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An Improved TOPSIS Method Based on a New Distance Measure and Its Application to the House Selection Problem

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ABSTRACT

It is difficult to choose an appropriate house for homebuyers. This is due to the difficulty of evaluating the multitude of factors, such as price, location, size, and so on. In order to help homebuyers in choosing an appropriate house, a method integrating the interval-valued Pythagorean FAHP and FTOPSIS is proposed. In the proposed approach, the evaluation criteria were determined by the experts, and the linguistic variables of interval-valued Pythagorean fuzzy numbers were used in the evaluations of the homebuyers and experts. A new distance between two IVPFNs is proposed. The weights of the evaluation criteria were determined by the interval-valued Pythagorean FAHP method by the homebuyers, and house selections were evaluated by interval-valued Pythagorean FTOPSIS method taking into account the new distance. Finally, a case study was executed to verify the feasibility of the proposed approach. The case study results reveal that the weights of criteria obtained by FAHP are not the same according to opinions of the different homebuyers.

KEYWORDS

FAHP, FTOPSIS, House Selection, Interval-Valued Pythagorean

INTRODUCTION

With the gradual improvement of people's living standards, more and more people buy houses for life in a certain city either for work or for education of children. But it is difficult for homebuyers to choose an appropriate house from the house resources by the real estate agents because they need to simultaneously consider factors such as price, value, size, and location. Some of the factors might be even contradictory. Therefore, house selection is a multi-criteria decision-making problem (MCDM).

MCDM approaches can be suitable tools to deal with the house selection problem. In the decades, researchers have proposed various methods regarding MCDM problems in fuzzy environments. For example, Prabhu and Ilangkumaran (2019) and Ahmet (2021) presented work on the analytic hierarchy process (AHP). Li (2010) and Li and Nan (2011) presented research on the technique for

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order preference by similarity to idea solution (TOPSIS). Other research was conducted on areas such as the relative ratio (RR) method (Li, 2009), fuzzy linear programming technique for multidimensional analysis of preference (FLINMAP) method (Li & Sun, 2007), linear programming (LP) (Yu et al., 2019), nonlinear programming approach (Li, 2011; Li & Liu, 2015), and game theory (Ye & Li, 2021; Liang et al., 2023).

The traditional AHP may not reflect the opinions of decision-makers. Therefore, new versions of AHP with fuzzy sets have been proposed. Zadeh (1965) proposed the fuzzy theory as an extension of the classical sets, and Atanassov (1986) proposed intuitionistic fuzzy sets (IFSs) as an extension of fuzzy sets. Atanassov (1989) also proposed interval-valued intuitionistic fuzzy sets (IVIFSs). However, the sum of the membership and non-membership of IVIFSs is equal to or less than one, and which may not be in line with people's way of thinking. To address this problem, Yager (2013) proposed Pythagorean fuzzy sets (PFSs) as an extension of the IFSs. Because PFSs allows the sum of membership and non-membership to exceed one, and the sum of squares to not exceed one, Pythagorean fuzzy sets theory are more powerful and flexible in solving problems involving uncertainty. Interval-valued Pythagorean fuzzy sets (IVPFSs) (Zhang, 2016), a generalization of PFSs, emerged as an effective tool to model the uncertain and imprecise information in the real-life decision evaluation process, and this method can be considered when decision-makers fail to employ crisp values, but use interval values to express their evaluation information. In the proposed method, linguistic variables of interval-valued Pythagorean fuzzy numbers (IVPFNs) are used in the evaluations by the homebuyers and experts.

Hwang (1981) first proposed the TOPSIS method. Regarding the uncertainty in real situations, many studies on fuzzy extensions have been completed to enrich the theory of TOPSIS method. Different versions of TOPSIS based on fuzzy sets have been developed for considering uncertainties and vagueness in MCDM problems, such as the fuzzy TOPSIS (Dwivedi et al., 2018), the weighted fuzzy TOPSIS (Prabhu & Ilangkumaran, 2019), the intuitionistic fuzzy TOPSIS (Li & Nan, 2011), the interval-valued intuitionistic fuzzy TOPSIS (Li, 2010), and the Pythagorean fuzzy TOPSIS (Zhang & Xu, 2014). Although many studies state that methods of TOPSIS with different fuzzy sets have been applied widely in various fields, relatively little attention has been paid to the extended TOPSIS dealing with house selection problems under complex uncertainty based on IVPFSs. From the aforementioned studies, we were inspired to use weighted fuzzy TOPSIS (FTOPSIS) to rank the houses.

This study proposes a new hybrid group decision-making approach with the fuzzy AHP (FAHP) and FTOPSIS methods based on IVPFSs for the house selection problem. To choose an appropriate house for homebuyers, we propose an integrated two-stage MCDM approach. In the first stage, a panel of experts is formed to gather the opinions. The criteria of house selection are then obtained according to literature review and experts' opinions from the house perspective. IVPFSs are an extension of IFSs, and they provide more freedom to express homebuyers' judgments on the uncertainty and vagueness in house selection problems. The identified criterion weights are obtained through interval-valued Pythagorean FAHP. In the second stage, experts provide the judgment matrices of houses. Based on the judgment matrices and weights, the houses are ranked with interval-valued Pythagorean FTOPSIS according to the new distance.

