a_2, a_3)(a_1^{'}, a_2, a_3^{'})$ that can be kept as it is. That is $(a_1, a_2, a_3)(a_1^{'}, a_2, a_3^{'}) = (a_1, a_2, a_3)(a_1^{'}, a_2, a_3^{'})$.

Step 4: After using step 3, transform the BFIFSTP into its equivalent crisp STP using the ranking procedure as mentioned in section 3.

Step 5: Now, the crisp STP having all the entries of supply, demand, unit transportation costs and conveyance capacities are in integers then kept as it is. Otherwise at least one or all of the supply, demand, unit transportation costs and conveyance capacities are not in integers then rewrite its nearest integer value.

Step 6: After using step 5 of the proposed method, now solve the crisp STP by using any one of the existing methods (Modified Distribution Method or Min Zero-Min Cost). This step yields the optimal allocation and optimal objective value of the crisp STP (The optimal allotted cell in crisp solid transportation table is referred as occupied cells. The remaining cells are called unoccupied cells. The number of occupied cells in crisp STP which are exactly $(m+n+l-2)$ and all has zero cost. Similarly, in FIFSTP also have the same $(m+n+l-2)$ number of occupied cells but its corresponding costs are intuitionistic fuzzy zeros). Now, construct the new fully intuitionistic fuzzy solid transportation table (FIFSTT) whose occupied cells costs are intuitionistic fuzzy zeros and the remaining cells costs are its original cost. Now, subtract the minimum cost of each source from all the elements of that source. Now, subtract the minimum cost of each destination from all the elements of that destination. Then subtract the minimum cost of each conveyance from all the elements of that conveyance. Clearly, each source, each destination and each conveyance of the resulting table has at least one intuitionistic fuzzy zero. Thus, the current resulting table is the allotment table.

Step 7: After using step 6 of the proposed method, now we check the allotment table if one or more an origin/a demand/a conveyance having exactly one occupied cell (intuitionistic fuzzy zero) then allot the maximum possible value (i.e., minimum of supply, demand and conveyance capacities) to that cell and adjust the corresponding supply and demand/demand and conveyance capacity/conveyance capacity and supply. Otherwise, if all the origins/destinations/conveyances having more than one occupied cells then select a cell in the α - origin, β - destination and γ-conveyance of the transportation table whose cost is maximum (If the maximum cost is more than one i.e., a tie occurs then select arbitrarily) and examine which one of the cell having minimum original cost (If the minimum original cost is more than one i.e., a tie occurs then select arbitrarily) among all the occupied cells in that origin/destination/conveyance then allot the maximum possible value to that cell. In this manner proceed selected origin, destination and conveyance entirely. If the entire origin, destination and conveyance of the occupied cells having fully allotted then select the next maximum cost of the transportation table and examine which one of the cells is minimum cost among all the occupied cells in that origin, destination and conveyance then allot the maximum possible value to that cell. Repeat this process until all intuitionistic fuzzy supply points are fully used, all intuitionistic fuzzy demand points are fully received and all conveyance capacities are fully used. This allotment yields the fully intuitionistic fuzzy solution to the given fully intuitionistic fuzzy solid transportation problem.

Remark: Allot the maximum possible value to the occupied cells in FIFSTP which is the most preferable origin/a destination/a conveyance having exactly one occupied cell. Further, we check the minimum number of zeros originwise, demandwise and conveyancewise in the allotment table and allot the maximum possible value to the zero cells having minimum original cost. If more than one occurs, then select arbitrarily.

Remark: From the MODI method, we conclude that the $m \times n \times l$ STP have exactly $m+n+l-2$ number of non-negative independent allocations.

Remark: From the Min-Zero Min-Cost Method, we can make exactly $m+n+l-2$ number of zeros (zero referred to as zero cost) in the cost matrix. All these zeros (costs) are in independent positions.

Remark: From the first two Remarks of this section, we can directly replace the intuitionistic fuzzy zeros instead of original costs in the occupied cells in the original FIFSTP. This modification does not affect the originality of the problem.

Now, we prove the following theorems which are used to derive the solution to an intuitionistic fuzzy solid transportation problem obtained by the PSK method is an intuitionistic fuzzy optimal solution to the fully intuitionistic fuzzy solid transportation problem.

Theorem

If $\left\{\tilde{x}_{ijk}^{I^O}, i=1,2,\ldots,m; j=1,2,\ldots,n; \text{ and } k=1,2\ldots,l\right\}$ is an optimal solution of the problem (Q):

$$\text{(Q) Minimum } \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\left(\tilde{c}_{ijk}^{I}\Theta\tilde{u}_i^{I}\Theta\tilde{v}_j^{I}\Theta\tilde{w}_k^{I}\right)\tilde{x}_{ijk}^{I}$$

subject to Equations (1)-(4) and:

$$\tilde{c}_{ijk}^{I}\Theta\tilde{u}_i^{I}\Theta\tilde{v}_j^{I}\Theta\tilde{w}_k^{I} \succcurlyeq 0, \text{ for all } i, j \text{ and } k \tag{9}$$

where $\tilde{u}_i^{I}$ (minimum cost of i^{th} source of the newly constructed transportation table $\tilde{c}_{ijk}^{I}$), $\tilde{v}_j^{I}$ (minimum cost of j^{th} destination of the resulting transportation table $\left[\tilde{c}_{ijk}^{I}\Theta\tilde{u}_i^{I}\right]$, $\tilde{w}_k^{I}$ (minimum cost of k^{th} conveyance of the resulting transportation table $\left[\tilde{c}_{ijk}^{I}\Theta\tilde{u}_i^{I}\Theta\tilde{v}_j^{I}\right]$, are any real values, then $\left\{\tilde{x}_{ijk}^{I^O}, i=1,2,\ldots,m; j=1,2,\ldots,n; \text{ and } k=1,2\ldots,l\right\}$ is an optimum solution to the problem (P).

Proof

Let $\tilde{u}_i^{I}$ be the minimum cost of ith source of the newly constructed transportation table $\left[\tilde{c}_{ijk}^{I}\right]$. Now, we subtract $\tilde{u}_i^{I}$ from the ith source entries so that the resulting table is $\left[\tilde{c}_{ijk}^{I}\Theta\tilde{u}_i^{I}\right]$. Let $\tilde{v}_j^{I}$ be the minimum cost of jth destination entries of the resulting table $\left[\tilde{c}_{ijk}^{I}\Theta\tilde{u}_i^{I}\right]$. Now, we subtract $\tilde{v}_j^{I}$ from the jth destination entries so that the resulting table is $\left(\tilde{c}_{ijk}^{I}\Theta\tilde{u}_i^{I}\Theta\tilde{v}_j^{I}\right)$. Let $\tilde{w}_k^{I}$ be the minimum cost of kth conveyance entries of the resulting table $\left(\tilde{c}_{ijk}^{I}\Theta\tilde{u}_i^{I}\Theta\tilde{v}_j^{I}\right)$. Now, we subtract $\tilde{w}_k^{I}$ from the kth conveyance entries so that the resulting table is $\left(\tilde{c}_{ijk}^{I}\Theta\tilde{u}_i^{I}\Theta\tilde{v}_j^{I}\Theta\tilde{w}_k^{I}\right)$. It may be noted that $\left(\tilde{c}_{ijk}^{I}\Theta\tilde{u}_i^{I}\Theta\tilde{v}_j^{I}\Theta\tilde{w}_k^{I}\right) \succcurlyeq \tilde{0}^{I}$, for all *i*, *j* and *k*. Further each source, each destination and each conveyance having at least one intuitionistic fuzzy zero.