In the rest of this study we include a literature review on IVPFSs, FAHP, and FTOPSIS; discuss the house selection problem; and share some basic concepts related to IVPFSs. We then present a novel distance measure of IVPFNs, and based on this new distance, propose an improved FTOPSIS method for house selection. To verify the proposed method's feasibility, we provide a numerical example of house selection. We then compare the proposed method and other methods and present our sensitivity analysis. We end the study with a conclusion and our suggestions for future areas to study.

LITERATURE REVIEW

Interval-Valued Pythagorean Fuzzy Sets

In recent decades, many studies involving PFSes theory in MCDM problems have been completed. Yager (2014) introduced a variety of aggregation operations for Pythagorean fuzzy numbers (PFNs). Peng and Yang (2016) proposed the basic concept of PFN, weighted operator, and score function. Zulqarnain et al. (2022) proposed the concept of interval Pythagorean fuzzy power-geometric-geometric Heronian mean operator. In the fuzzy sets, the problem of measurement of differences or distances is unavoidable. To manifest the distances properly, many methods of measuring distances have been proposed, and some of them have had an ideal effect on classification. Szmidt and Kacprzyk, (2000) put forward the Hamming distance, Li and Cheng (2002) proposed similarity measures of intuitionistic fuzzy sets. Li (2004) demonstrated measures of dissimilarity in intuitionistic fuzzy structures. Li and Wan (2017) introduced minimum weighted Minkowski distance power models for intuitionistic fuzzy multi-attribute decision-making (MADM). Similarly, many methods of measuring distances of PFNs were proposed and used widely in the MCDM problem. Zhang and Xu (2014) proposed a distance between PFNs, and Han et al. (2019) put forward a distance measure for linguistic Pythagorean fuzzy sets. Fei and Deng (2020) proposed a distance measure between PFNs and IVPFNs. Paul et al. (2023) proposed a new Pythagorean fuzzy-based distance operator. We propose a new distance measure of IVPFSs for house selection problem that is based on these studies.

FAHP AND FTOPSIS

The application of MCDM in various fields has attracted the interest of many scholars. In recent years, many researchers have shared results about MCDM. For example, Prabhu (2019) used FAHP and GRA-TOPSIS methods for 3D printer selection problems, and Ahmet (2021) employed the FAHP-FTOPSIS method to solve green supplier selection problems. Table 1 provides the details of these studies that are constructed in the fields of MCDM problems and methods. As Table 1 indicates, AHP, ARAS, TOPSIS, MCGP, CPT, and weighted operator methods are used frequently in MCDM problems.

Table 1. A brief summary of MCDM studies

Authors	Method	Illustrative Example
Yu et al. (2019)	Linear programming	A strategy partner selection
Fu (2019)	AHP-ARAS-MCGP	Catering supplier selection
Ho et al. (2020)	TOPSIS	Stroke rehabilitation treatments
Chen et al. (2020)	Minimum trust discount coefficient model	Car selection
Yu et al. (2021)	IFMOLP	Portfolio selection
Ho et al. (2021)	MCGP	Smart phone selection
Habib et al. (2022)	Similarity measure	Functional brain networks
Alrasheedi et al. (2022)	Entropy-SWARA-WASPAS	Sustainable supplier
Mandal and Seikh (2022)	Fuzzy TOPSIS	Sustainable development
Zulqarnain et al. (2023)	Weighted geometric operator	Material selection
Rahim et al. (2023)	Distance measures-based TOPSIS method	Medical diagnosis
Habib at al. (2023)	Fuzzy TOPSIS, PF-Entropy, PFPWG	Risk assessment of childhood cancer

House selection and evaluation have been the focus of many studies concerning criteria such as price, location, house size, and transportation. Table 2 shows the details of the studies that are made for the solution and evaluation of the house selection problem.

The house selection problem has been investigated, as discussed in the *Literature Review* section, and shown in Tables 1 and 2. However, to the best of our knowledge, only a few studies have been completed on using the linguistic variables of IVPFSs in the house selection problem. Therefore, integrating the interval-valued Pythagorean FAHP and interval-valued Pythagorean FTOPSIS for the house selection problem is a meaningful undertaking.

In this study, we propose a group decision-making approach of FAHP and FTOPSIS methods under IVPFSs for the house selection problem. In the first stage, the criteria of the house selection problem are identified from literature review and experts. Based on the opinions of homebuyers, the weights of identified criteria are obtained by using interval-valued Pythagorean FAHP. In the second stage, houses are ranked by using interval-valued Pythagorean FTOPSIS according to the new distance measure. Figure 1 shows the flow chart of the proposed approach. The salient features of the proposed method are as follows. First, the proposed distance measure considers the membership degrees, non-membership degrees, and the degree of indeterminacy simultaneously. Hence, the new distance measure ensures the integrity of the information. Second, the weights of criteria are calculated according to homebuyers' opinions. Third, the rank of houses is calculated according to experts' opinions using the FTOPSIS method based on the new distance measure.

IVPFN

In this section, the general definition, and some basic operations of IVPFSs are introduced.

Definition 1 (Zhang, 2016): Let set X be a universe of discourse, and the interval-valued Pythagorean set (IVPFSs) P in X is defined as follows:

$$P = \{< x, \mu_P(x), \nu_P(x) > \big| x \in X\},$$

where $\mu_p : X \to [\mu_p^-(x), \mu_p^+(x)] \subset [0,1]$ represents the membership degree and holds that $0 \leq (\mu_P^+(x))^2 + (\nu_P^+(x))^2 \leq 1$, and $\nu_p : X \to [\nu_p^-(x), \nu_p^+(x)] \subset [0,1]$ represents the non-membership degree of the element $x \in X$ to P. In addition, $\pi_P(x) = \left[\pi_P^-(x), \pi_P^+(x)\right] = \left[1 - (\mu_P^+(x))^2 - (\nu_P^+(x))^2, 1 - (\mu_P^-(x))^2 - (\nu_P^-(x))^2\right]$ is named the degree of indeterminacy. For the convenience, the interval-valued Pythagorean fuzzy number (IVPFNs) is defined as $P = ([\mu^-, \mu^+], [\nu^-, \nu^+])$.

To compare the magnitude of two IVPFNs, Peng and Yang (2016) introduced a score function and distance.

Table 2. A brief summary of house selection studies

Method	Criteria	Authors
AHP and MCGP	Value (price), construction (size, number of rooms, number of floor levels), neighbor (pollution level, safety, landscape, recreational facilities), location (distance to downtown, distance of workplace, distance of children's school, public transportation)	Ho et al. (2015)
AHP	Value, building quality, surrounding facility, transportation	Wang (2013)
AHP	Location, price, design style, landscape, property service, surrounding facility, building quality, developer reputation, transportation	Sun et al. (2013)

Definition 2 (Zhang, 2016): Let $P_i = ([\mu_i^-, \mu_i^+], [\nu_i^-, \nu_i^+])$ $(i = 1,2)$ be two IVPFNs, a nature quasi-ordering on the IVPFNs is defined as follows: $P_1 \geq P_2$, if and only if $\mu_1^- \geq \mu_2^-, \mu_1^+ \geq \mu_2^+, \nu_1^- \leq \nu_2^-, \nu_1^+ \leq \nu_2^+$.

Definition 3 (Peng & Yang, 2016): Let $P = ([\mu^-, \mu^+], [\nu^-, \nu^+])$ be an IVPFNs, the score function of $\tilde{P}$ is defined as shown in equation (1).

$$s(P) = \frac{1}{2}\left[\left(\mu^-\right)^2 + \left(\mu^+\right)^2 - \left(\nu^-\right)^2 - \left(\nu^+\right)^2\right]. \tag{1}$$

Peng and Yang (2016) introduced the distance measure between two IVPFSs.

Definition 4 (Peng & Yang, 2016): Let $P_i = ([\mu_i^-, \mu_i^+], [\nu_i^-, \nu_i^+])$ $(i = 1,2)$ be two IVPFNs, the distance between P_1 and P_2 is defined as shown in equation (2).

$$d_{XP}(P_1, P_2) = \frac{1}{4}\left[\left|\left(\mu_1^-\right)^2 - \left(\mu_2^-\right)^2\right| + \left|\left(\mu_1^+\right)^2 - \left(\mu_2^+\right)^2\right| + \left|\left(\nu_1^-\right)^2 - \left(\nu_2^-\right)^2\right| + \left|\left(\nu_1^+\right)^2 - \left(\nu_2^+\right)^2\right| + \left|\left(\pi_1^-\right)^2 - \left(\pi_2^-\right)^2\right| + \left|\left(\pi_1^+\right)^2 - \left(\pi_2^+\right)^2\right|\right]. \tag{2}$$

Example 1, which we present later in this paper, shows that Peng and Yang's method fails to measure IVPFS's distance in some cases. To avoid this issue, Fei and Deng (2020) developed a new distance measure.

Definition 5 (Fei & Deng, 2020): Let $P_i = ([\mu_i^-, \mu_i^+], [\nu_i^-, \nu_i^+])$ $(i = 1,2)$ be two IVPFNs, the distance between P_1 and P_2 is defined as shown in equation (3).

$$d_{LF}(P_1, P_2) = \frac{\sqrt{2}}{4}\left(\sqrt{(\mu_1^- - \mu_2^-)^2 + (\nu_1^- - \nu_2^-)^2} + \sqrt{(\mu_1^+ - \mu_2^+)^2 + (\nu_1^+ - \nu_2^+)^2}\right). \tag{3}$$

This approach considers the membership and nonmembership degrees, but the degree of indeterminacy is not considered. This omission leads to information loss and thus affects the interpretation (Fei & Deng, 2020). To solve MCDM problems in a Pythagorean fuzzy environment, Zhang (2016) developed the Pythagorean fuzzy weighted averaging aggregation operator.