From the statement of the theorem, clearly:

$$\left\{\tilde{x}_{ijk}^{I^O}, i=1,2,\ldots,m; j=1,2,\ldots,n; \text{ and } k=1,2\ldots,l\right\}$$

is a feasible solution of (P).

Suppose that:

$$\left\{\tilde{x}_{ijk}^{I^O}, i=1,2,\ldots,m; j=1,2,\ldots,n; \text{ and } k=1,2\ldots,l\right\}$$

is not an optimal solution of (P). Then, there exists a feasible solution:

$$\left\{y_{ijk}^{I}, i=1,2,\ldots,m; j=1,2,\ldots,n; \text{ and } k=1,2\ldots,l\right\}$$

such that:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\tilde{c}_{ijk}^{I}\, y_{ijk}^{I} < \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\tilde{c}_{ijk}^{I}\, \tilde{x}_{ijk}^{I^{O}}.$$

Clearly, $\left\{y_{ijk}^{I}, i=1,2,\ldots,m; j=1,2,\ldots,n; \text{ and } k=1,2\ldots,l\right\}$ is also a feasible solution of the problem (Q).

Now:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\left(\tilde{c}_{ijk}^{I}\Theta\tilde{u}_{i}^{I}\Theta\tilde{v}_{j}^{I}\Theta\tilde{w}_{k}^{I}\right)y_{ijk}^{I} \approx \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\tilde{c}_{ijk}^{I}\otimes$$
$$y_{ijk}^{I}\Theta\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\tilde{u}_{i}^{I}\otimes y_{ijk}^{I}\Theta\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\tilde{v}_{j}^{I}\otimes$$
$$y_{ijk}^{I}\Theta\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\tilde{w}_{k}^{I}\otimes y_{ijk}^{I} < \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\tilde{c}_{ijk}^{I}\otimes\tilde{x}_{ijk}^{I^{O}}\Theta\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\tilde{u}_{i}^{I}\otimes\tilde{a}_{i}^{I}\Theta\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\tilde{v}_{j}^{I}\otimes\tilde{b}_{j}^{I}\Theta\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\tilde{w}_{k}^{I}\otimes\tilde{e}_{k}^{I}$$

by (1) to (3):

$$\approx\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\left(\tilde{c}_{ijk}^{I}\Theta\tilde{u}_{i}^{I}\Theta\tilde{v}_{j}^{I}\Theta\tilde{w}_{k}^{I}\right)\tilde{x}_{ijk}^{I^{O}}$$

by (1) to (3), which contradicts:

$$\left\{\tilde{x}_{ijk}^{I^{O}}, i=1,2,\ldots,m; j=1,2,\ldots,n; \text{ and } k=1,2\ldots,l\right\}$$

is optimal solution of the problem (Q). Hence, we can conclude that any optimal solution:

$$\left\{\tilde{x}_{ijk}^{I^{O}}, i=1,2,\ldots,m; j=1,2,\ldots,n; \text{ and } k=1,2\ldots,l\right\}$$

to the problem (Q) is also an intuitionistic fuzzy optimal solution to the problem (P).

Hence proved the theorem.

Corollary

If $\left\{\tilde{x}_{ijk}^{oI}, i=1,2,\ldots,m; j=1,2,\ldots n; \text{ and } k=1,2,\ldots l\right\}$ is a feasible solution to the problem (P) and $\left(\tilde{c}_{ijk}^{I}\Theta\tilde{u}_{i}^{I}\Theta\tilde{v}_{j}^{I}\Theta\tilde{w}_{k}^{I}\right)\succcurlyeq\tilde{0}^{I}$, for all *i*, *j* and *k* where $\tilde{u}_{i}^{I}$, $\tilde{v}_{j}^{I}$ and $\tilde{w}_{k}^{I}$ are some real TIFNs, such that the minimum:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\left(\tilde{c}_{ijk}^{I}\Theta\tilde{u}_{i}^{I}\Theta\tilde{v}_{j}^{I}\Theta\tilde{w}_{k}^{I}\right)\tilde{x}_{ijk}^{I}$$

subject to Equations (1)-(4) are satisfied, is intuitionistic fuzzy zero, then:

$$\left\{\tilde{x}_{ijk}^{oI}, i=1,2,\ldots,m; j=1,2,\ldots n; \text{ and } k=1,2,\ldots l\right\}$$

is an intuitionistic fuzzy optimal solution to the problem (P).

Proof

Let $\left\{x_{ijk}^{oI}, i=1,2,\ldots,m; j=1,2,\ldots n; \text{ and } k=1,2,\ldots l\right\}$ be the feasible solution to the problem (P). Now, consider the problem (P) with $\left(\tilde{c}_{ijk}^{I}\Theta\tilde{u}_{i}^{I}\Theta\tilde{v}_{j}^{I}\Theta\tilde{w}_{k}^{I}\right)\succcurlyeq\tilde{0}^{I}$, for all *i*, *j* and *k* denoted by problem (P_1). From the first Theorem of in this section, Clearly, (P) is a original problem and (P_1) is a reduced problem of problem (P). Further, in a problem (P_1) there is no possibility to minimize the cost/time below intuitionistic fuzzy zero. Hence proved the Corollary.

Now, the author proves that the solution to the fully intuitionistic fuzzy solid transportation problem (or mixed intuitionistic fuzzy solid transportation problem) obtained by the PSK method is a fully intuitionistic fuzzy optimal solution (or mixed intuitionistic fuzzy optimal solution) to the fully intuitionistic fuzzy solid transportation problem (or mixed intuitionistic fuzzy solid transportation problem).

Theorem (PSK Theorem in MIFSTP)

A solution obtained by the PSK's method for a fully intuitionistic fuzzy solid transportation problem (or mixed intuitionistic fuzzy solid transportation problem) with equality constraints (P) is a fully intuitionistic fuzzy optimal solution (or mixed intuitionistic fuzzy optimal solution) for the fully intuitionistic fuzzy solid transportation problem (P) (or mixed intuitionistic fuzzy solid transportation problem).

Proof

Let us, now describe the PSK's method in detail.