Definition 6 (Zhang, 2016): Let $P_i = ([\mu_i^-, \mu_i^+], [\nu_i^-, \nu_i^+])$ $(i = 1,2,\cdots,n)$ be n IVPFNs, these IVPFS are aggregated using the interval-valued Pythagorean fuzzy weighted geometric (IVPFWG) operator in equation (4): IVPFWG: $P^n \to P$,

$$\text{IVPFWG}(P_1, P_2, \cdots, P_n) = \left(\left[\prod_{i=1}^{n}(\mu_i^-)^{w_i}, \prod_{i=1}^{n}(\mu_i^+)^{w_i}\right], \left[\prod_{i=1}^{n}(\nu_i^-)^{w_i}, \prod_{i=1}^{n}(\nu_i^+)^{w_i}\right]\right), \tag{4}$$

where $\mathbf{w} = \left[w_1, w_2, \cdots, w_n\right]^{\mathrm{T}}$ is the weight of P_i $(i = 1,2,\cdots,n)$, and $\sum_{i=1}^{n} w_i = 1$.

The Proposed Method

In this section, we propose two steps to help homebuyers choose an appropriate house from many houses. First, we calculate the weights of criteria of the house selection problem using the FAHP method. Second, we propose a new distance between IVPFNs and rank houses using the FTOPSIS method according to the proposed distance measure.

FAHP

To help homebuyers choose an appropriate house, we read a lot of literatures and consulted with five real estate experts. We determined four criteria of the house selection problem, each of which was found to include some sub-criteria. We constructed a FAHP of the house selection problem, as shown in Figure 1.

The procedures of the AHP approach in the interval-valued Pythagorean fuzzy environment are presented as follows.

Step 1: The pairwise comparison matrix $\mathbf{A} = (a_{ij})_{m\times m}$ is constructed based on the linguistic evaluation of house homebuyer, which adopts IVPFNs. The linguistic terms given by Karasan et al. (2018) are shown in Table 3.

The difference matrix $\mathbf{B} = (B_{ij})_{m\times m}$ between the lower and upper values of the membership and non-membership functions is calculated using equation (5).

$$b_{ij}^{-} = \left(\mu_{ij}^{-}\right)^2 - \left(v_{ij}^{+}\right)^2,\ b_{ij}^{+} = \left(\mu_{ij}^{+}\right)^2 - \left(v_{ij}^{-}\right)^2. \tag{5}$$

The interval multiplicative matrix $\mathbf{S} = (s_{ij})_{m\times m}$ is computed using equation (6).

$$s_{ij}^{-} = \sqrt{1000^{b_{ij}^{-}}},\ s_{ij}^{+} = \sqrt{1000^{b_{ij}^{+}}}. \tag{6}$$

Figure 1. The FAHP hierarchy for the house selection problem

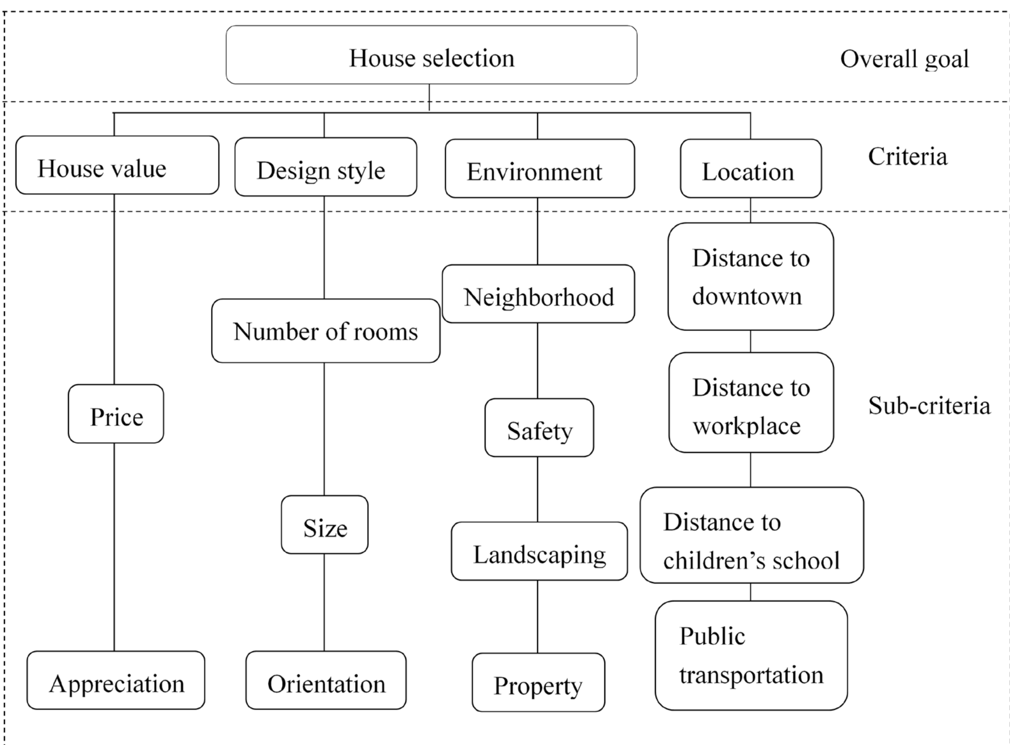


Table 3. Linguistic terms for importance weights of criteria

Linguistic Variables	IVPFNs			
	μ^-	μ^+	ν^-	ν^+
Extremely unimportant (EUI)	0.10	0.20	0.80	0.90
Very unimportant (VUI)	0.20	0.30	0.70	0.80
Unimportant (UI)	0.30	0.40	0.60	0.70
Middle (M)	0.50	0.60	0.40	0.50
Important (I)	0.60	0.70	0.30	0.40
Very important (VI)	0.70	0.80	0.20	0.30
Extremely important (EI)	0.80	0.90	0.10	0.20

The determinacy value $\ddot{\boldsymbol{A}} = (\tau_{ij})_{m\times m}$ of the houses is calculated using equation (7).

$$\tau_{ij} = 1 - \left(\left(\mu_{ij}^+\right)^2 - \left(\mu_{ij}^-\right)^2\right) - \left(\left(v_{ij}^+\right)^2 - \left(v_{ij}^-\right)^2\right). \quad (7)$$

To obtain the weight matrices of houses, the determinacy degrees are multiplied with matrix $\mathbf{S}$, and the weight matrix $\mathbf{T} = (t_{ij})_{m\times m}$ is calculated using equation (8).

$$t_{ij} = \left(\frac{s_{ij}^- + s_{ij}^+}{2}\right)\tau_{ij}. \quad (8)$$

The weights w_i of houses are normalized using equation (9).

$$w_i = \frac{\sum_{j=1}^{m} t_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{m} t_{ij}}. \quad (9)$$

FTOPSIS

The careful analysis shows that the method proposed by Zhang and Xu (2016) fails to measure IVPFS's distance in some cases, and the distance proposed by Fei and Deng (2020) considers only membership and non-membership degrees, neglecting the degree of indeterminacy. This omission leads to information loss and thus affects interpretations. Based on the discussion above, a new distance measure between IVPFNs is defined and proved to satisfy all the axioms for distance.

Definition 7: Let $P_i = ([\mu_i^-, \mu_i^+], [\nu_i^-, \nu_i^+])$ $(i = 1,2)$ be two IVPFNs, then the distance between P_1 and P_2 is defined as shown in equation (10).

$$d(P_1, P_2) = \frac{1}{2\sqrt{3}}\left(\sqrt{(\mu_1^- - \mu_2^-)^2 + (\nu_1^- - \nu_2^-)^2 + (\pi_1^- - \pi_2^-)^2}\right.$$
$$\left.+\sqrt{(\mu_1^+ - \mu_2^+)^2 + (\nu_1^+ - \nu_2^+)^2 + (\pi_1^+ - \pi_2^+)^2}\right). \tag{10}$$

Theorem 1: Let $P_i = ([\mu_i^-, \mu_i^+], [\nu_i^-, \nu_i^+])$ $(i = 1,2)$ be two IVPFNs, then
(1) $0 \le d(P_1, P_2) \le 1$;
(2) $d(P_1, P_2) = 0$ if and only if $P_1 = P_2$;
(3) $d(P_1, P_2) = d(P_2, P_1)$;
(4) If $P_1 \le P_2 \le P_3$, then $d(P_1, P_2) \le d(P_1, P_3)$ and $d(P_2, P_3) \le d(P_1, P_3)$.

Proof (1): Because $0 \le \mu_i^-, \mu_i^+, \nu_i^-, \nu_i^+ \le 1$ $(i = 1,2)$, so $0 \le \left|\mu_1^- - \mu_2^-\right|, \left|\nu_1^- - \nu_2^-\right|, \left|\pi_1^- - \pi_2^-\right|,$ $\left|\mu_1^+ - \mu_2^+\right|, \left|\nu_1^+ - \nu_2^+\right|, \left|\pi_1^+ - \pi_2^+\right| \le 1$, then $0 \le d(P_1, P_2) \le \frac{1}{2\sqrt{3}}(\sqrt{3} + \sqrt{3}) = 1$.

(2) Because $0 \le \left|\mu_1^- - \mu_2^-\right|, \left|\nu_1^- - \nu_2^-\right|, \left|\pi_1^- - \pi_2^-\right|, \left|\mu_1^+ - \mu_2^+\right|, \left|\nu_1^+ - \nu_2^+\right|, \left|\pi_1^+ - \pi_2^+\right| \le 1$, then $d(\tilde{P}_1, \tilde{P}_2) = 0$ if and only if $\left|\mu_1^- - \mu_2^-\right| = \left|\nu_1^- - \nu_2^-\right| = \left|\pi_1^- - \pi_2^-\right| = \left|\mu_1^+ - \mu_2^+\right| = \left|\nu_1^+ - \nu_2^+\right| = \left|\pi_1^+ - \pi_2^+\right| = 0$, if and only if $\mu_1^- = \mu_2^-, \nu_1^- = \nu_2^-, \mu_1^+ = \mu_2^+, \nu_1^+ = \nu_2^+$, if and only if $P_1 = P_2$.