We construct a solid transportation table in which costs, supplies, demands and conveyance capacities are must be a mixture of crisp numbers, triangular fuzzy numbers and triangular intuitionistic fuzzy numbers such transportation problem is called MIFSTP. Next, transform the MIFSTP into a balanced mixed intuitionistic fuzzy solid transportation problem (BMIFSTP), if it is not balanced, by using ranking method and convert BMIFSTP into balanced fully intuitionistic fuzzy solid transportation problem (BFIFSTP) using the following steps:

1. If any one or more in the supplies/demands/conveyance capacities/costs of a transportation problem having a real number say (a_1) that can be expanded as a TIFN $a_1=(a_1,a_1,a_1)(a_1,a_1,a_1)$;
2. If any one or more in the supplies/demands/conveyance capacities/costs of a transportation problem having a triangular fuzzy number say (a_1,a_2,a_3) that can be expanded as a TIFN $(a_1,a_2,a_3)=(a_1,a_2,a_3)(a_1,a_2,a_3)$;

3. If any one or more in the supplies/demands/conveyance capacities/costs of a transportation problem having a TIFN say $\left(a_1, a_2, a_3\right)\left(a_1^{'}, a_2, a_3^{'}\right)$ that can be kept as it is. That is $\left(a_1, a_2, a_3\right)\left(a_1^{'}, a_2, a_3^{'}\right) = \left(a_1, a_2, a_3\right)\left(a_1^{'}, a_2, a_3^{'}\right)$.

After using above steps, we get the fully intuitionistic fuzzy solid transportation table $\left[\tilde{c}_{ijk}^{I}\right]$ then, transform the FIFSTP into its equivalent crisp STP using the ranking procedure of Varghese and Kuriakose.

Now, the crisp STP having all the entries of supply, demand, unit transportation costs and conveyance capacities are integers then kept as it is. Otherwise at least one or all of the supply, demand, unit transportation costs and conveyance capacities are not in integers then rewrite its nearest integer value because decimal values in solid transportation problem has no physical meaning (such a transportation problem referred as crisp STP).

Now, solve the crisp STP by using any one of the existing methods (Modified Distribution Method, Min Zero-Min Cost). This process will yield the optimal allotment and optimal objective value of the crisp STP (The optimal allotted cells in crisp solid transportation table is referred to as occupied cells which are exactly $\left(m+n+l-2\right)$. All the decision variables in these occupied cells are basic feasible with zero cost. Clearly, each supplies of sources, each demand of destinations and each capacity of conveyance have at least one zero cost which corresponds to the occupied cells. The remaining cells are called unoccupied cells. All the decision variables in these unoccupied cells are non-basic. The value of decision variables in these unoccupied cells are at zero level).

By the definitions, occupied cells in crisp STP is same as that of occupied cells in FIFSTP but the value of occupied cells for FIFSTP is the maximum possible value of intuitionistic fuzzy supply, intuitionistic fuzzy demand and intuitionistic fuzzy conveyance capacity. Therefore, we need not further investigate the occupied cells in FIFSTP. But only we claim that how much quantity (intuitionistic fuzzy supply, intuitionistic fuzzy demand and intuitionistic fuzzy conveyance capacity) to allot the occupied cells subject to (1), (2), (3) and (4) are satisfied. The occupied cells in crisp STP is exactly $\left(m+n+l-2\right)$ and all are having zero cost. Similarly, in FIFSTP also have the same $\left(m+n+l-2\right)$ number of occupied cells but its corresponding cost is intuitionistic fuzzy zeros. Now, construct the new FIFSTT whose occupied cells costs are intuitionistic fuzzy zeros and the remaining cells costs are its original cost. Let $\tilde{u}_i^I$ be the minimum cost of i^th^ source of the current table $\left[\tilde{c}_{ijk}^{I}\right]$. Now, we subtract $\tilde{u}_i^I$ from the i^th^ source entries so that the resulting table is $\left[\tilde{c}_{ijk}^{I}\Theta\tilde{u}_i^I\right]$. Let $\tilde{v}_j^I$ be the minimum cost of j^th^ destination of the resulting table $\left[\tilde{c}_{ijk}^{I}\Theta\tilde{u}_i^I\right]$. Now, we subtract $\tilde{v}_j^I$ from the j^th^ destination entries so that the resulting table is $\left(\tilde{c}_{ijk}^{I}\Theta\tilde{u}_i^I\Theta\tilde{v}_j^I\right)$. Let $\tilde{w}_k^I$ be the minimum cost of k^th^ conveyance capacity of the resulting table $\left(\tilde{c}_{ijk}^{I}\Theta\tilde{u}_i^I\Theta\tilde{v}_j^I\right)$. Now, we subtract $\tilde{w}_k^I$ from the k^th^ conveyance capacity entries so that the resulting table is $\left(\tilde{c}_{ijk}^{I}\Theta\tilde{u}_i^I\Theta\tilde{v}_j^I\Theta\tilde{w}_k^I\right)$. It may be noted that $\left(\tilde{c}_{ijk}^{I}\Theta\tilde{u}_i^I\Theta\tilde{v}_j^I\Theta\tilde{w}_k^I\right) \succcurlyeq \tilde{0}^I$, for all *i*, *j* and *k*. Clearly, each source, each destination and each conveyance have at least one intuitionistic fuzzy zero. Hence the current resulting table is the allotment table.

Now, we check the allotment table if one or more sources/destinations/conveyances having exactly one occupied cell then allot the maximum possible value to that cell and adjust the corresponding supply and demand/demand and conveyance capacity/conveyance capacity and supply with a positive difference of supply, demand and conveyance capacity. Otherwise, if all the supply of sources, demand of destinations/capacity of conveyance having more than one occupied cells then select a cell in the

α -source, β -destination and γ-conveyance of the transportation table whose cost is maximum (If the maximum cost is more than one i.e., a tie occurs then select arbitrarily) and examine which one of the cells is minimum cost (If the minimum cost is more than one i.e., a tie occurs then select arbitrarily) among all the occupied cells in that source, destination and conveyance capacity then allot the maximum possible value to that cell. In this manner proceed selected source, destination/conveyance entirely. If the entire source, destination/conveyance of the occupied cells having fully allotted then select the next maximum cost of the transportation table and examine which one of the cells is minimum cost among all the occupied cells in that supply of sources, demand of destination and capacity of conveyance then allot the maximum possible value to that cell. Repeat this process until all the intuitionistic fuzzy supply points are fully used, all the intuitionistic fuzzy demand points are fully received and all the intuitionistic fuzzy conveyance capacity are fully used. This step yields the optimum intuitionistic fuzzy allotment.

Clearly, the above process satisfies all the rim requirements (supply, demand and conveyance capacity restriction). If all the rim requirements are satisfied then automatically it satisfies, the total intuitionistic fuzzy supply and total intuitionistic fuzzy demand is equal to total intuitionistic fuzzy conveyance capacity i.e., the necessary and sufficient condition for a FIFSTP is satisfied.