(3) Based on definition 7, $d(P_1, P_2) = d(P_2, P_1)$.

(4) Because $P_1 \le P_2 \le P_3$, by definition 2, then $\mu_1^- \le \mu_2^- \le \mu_3^-$, $\mu_1^+ \le \mu_2^+ \le \mu_3^+$, $\nu_1^- \ge \nu_2^- \ge \nu_3^-$, $\nu_1^+ \ge \nu_2^+ \ge \nu_3^+$, so we get $(\mu_1^- - \mu_2^-)^2 \le (\mu_1^- - \mu_3^-)^2$, $(\nu_1^- - \nu_2^-)^2 \le (\nu_1^- - \nu_3^-)^2$, $(\mu_1^+ - \mu_2^+)^2 \le (\mu_1^+ - \mu_3^+)^2$, and $(\nu_1^+ - \nu_2^+)^2 \le (\nu_1^+ - \nu_3^+)^2$, then $d(P_1, P_2) \le d(P_1, P_3)$. The proof of $d(P_2, P_3) \le d(P_1, P_3)$ is similar to the above.

Now, consider the following example.

Example 1: Let $P_1 = ([0.1, 0.2], [0.6, 0.7])$, $P_2 = ([0.3, 0.4], [0.8, 0.9])$ be two IVPFNs. Distance $d_{XP}(P_1, P_2) = 0$ based on equation (2). Hence, Peng and Yang's (2016) method fails to measure the distance between P_1 and P_2. It can be calculated that $d_{LF}(P_1, P_2) = 0.3$ and $d(P_1, P_2) = 0.2976$ based on equations (3) and (10), respectively. In addition, the omission in Fei and Deng's (2020) method leads to information loss and thus affects interpretations.

We propose an improved FTOPSIS method that is based on the new distance measure between IVPFNs. It is best to obtain through the TOPSIS method a satisfactory solution that should be as close as possible to the positive ideal solution and as far as possible to the negative ideal solution. The procedures of the FTOPSIS approach in the interval-valued Pythagorean fuzzy environment are presented as follows:

The decision matrix $\mathbf{R} = (P(C_j(x_i)))_{m \times n}$ is constructed based on IVPFNs, where $P(C_j)$ $(j = 1, 2, \cdots, n)$ and $x_i (i = 1, 2, \cdots, m)$ refer to the values of the criteria and housing resources, respectively. The matrix is denoted as follows:

$$\mathbf{R} = \begin{pmatrix} P_{11} & P_{12} & \cdots & P_{1n} \\ P_{21} & P_{22} & \cdots & P_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ P_{m1} & P_{m2} & \cdots & P_{mn} \end{pmatrix},$$

where $P_{ij} = ([\mu_{ij}^-, \mu_{ij}^+], [\nu_{ij}^-, \nu_{ij}^+])$.
The positive ideal solution (PIS) and negative ideal solution (NIS) of criteria $C_j (j = 1, 2, \cdots, n)$ of the houses $x_i (i = 1, 2, \cdots, m)$ are determined using equations (11) and (12).

$$\mathbf{r}^+ = \left(r_1^+, r_2^+, \cdots, r_n^+\right), \tag{11}$$

and

$$\mathbf{r}^- = \left(r_1^-, r_2^-, \cdots, r_n^-\right), \tag{12}$$

where $r_j^+ = \left\{P_{ij} \middle| \max_i \left\{s(P_{ij})\right\}, j = 1, 2, \cdots, n\right\}$, $r_j^- = \left\{P_{ij} \middle| \min_i \left\{s(P_{ij})\right\}, j = 1, 2, \cdots, n\right\}$.
The distance between the values of criteria of the house $x_i (i = 1, 2, \cdots, m)$ and PIS are calculated using equation (13), and the distance between the values of criteria of the house $x_i (i = 1, 2, \cdots, m)$ and NIS are calculated using equation (14).

$$d_i^+ = \sum_{j=1}^{n} w_j d\left(P_{ij}, r_j^+\right), \tag{13}$$

and

$$d_i^- = \sum_{j=1}^{n} w_j d\left(P_{ij}, r_j^-\right), \tag{14}$$

where the distance d is calculated using definition 7.
In the TOPSIS method, the relative closeness RC_i of all houses $x_i \left(i = 1, 2, \cdots, m\right)$ and the optimal house is calculated using equation (15).

$$RC_i = \frac{d_i^-}{d_i^- + d_i^+}. \tag{15}$$

However, the relative closeness cannot achieve the aim that the optimal solution should have the shortest distance from the PIS and the farthest distance from the NIS simultaneously. To overcome this problem, Li (2009) proposed first the relative ratio (RR) method, which is used in this study. The relative ratio ξ_i of all houses $x_i (i = 1, 2, \cdots, m)$ is calculated using equation (16).

$$\xi_i = \frac{d_i^-}{d_{\max}^-} - \frac{d_i^+}{d_{\min}^+}, \tag{16}$$

where $d_{\min}^+ = \min_{1 \le i \le m} \left\{ d_i^+ \right\}$, $d_{\max}^- = \max_{1 \le i \le m} \left\{ d_i^- \right\}$.