Finally, we have a solution:

$$\left\{\tilde{x}_{ijk}^{I}, i=1,2,\ldots,m; j=1,2,\ldots n; \text{ and } k=1,2,\ldots l\right\}$$

for the FIFSTP whose cost matrix is [$\tilde{c}_{ijk}^{I}\Theta\tilde{u}_{i}^{I}\Theta\tilde{v}_{j}^{I}\Theta\tilde{w}_{k}^{I}$] such that $\tilde{x}_{ijk}^{I} \approx \tilde{0}^{I}$ for ($\tilde{c}_{ijk}^{I}\Theta\tilde{u}_{i}^{I}\Theta\tilde{v}_{j}^{I}\Theta\tilde{w}_{k}^{I}$) $\succeq \tilde{0}^{I}$ and $\tilde{x}_{ijk}^{I} \succ \tilde{0}^{I}$ for ($\tilde{c}_{ijk}^{I}\Theta\tilde{u}_{i}^{I}\Theta\tilde{v}_{j}^{I}\Theta\tilde{w}_{k}^{I}$) $\approx \tilde{0}^{I}$.

Therefore, the minimum:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\left(\tilde{c}_{ijk}^{I}\Theta\tilde{u}_{i}^{I}\Theta\tilde{v}_{j}^{I}\Theta\tilde{w}_{k}^{I}\right)\tilde{x}_{ijk}^{I}$$

subject to Equations (1)-(4) are satisfied, is intuitionistic fuzzy zero. Thus, from the Corollary of in this section, the solution:

$$\left\{\tilde{x}_{ijk}^{I}, i=1,2,\ldots,m; j=1,2,\ldots n; \text{ and } k=1,2,\ldots l\right\}$$

is obtained by the PSK method for a fully intuitionistic fuzzy transportation problem (or mixed intuitionistic fuzzy solid transportation problem) with equality constraints is a fully intuitionistic fuzzy optimal solution (or mixed intuitionistic fuzzy optimal solution) for the fully intuitionistic fuzzy solid transportation problem (or mixed intuitionistic fuzzy solid transportation problem). Hence proved the theorem.

Theorem

Let $\left\{\tilde{x}_{ijk}^{I}, i=1,2,\ldots,m; j=1,2,\ldots n; \text{ and } k=1,2,\ldots l\right\}$ be the vector of feasible (optimal) solutions to the IFSTP. Then any other vector:

$$\left\{\tilde{y}_{ijk}^{I}, i=1,2,\ldots,m; j=1,2,\ldots n; \text{ and } k=1,2,\ldots l\right\}$$

(say) with the same ranking values is also feasible (optimal). That is, given a vector of feasible (optimal) solutions to the IFSTP, any other vector with the same ranking values is also feasible (optimal).

Proof

The proof is trivial.

Theorem

A vector of intuitionistic fuzzy numbers is a feasible (optimal) solution to the IFSTP if and only if the crisp vector of its ranking values is a feasible (optimal) solution of the crisp one.

Proof

The proof is trivial.

Theorem

If some of the intuitionistic fuzzy numbers in the IFSTP (type-1, type-2, type-3 and type-4 IFSTP) are replaced by equivalent intuitionistic fuzzy numbers (their ranking values, for example), the new IFSTP has the same set of feasible (optimal) solutions.

Proof

The proof is straightforward.

The proposed method, namely, PSK method for solving a mixed and fully intuitionistic fuzzy solid transportation problems are illustrated by the following examples.

ILLUSTRATIVE EXAMPLES

Example 1: Real Life MIFSTP

A company has three factories O_1, O_2, and O_3 that manufacture the same product of umbrellas in three different places. The company manager would like to transport umbrellas from three different factories to three different retail stores D_1, D_2, and D_3. All the factories are connected to all the retail stores by the three different mediums called land, water, space and umbrellas are transported by means of motorcycle (E_1), ship (E_2) and aircraft (E_3). The availability(availability of umbrellas are depends on its production but production depends on men, machine, etc.) of umbrellas are not known exactly due to long power cut, labour's over time work, unexpected failures in machine etc. The demand of an umbrella is not known exactly due to seasonal changes (In rainy days the sale of an umbrella is more when compared to sunny days). The transportation cost is not known exactly due to variations in rates of petrol, traffic jams, weather in hilly areas etc. Similarly, the capacity of different modes of transport is not known exactly (Since it depends on size of the transport, structure of the transport, efficiency of the transport etc.). So, all the parameters of the STP are given in mixture of crisp numbers, triangular fuzzy numbers and triangular intuitionistic fuzzy numbers. The transportation cost for an umbrella from three different factories to three different retail stores by means of three different transports are given in Table 3 from the past experience.

In this table, $c_{111} = 4$; $\tilde{c}_{112} = (5,7,9)$; $\tilde{c}^I_{113} = (7,8,9)(6,8,10)$; $\tilde{c}^I_{121} = (1,3,5)(0,3,6)$; $c_{122} = 9$; $\tilde{c}_{123} = (5,7,9)$; $\tilde{c}_{131} = (4,6,8)$; $\tilde{c}^I_{132} = (5,8,10)(1,8,11)$; $c_{133} = 2$; $\tilde{c}_{211} = (2,4,6)$; $\tilde{c}^I_{212} = (1,2,3)(0,2,4)$; $c_{213} = 6$; $c_{221} = 1$; $\tilde{c}_{222} = (2,3,4)$; $\tilde{c}^I_{223} = (4,8,12)(3,8,13)$; $\tilde{c}^I_{231} = (7,8,9)(6,8,10)$; $c_{232} = 4$; $\tilde{c}_{233} = (4,5,6)$; $\tilde{c}^I_{311} = (4,8,12)(3,8,13)$; $c_{312} = 1$; $\tilde{c}_{313} = (2,3,4)$; $\tilde{c}_{321} = (2,4,6)$; $\tilde{c}^I_{322} = (5,8,10)$ (1,8,11); $c_{323} = 3$; $c_{331} = 5$; $\tilde{c}_{332} = (4,6,8)$; $\tilde{c}^I_{333} = (3,4,5)(2,4,6)$.

Table 3. Tabular representation of real life 3 × 3 × 3 MIFSTP

										Capacity e_k
	E_1			E_1			E_1			e_1
		E_2			E_2			E_2		$\tilde{e}_2$
			E_3			E_3			E_3	$\tilde{e}_3^I$
Retail Stores → Factories ↓	D_1			D_2			D_3			**Supply** a_i
O_1	c_{111}	$\tilde{c}_{112}$	$\tilde{c}_{113}^I$	$\tilde{c}_{121}^I$	c_{122}	$\tilde{c}_{123}$	$\tilde{c}_{131}$	$\tilde{c}_{132}^I$	c_{133}	$\tilde{a}_1$
O_2	$\tilde{c}_{211}$	$\tilde{c}_{212}^I$	c_{213}	c_{221}	$\tilde{c}_{222}$	$\tilde{c}_{223}^I$	$\tilde{c}_{231}^I$	c_{232}	$\tilde{c}_{233}$	a_2
O_3	$\tilde{c}_{311}^I$	c_{312}	$\tilde{c}_{313}$	$\tilde{c}_{321}$	$\tilde{c}_{322}^I$	c_{323}	c_{331}	$\tilde{c}_{332}$	$\tilde{c}_{333}^I$	$\tilde{a}_3^I$
Demand	$\tilde{b}_1^I$			$\tilde{b}_2$			b_3			

Supply: $\tilde{a}_1 = (10,11,12),\ a_2 = 13,\ \tilde{a}_3^I = (4,10,16)(2,10,18)$
Demand: $\tilde{b}_1^I = (5,8,10)(1,8,11),\ \tilde{b}_2 = (10,15,20),\ b_3 = 12$
Conveyance: $e_1 = 11,\ \tilde{e}_2 = (12,14,16),\ \tilde{e}_3^I = (3,8,16)(0,8,19)$

Find the optimal allocation which minimizes total intuitionistic fuzzy transportation cost.

Solution: For each mixed intuitionistic fuzzy number, its ranking indices is obtained by using the Varghese and Kuriakose (2012) ranking procedure as follows:

$$\Re\left(\tilde{c}_{111}^I\right)=4,\ \Re\left(\tilde{c}_{112}^I\right)=7,\ \Re\left(\tilde{c}_{113}^I\right)=8,\ \Re\left(\tilde{c}_{121}^I\right)=3,\ \Re\left(\tilde{c}_{122}^I\right)=9,\ \Re\left(\tilde{c}_{123}^I\right)=7,$$
$$\Re\left(\tilde{c}_{131}^I\right)=6,\ \Re\left(\tilde{c}_{132}^I\right)=7,\ \Re\left(\tilde{c}_{133}^I\right)=2,\ \Re\left(\tilde{c}_{211}^I\right)=4,\ \Re\left(\tilde{c}_{212}^I\right)=2,\ \Re\left(\tilde{c}_{213}^I\right)=6,$$
$$\Re\left(\tilde{c}_{221}^I\right)=1,\ \Re\left(\tilde{c}_{222}^I\right)=3,\ \Re\left(\tilde{c}_{223}^I\right)=8,\ \Re\left(\tilde{c}_{231}^I\right)=8,\ \Re\left(\tilde{c}_{232}^I\right)=4,\ \Re\left(\tilde{c}_{233}^I\right)=5,$$
$$\Re\left(\tilde{c}_{311}^I\right)=8,\ \Re\left(\tilde{c}_{312}^I\right)=1,\ \Re\left(\tilde{c}_{313}^I\right)=3,\ \Re\left(\tilde{c}_{321}^I\right)=4,\ \Re\left(\tilde{c}_{322}^I\right)=7,\ \Re\left(\tilde{c}_{323}^I\right)=3,$$
$$\Re\left(\tilde{c}_{331}^I\right)=5,\ \Re\left(\tilde{c}_{332}^I\right)=6,\ \Re\left(\tilde{c}_{333}^I\right)=4$$

Supply: $\Re\left(\tilde{a}_1\right)= 11,\ \Re\left(a_2\right)= 13,\ \Re\left(\tilde{a}_3^I\right)= 10$

Demand: $\Re\left(\tilde{b}_1^I\right)= 7,\ \Re\left(\tilde{b}_2\right)= 15,\ \Re\left(b_3\right)= 12$

Conveyance: $\Re\left(e_1\right)=11,\ \Re\left(\tilde{e}_2\right)= 14,\ \Re\left(\tilde{e}_3^I\right)= 9$

Now using step 2 of the proposed method, we get:

$$\sum_{i=1}^{m}\Re\left(a_i\right)=\sum_{j=1}^{n}\Re\left(b_j\right)=\sum_{k=1}^{l}\Re(e_k)=34$$

Hence, the given problem is a BMIFSTP.

Now, using step 3 and step 4 of the proposed method, in conformation to Model (P*) mixed intuitionistic fuzzy solid transportation problem can be transformed into its equivalent crisp solid transportation problem (refer to Table 4) by using the ranking method of Varghese and Kuriakose.

After using step 5 of the proposed method, the optimal allotment (refer to Table 5) of the above problem is given.

The minimum objective value Z= (2×0) + (1×7) + (3×2) + (1×9) + (3×4) + (6×3) + (2×9) = 70 and the optimal solution is:

$$x_{121}=2,\ x_{133}=9,\ x_{221}=9,\ x_{222}=4,\ x_{232}=0,\ x_{312}=7,\ x_{332}=3$$

After using step 6 of the proposed method, now using step 7, we get the optimal solution directly of the MIFSTP is as follows:

$$x_{121}=\left(2,2,2\right)\left(2,2,2\right),\ x_{133}=\left(3,8,16\right)\left(0,8,19\right),\ x_{221}=\left(9,9,9\right)\left(9,9,9\right),$$
$$x_{222}=\left(2,4,6\right)\left(2,4,6\right),\ x_{232}=\left(0,0,0\right)\left(0,0,0\right),\ x_{312}=\left(5,8,10\right)\left(1,8,11\right),$$
$$x_{332}=\left(3,3,3\right)\left(3,3,3\right)$$

The minimum objective value is denoted by Min $\tilde{Z}^{\mathrm{I}}$ and is equal to:

Table 4. Crisp version of real life MIFSTP (Example 1)

										Capacity e_k
	E_1			E_1			E_1			11
		E_2			E_2			E_2		14
			E_3			E_3			E_3	9
Retail Stores → Factories ↓	D_1			D_2			D_3			**Supply** a_i
O_1	4	7	8	3	9	7	6	7	2	11
O_2	4	2	6	1	3	8	8	4	5	13
O_3	8	1	3	4	7	3	5	6	4	10
Demand b_j	7			15			12			

Table 5. Crisp optimal table of real life MIFSTP (Example 1)