Step 5: Finally, the best ranking of houses is determined. The house with the highest revised coefficient value is the best house.

The Proposed Method

In this section, we propose a method integrating the interval-valued Pythagorean FAHP and interval-valued Pythagorean FTOPSIS for the house selection problem. The details of this method for the house selection problem are shown in Figure 2. The procedure can be summarized as follows:

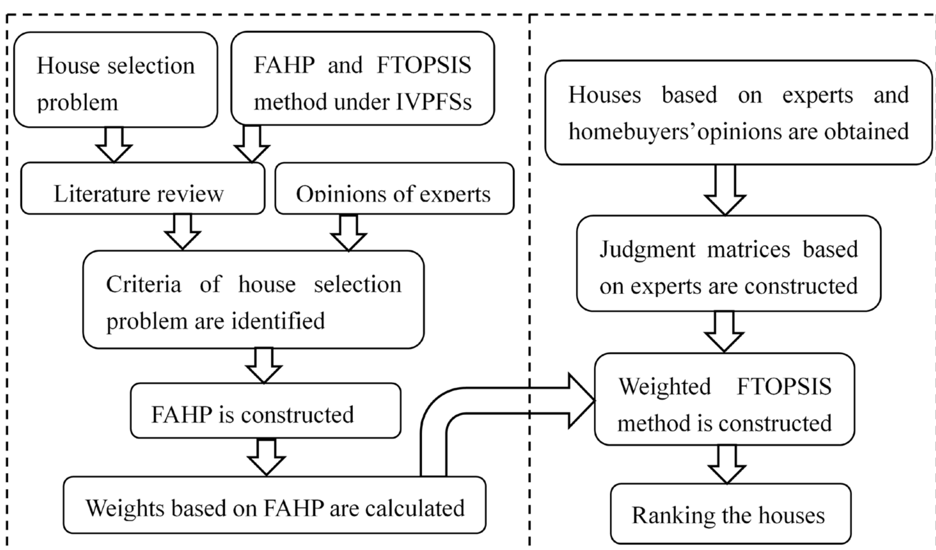
1. The criteria of house selection problem is determined based on literature review and opinions from experts.
2. The FAHP is constructed to calculate the weights of criteria based on the judgment matrices of homebuyers.
3. The rank of the houses is obtained through the weighted FTOPSIS method based on the judgment matrices of the experts.

NUMERICAL EXAMPLE

An Evaluation Case of House Selection

The aim of this section is to demonstrate the effectiveness of the proposed method through a case study. Mr. Wang and Mrs. Chen, a young couple, decide to buy a house, emphasizing their consideration of money, number of rooms, and children's education. They want to buy a house at the price of 0.9 million to 1.4 million yuan. The house should be in a neighborhood with a good school, and the house should have three rooms. Based on these conditions, the housing resources $x_i (i = 1, 2, \cdots, 8)$ obtained from

Figure 2. Steps of the method integrating FAHP and FTOPSIS for house selection



several real estate agencies are shown in Table 4. Mr. Wang and Mrs. Chen have difficulty selecting an appropriate house among this listing of houses. The proposed method can help them to make a decision because it creates a personalized ranking list according to the coefficient value of their fuzzy preferences.

First, the judgment matrix is finished by Mr. Wang and Mrs. Chen and shown in Table 5.

Second, according to Table 2 and the IVPFWG operator, the interval-valued Pythagorean fuzzy matrix is obtained and shown in Table 6.

Finally, the weights of criteria are determined according to the judgment matrix of the young couple using the FAHP method. Similarly, operations are carried out within the sub-criteria. The local weights and global weights of the criteria and the sub-criteria are calculated and shown in Table 7. In Table 7, the weights of C_{11}, C_{42} and C_{43} are 0.341, 0.105 and 0.177, respectively, indicating that young couples pay more attention to the price of the house, the distance to workplace, and the distance to children's school. The result is consistent with their requirements.

Table 4. Information of housing resources

Houses	Prices (Million Yuan)	Area (Square Meters)	Number of Rooms	Locations	Distances to Children's School (Meters)
x_1	1.1	95.86	3	Wandahuafu	1,300
x_2	1.4	88	3	Mudanxincun	530
x_3	0.9	100.72	3	Yujiangshoufu	537
x_4	1.0	91	3	Biguiyuan	583
x_5	0.9	91	3	Shuixiexincheng	828
x_6	1.4	83.17	3	Hongyanxincun	253
x_7	0.9	90	3	Kangchengshuidu	715
x_8	1.3	94.29	3	Meididadao	790

Table 5. Judgment matrix of the young couple

Criteria	C_1	C_2	C_3	C_4
C_1	M, M	UI, VUI	VUI, VUI	M, M
C_2	I, VI	M, M	VUI, M	I, VI
C_3	VI, VI	VI, M	M, M	VI, VI
C_4	M, M	UI, VUI	VUI, VUI	M, M