										Capacity e_k
	E_1			E_1			E_1			11
		E_2			E_2			E_2		14
			E_3			E_3			E_3	9
Retail Stores → Factories ↓	D_1			D_2			D_3			**Supply** a_i
O_1	4	7	8	3(2)	9	7	6	7	2(9)	11
O_2	4	2	6	1(9)	3(4)	8	8	4(0)	5	13
O_3	8	1(7)	3	4	7	3	5	6(3)	4	10
Demand b_j	7			15			12			

$$\text{Min } \tilde{Z}^{I} = (1, 3, 5)(0,3,6) \times (2,2,2)(2,2,2)+2\times(3,8,16)(0,8,19)+1\times(9,9,9)(9,9,9)+(2,3,4) \times(2,4,6)(2,4,6)+4\times(0,0,0)(0,0,0)+1\times(5,8,10)(1,8,11)+ (4,6,8) \times(3,3,3)(3,3,3)$$

$$\text{Min } \tilde{Z}^{I} = (1, 3, 5)(0,3,6) \times \Re[(2,2,2)(2,2,2)]+(2,2,2)(2,2,2)\times \Re[(3,8,16)(0,8,19)] + (1,1,1)(1,1,1)\times \Re[(9,9,9)(9,9,9)]+(2,3,4) (2,3,4) \times \Re[(2,4,6)(2,4,6)]+(4,4,4)(4,4,4) \times \Re[(0,0,0)(0,0,0)]+(1,1,1)(1,1,1)\times \Re[(5,8,10)(1,8,11)]+ (4,6,8)(4,6,8) \times \Re[(3,3,3)(3,3,3)]$$

$$\text{Min } \tilde{Z}^{I} =(1,3,5)(0,3,6) \times 2+(2,2,2)(2,2,2)\times 9+(1,1,1)(1,1,1)\times 9+(2,3,4) (2,3,4)\times 4 + (4,4,4)(4,4,4)\times 0+(1,1,1)(1,1,1)\times 7 + (4,6,8)(4,6,8)\times 3$$

$$\text{Min } \tilde{Z}^{I} =(2,6,10)(0,6,12)+(18,18,18)(18,18,18)+(9,9,9)(9,9,9)+(8,12,16)(8,12,16) + (0,0,0)(0,0,0)+(7,7,7)(7,7,7)+(12,18,24)(12,18,24)$$

$$\text{Min } \tilde{Z}^{I} = (56, 70,84) (54, 70,86)$$

Hence, the total minimum intuitionistic fuzzy transportation cost is:

$$\text{Min } \tilde{Z}^{I} = (56, 70,84) (54, 70,86)$$

$$\Re\left(\tilde{Z}^{I}\right) = \Re\left(56,70,84\right)\left(54,70,86\right) = 70 \tag{10}$$

Example 2: Real Life Type-4 IFSTP

A firm has three factories S_1, S_2, and S_3 that manufacture the same product of air coolers in three different places. The firm manager would like to transport air coolers from three different factories to three different warehouses W_1, W_2, and W_3. All the factories are connected to all the warehouses by the three different mediums called land, water, space and air coolers are transported by means of motorcycle (E_1), ship (E_2) and aircraft (E_3). The availability (availability of air coolers depends on its production but production depends on machine, men, etc.) of air coolers are not known exactly due to labour's over time work, long power cut, unexpected failures in machine etc. The demand of air coolers is not known exactly due to seasonal changes (In sunny days the sale of air coolers is greater when compared to rainy days). The transportation cost is not known exactly due to variations in rates of petrol, weather in hilly areas, traffic jams etc. So, all the parameters of the STP are in uncertain quantities which are given in terms of TIFN. The transportation costs (rupees in hundreds) for an air cooler from different factories to different warehouses by means of different modes of transport are given in Table 6 from the past experience.

In this table, $\tilde{c}^I_{111} = (3,4,5)(2,4,6)$; $\tilde{c}^I_{112} = (5,8,10)(1,8,11)$; $\tilde{c}^I_{113} = (7,8,9)(6,8,10)$; $\tilde{c}^I_{121} = (1,3,5)(0,3,6)$; $\tilde{c}^I_{122} = (3,8,16)(0,8,19)$; $\tilde{c}^I_{123} = (5,8,10)(1,8,11)$; $\tilde{c}^I_{131} = (4,6,8)(3,6,9)$; $\tilde{c}^I_{132} = (5,8,10)(1,8,11)$; $\tilde{c}^I_{133} = (1,2,3)(0,2,4)$; $\tilde{c}^I_{211} = (3,4,5)(2,4,6)$; $\tilde{c}^I_{212} = (1,2,3)(0,2,4)$; $\tilde{c}^I_{213} = (2,6,10)(1,6,11)$; $\tilde{c}^I_{221} = (0.5,1,1.5)(0,1,2)$; $\tilde{c}^I_{222} = (1,3,5)(0,3,6)$; $\tilde{c}^I_{223} = (4,8,12)(3,8,13)$; $\tilde{c}^I_{231} = (7,8,9)(6,8,10)$; $\tilde{c}^I_{232} = (3,4,5)(2,4,6)$; $\tilde{c}^I_{233} = (2,5,8)(1,5,9)$; $\tilde{c}^I_{311} = (4,8,12)(3,8,13)$; $\tilde{c}^I_{312} = (0.5,1,1.5)(0,1,2)$; $\tilde{c}^I_{313} = (1,3,5)(0,3,6)$; $\tilde{c}^I_{321} = (3,4,5)(2,4,6)$; $\tilde{c}^I_{322} = (5,8,10)(1,8,11)$; $\tilde{c}^I_{323} = (1,3,5)(0,3,6)$; $\tilde{c}^I_{331} = (2,5,8)(1,5,9)$; $\tilde{c}^I_{332} = (4,6,8)(3,6,9)$; $\tilde{c}^I_{333} = (3,4,5)(2,4,6)$.

Supply: $\tilde{a}^I_1 = (3,10,20)(0,10,23)$, $\tilde{a}^I_2 = (6,12,21)(2,12,25)$, $\tilde{a}^I_3 = (4,10,16)(2,10,18)$
Demand: $\tilde{b}^I_1 = (5,8,10)(1,8,11)$, $\tilde{b}^I_2 = (12,16,19)(7,16,21)$, $\tilde{b}^I_3 = (4,12,20)(2,12,22)$
Conveyance: $\tilde{e}^I_1 = (3,10,20)(0,10,23)$, $\tilde{e}^I_2 = (10,16,20)(2,16,22)$, $\tilde{e}^I_3 = (3,8,16)(0,8,19)$

Table 6. Tabular representation of real-life type-4 IFSTP

										Capacity $\tilde{e}^I_k$
	E_1			E_1			E_1			$\tilde{e}^I_1$
		E_2			E_2			E_2		$\tilde{e}^I_2$
			E_3			E_3			E_3	$\tilde{e}^I_3$
Warehouses → Factories ↓	W_1			W_2			W_3			**Supply** $\tilde{a}^I_i$
S_1	$\tilde{c}^I_{111}$	$\tilde{c}^I_{112}$	$\tilde{c}^I_{113}$	$\tilde{c}^I_{121}$	$\tilde{c}^I_{122}$	$\tilde{c}^I_{123}$	$\tilde{c}^I_{131}$	$\tilde{c}^I_{132}$	$\tilde{c}^I_{133}$	$\tilde{a}^I_1$
S_2	$\tilde{c}^I_{211}$	$\tilde{c}^I_{212}$	$\tilde{c}^I_{213}$	$\tilde{c}^I_{221}$	$\tilde{c}^I_{222}$	$\tilde{c}^I_{223}$	$\tilde{c}^I_{231}$	$\tilde{c}^I_{232}$	$\tilde{c}^I_{233}$	$\tilde{a}^I_2$
S_3	$\tilde{c}^I_{311}$	$\tilde{c}^I_{312}$	$\tilde{c}^I_{313}$	$\tilde{c}^I_{321}$	$\tilde{c}^I_{322}$	$\tilde{c}^I_{323}$	$\tilde{c}^I_{331}$	$\tilde{c}^I_{332}$	$\tilde{c}^I_{333}$	$\tilde{a}^I_3$
Demand	$\tilde{b}^I_1$			$\tilde{b}^I_2$			$\tilde{b}^I_3$			

Find the optimal allocation which minimizes total intuitionistic fuzzy transportation cost.

Solution: For each fully intuitionistic fuzzy number, its ranking indices is obtained by using the Varghese and Kuriakose (2012) ranking procedure as follows:

$$\Re\left(\tilde{c}_{111}^{I}\right)=,\ \Re\left(\tilde{c}_{112}^{I}\right)=7,\ \Re\left(\tilde{c}_{113}^{I}\right)=8,\ \Re\left(\tilde{c}_{121}^{I}\right)=3,\ \Re\left(\tilde{c}_{122}^{I}\right)=9,\ \Re\left(\tilde{c}_{123}^{I}\right)=7,\ \Re\left(\tilde{c}_{131}^{I}\right)=6,$$
$$\Re\left(\tilde{c}_{132}^{I}\right)=7,\ \Re\left(\tilde{c}_{133}^{I}\right)=2,\ \Re\left(\tilde{c}_{211}^{I}\right)=4,\ \Re\left(\tilde{c}_{212}^{I}\right)=2,\ \Re\left(\tilde{c}_{213}^{I}\right)=6,\ \Re\left(\tilde{c}_{221}^{I}\right)=1,$$
$$\Re\left(\tilde{c}_{222}^{I}\right)=3,\ \Re\left(\tilde{c}_{223}^{I}\right)=8,\ \Re\left(\tilde{c}_{231}^{I}\right)=8,\ \Re\left(\tilde{c}_{232}^{I}\right)=4,\ \Re\left(\tilde{c}_{233}^{I}\right)=5,\ \Re\left(\tilde{c}_{311}^{I}\right)=8,$$
$$\Re\left(\tilde{c}_{312}^{I}\right)=1,\ \Re\left(\tilde{c}_{313}^{I}\right)=3,\ \Re\left(\tilde{c}_{321}^{I}\right)=4,\ \Re\left(\tilde{c}_{322}^{I}\right)=7,\ \Re\left(\tilde{c}_{323}^{I}\right)=3,\ \Re\left(\tilde{c}_{331}^{I}\right)=5,$$
$$\Re\left(\tilde{c}_{332}^{I}\right)=6,\ \Re\left(\tilde{c}_{333}^{I}\right)=44$$

Supply: $\Re\left(\tilde{a}_1^{I}\right)=11,\ \Re\left(\tilde{a}_2^{I}\right)=13,\ \Re\left(\tilde{a}_3^{I}\right)=10$

Demand: $\Re\left(\tilde{b}_1^{I}\right)=7,\ \Re\left(\tilde{b}_2^{I}\right)=15,\ \Re\left(\tilde{b}_3^{I}\right)=12$

Conveyance: $\Re\left(\tilde{e}_1^{I}\right)=11,\ \Re\left(\tilde{e}_2^{I}\right)=14,\ \Re\left(\tilde{e}_3^{I}\right)=9$

Now using step 2 of the proposed method, we get:

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = \sum_{k=1}^{l} e_k = 34$$

Therefore, the given problem is a balanced type-4 IFSTP.

Now, using step 3 and step 4 of the proposed method, in conformation to Model (P*) type-4 intuitionistic fuzzy solid transportation problem can be transformed into its equivalent crisp solid transportation problem (refer to Table 7) by using the ranking method of Varghese and Kuriakose.

Table 7. Crisp version of real-life type-4 IFSTP (Example 2)

										Capacity e_k
	E_1			E_1			E_1			11
		E_2			E_2			E_2		14
			E_3			E_3			E_3	9
Warehouses → Factories ↓	W_1			W_2			W_3			Supply a_i
S_1	4	7	8	3	9	7	6	7	2	11
S_2	4	2	6	1	3	8	8	4	5	13
S_3	8	1	3	4	7	3	5	6	4	10
Demand b_j	7			15			12			

After using step 5 of the proposed method, the optimal allotment (refer to Table 8) of the above problem.

The minimum objective value:

Z= (2×0) + (1×7) + (3×2) + (1×9) + (3×4) + (6×3) + (2×9) = 70

$x_{121} = 2,\ x_{133} = 9,\ x_{221} = 9,\ x_{222} = 4,\ x_{232} = 0,\ x_{312} = 7,\ x_{332} = 3$

After using step 6 of the proposed method, now using step 7, we get the optimal solution directly of the IFSTP of type-4 is as follows:

$x_{121} = \left(1,2,3\right)\left(0,2,4\right),\ x_{133} = \left(3,8,16\right)\left(0,8,19\right),\ x_{221} = \left(3,8,16\right)\left(0,8,19\right),\ x_{222} = \left(3,4,5\right)\left(2,4,6\right),$
$x_{232} = \left(0,0,0\right)\left(0,0,0\right)\ x_{312} = \left(5,8,10\right)\left(1,8,11\right),\ x_{332} = \left(1,3,5\right)\left(0,3,6\right).$

The minimum objective value:

$\tilde{Z}^{I}$ = (1,3,5)(0,3,6) × (1,2,3)(0,2,4) + (1,2,3)(0,2,4) × (3,8,16)(0,8,19) + (0.5,1,1.5)(0,1,2) × (3,8,16)(0,8,19) + (1,3,5)(0,3,6) × (3,4,5)(2,4,6)+ (3,4,5)(2,4,6) × (0,0,0)(0,0,0) + (0.5,1,1.5)(0,1,2) × (5,8,10)(1,8,11) + (4,6,8)(3,6,9) × (1,3,5)(0,3,6)

The minimum objective value:

$\tilde{Z}^{I}$ = (1,3,5)(0,3,6) × $\Re$ [(1,2,3)(0,2,4)]+ (1,2,3)(0,2,4) × $\Re$ [(3,8,16)(0,8,19)] + (0.5,1,1.5)(0,1,2) × $\Re$ [(3,8,16)(0,8,19)] + (1,3,5)(0,3,6) × $\Re$ [(3,4,5)(2,4,6)] + (3,4,5)(2,4,6) × $\Re$ [(0,0,0)(0,0,0)] + (0.5,1,1.5)(0,1,2) × $\Re$ [(5,8,10)(1,8,11)]+ (4,6,8)(3,6,9) × $\Re$ [(1,3,5)(0,3,6)]

The minimum objective value:

Table 8. Crisp optimal table of real-life type-4 IFSTP (Example 2)

										Capacity e_k
	E_1			E_1			E_1			11
		E_2			E_2			E_2		14
			E_3			E_3			E_3	9
Warehouses → Factories ↓	W_1			W_2			W_3			**Supply** a_i
S_1	4	7	8	3(**2**)	9	7	6	7	2(**9**)	11
S_2	4	2	6	1(**9**)	3(**4**)	8	8	4(**0**)	5	13
S_3	8	1(**7**)	3	4	7	3	5	6(**3**)	4	10
Demand b_j	7			15			12			

$\tilde{Z}^{I}$ =(1,3,5)(0,3,6)×(2)+(1,2,3)(0,2,4)×(9)+(0.5,1,1.5)(0,1,2)×(9)+(1,3,5)(0,3,6)×(4) +(3,4,5)(2,4,6)×(0)+(0.5,1,1.5)(0,1,2)×(7)+(4,6,8)(3,6,9)×3

Min $\tilde{Z}^{I}$ =(2,6,10)(0,6,12)+ (9, 18, 27)(0, 18, 36)+ (4.5,9,13.5)(0,9,18) +(4,12,20)(0,12,24) +(0,0,0)(0,0,0) +(3.5,7,10.5)(0,7,14)+ (12, 18, 24) (9, 18, 27)

Min $\tilde{Z}^{I}$ = (35, 70,105) (9, 70,131)

Hence, the total intuitionistic fuzzy transportation minimum cost is:

Min $\tilde{Z}^{I}$ = (35, 70,105) (9, 70,131)

$$\Re\left(\tilde{Z}^{I}\right) = \Re\left(35, 70,105\right)\left(9, 70,131\right) = 70$$

RESULTS AND DISCUSSION

The minimum total intuitionistic fuzzy transportation cost of problem 1 is:

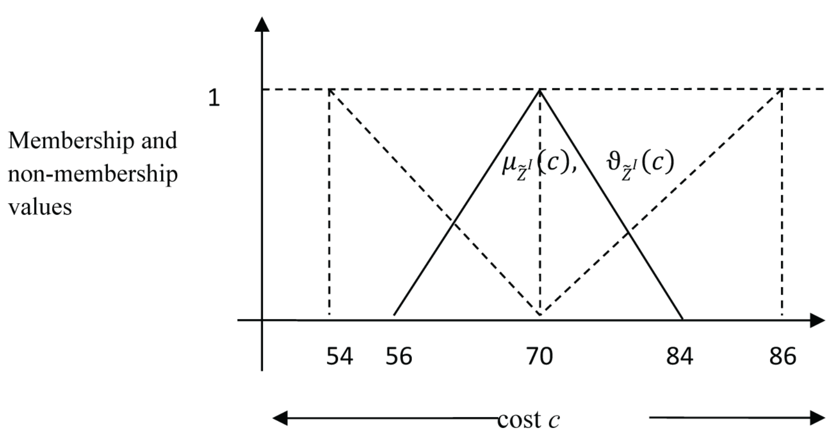
$$\text{Min } \tilde{Z}^{I} = (56, 70,84)\ (54, 70,86) \quad (11)$$

The result in (11) can be explained in Figure1.

The degree of acceptance of the transportation cost for the DM increases if the cost increases from 54 to 56; while it decreases if the cost increases from 56 to 70. Beyond (56,86), the level of acceptance or the level of satisfaction for the DM is zero. The DM is totally satisfied or the transportation cost is totally acceptable if the transportation cost is 70. The degree of non-acceptance of the transportation cost for the DM decreases if the cost increases from 54 to 70 while it increases if the cost increases from 70 to 86. Beyond (54,86), the cost is totally un-acceptable.

Assuming that $\mu_{\tilde{Z}^{I}}\left(c\right)$ is membership value (degree of acceptance or level of satisfaction) and $\vartheta_{\tilde{Z}^{I}}\left(c\right)$ is non-membership value (degree of non-acceptance) of transportation cost c. Then the degree

Figure 1. Graphical representation of MIFSTC



of acceptance of the transportation cost c is $100\ \mu_{\tilde{Z}^I}\left(c\right)\%$ for the DM and the degree of non-acceptance is $100\vartheta_{\tilde{Z}^I}\left(c\right)\%$ for the DM. The degree of hesitation for the acceptance of the transportation cost c is given by $100\,\pi_{\tilde{Z}^I}\left(c\right)\%$ where $\pi_{\tilde{Z}^I}=\left(1-\mu_{\tilde{Z}^I}\left(c\right)-\vartheta_{\tilde{Z}^I}\left(c\right)\right)$ represents the hesitation index.

Values of $\mu_{\tilde{Z}^I}\left(c\right)$ and $\vartheta_{\tilde{Z}^I}\left(c\right)$ at different values of c can be determined using equations given below:

$$\mu_{\tilde{Z}^I}\left(c\right)=\begin{cases}0, & \text{for } c<56\\ \dfrac{c-56}{14}, & \text{for } 56\le c\le 70\\ 1, & \text{for } c=70\\ \dfrac{84-c}{14}, & \text{for } 70\le c\le 84\\ 0, & \text{for } c>84\end{cases}$$

and:

$$\vartheta_{\tilde{Z}^I}\left(c\right)=\begin{cases}1, & \text{for } c<54\\ \dfrac{70-c}{16}, & \text{for } 54\le c\le 70\\ 0, & \text{for } c=70\\ \dfrac{c-70}{16}, & \text{for } 70\le c\le 86\\ 1, & \text{for } c>86\end{cases}$$

Advantages of the Proposed Method

By using the proposed method a decision maker has the following advantages:

1. We need not find out the basic feasible solution and we need not apply the optimality test because the solution obtained by proposed method is always optimal;
2. The proposed method is a single step method. So, the use of intuitionistic fuzzy modified distribution is not required.

CONCLUSION

The MIFSTPs and type-4 IFSTPs are solved by the proposed method which differs from the existing methods namely, intuitionistic fuzzy modified distribution method and intuitionistic fuzzy zero point method. Basically, intuitionistic fuzzy modified distribution method having too many steps. Also, it depends on the intuitionistic fuzzy initial basic feasible solution. Similarly, intuitionistic fuzzy zero point method also has the numerous number of steps. But, the proposed method doesn't depend on the intuitionistic fuzzy initial basic feasible solution and also it has a minimum number of steps. In intuitionistic fuzzy modified distribution method, the optimality test is required whereas in intuitionistic fuzzy zero point method the optimality test is not required. The main advantage of the PSK method is that the obtained solution is always optimal. To apply this method, there is no necessity to have $\left(m+n+l-2\right)$ number of non-negative allotted entries (i.e., basic feasible solution). Also, we need not test the optimality condition. It is applicable to type-1, type-2, type-3 and type-4 IFSTPs. The proposed method can help decision-makers in the logistics related issues of real-life problems by aiding them in the decision-making process and providing an optimal solution in a simple and effective manner. This proposed method will reduce the decision-makers computation time. Further, it can be served as an important tool for a decision-maker when he/she handles various types of logistic problems having different types of parameters.

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