Table 6. Aggregated pairwise comparison matrix of main criteria

Criteria	C_1	C_2	C_3	C_4
C_1	$([0.500, 0.600], [0.400, 0.500])$	$([0.251, 0.353], [0.652, 0.742])$	$([0.200, 0.300], [0.700, 0.800])$	$([0.500, 0.600], [0.400, 0.500])$
C_2	$([0.652, 0.742], [0.251, 0.353])$	$([0.500, 0.600], [0.400, 0.500])$	$([0.299, 0.406], [0.548, 0.652])$	$([0.652, 0.742], [0.251, 0.353])$
C_3	$([0.700, 0.800], [0.200, 0.300])$	$([0.548, 0.652], [0.299, 0.406])$	$([0.500, 0.600], [0.400, 0.500])$	$([0.700, 0.800], [0.200, 0.300])$
C_4	$([0.500, 0.600], [0.400, 0.500])$	$([0.251, 0.353], [0.652, 0.742])$	$([0.200, 0.300], [0.700, 0.800])$	$([0.500, 0.600], [0.400, 0.500])$

Table 7. Local and global weights of the criteria

Criteria	Sub-Criteria	Local Weights	Global Weights w_j
$C_1(0.386)$	C_{11}	0.884	0.341
	C_{12}	0.116	0.045
$C_2(0.158)$	C_{21}	0.376	0.059
	C_{22}	0.506	0.080
	C_{23}	0.117	0.019
$C_3(0.070)$	C_{31}	0.126	0.009
	C_{32}	0.448	0.031
	C_{33}	0.244	0.016
	C_{34}	0.181	0.013
$C_4(0.386)$	C_{41}	0.109	0.042
	C_{42}	0.271	0.105
	C_{43}	0.459	0.177
	C_{44}	0.161	0.062

In the second stage, the judgment matrix of houses $x_i(i = 1,2,\cdots,8)$ are finished by three experts. According to Table 2 and IVPFWG operator, the interval-valued Pythagorean fuzzy matrix is obtained. Positive and negative ideal solutions are then calculated using equations (11) and (12) and shown in Table 8.

The distances between houses $x_i(i = 1,2,\cdots,8)$ and the positive and negative ideal solution are calculated respectively using equations (13) and (14) and shown in Table 9. Closeness coefficients of each house $x_i(i = 1,2,\cdots,8)$ are then calculated using equation (16), and their rankings are obtained and shown in Table 9. The priority order of the houses is $x_8 \succ x_4 \succ x_1 \succ x_2 \succ x_6 \succ x_7 \succ x_3 \succ x_5$. This order suggests that the most appropriate house for the young couple is Meididadao, which is in downtown and close to children's school.

SENSITIVITY ANALYSIS

Sensitivity analysis is performed to test the results of the criteria weights. In fact, because different homebuyers have different requirements, the weights of criteria obtained by fuzzy AHP are not the

Table 8. Positive and negative ideal solutions of criteria

Criteria	Positive Ideal Solutions	Negative Ideal Solutions
C_{11}	$([0.695, 0.796], [0.182, 0.289])$	$([0.271, 0.378], [0.581, 0.684])$
C_{12}	$([0.383, 0.458], [0.438, 0.552])$	$([0.665, 0.765], [0.229, 0.331])$
C_{21}	$([0.477, 0.581], [0.378, 0.482])$	$([0.626, 0.727], [0.252, 0.256])$
C_{22}	$([0.695, 0.796], [0.182, 0.289])$	$([0.229, 0.331], [0.665, 0.765])$
C_{23}	$([0.732, 0.832], [0.159, 0.262])$	$([0.331, 0.483], [0.502, 0.608])$
C_{31}	$([0.565, 0.665], [0.331, 0.431])$	$([0.500, 0.600], [0.400, 0.500])$
C_{32}	$([0.765, 0.865], [0.126, 0.229])$	$([0.229, 0.331], [0.665, 0.765])$
C_{33}	$([0.765, 0.865], [0.126, 0.229])$	$([0.293, 0.416], [0.504, 0.608])$
C_{34}	$([0.765, 0.865], [0.126, 0.229])$	$([0.289, 0.398], [0.528, 0.635])$
C_{41}	$([0.800, 0.900], [0.100, 0.200])$	$([0.500, 0.600], [0.400, 0.500])$
C_{42}	$([0.665, 0.765], [0.229, 0.331])$	$([0.145, 0.252], [0.727, 0.828])$
C_{43}	$([0.732, 0.832], [0.159, 0.262])$	$([0.449, 0.552], [0.416, 0.520])$
C_{44}	$([0.695, 0.796], [0.182, 0.289])$	$([0.331, 0.416], [0.552, 0.655])$

Table 9. Distances house x_i and PIS and NIS, their closeness coefficients, and ranking

Houses	d_i^+	d_i^-	ξ_i	Ranking
x_1	0.101	0.257	−0.221	3
x_2	0.120	0.229	−0.556	4
x_3	0.228	0.127	−2.248	7
x_4	0.088	0.255	−0.064	2
x_5	0.241	0.118	−2.439	8
x_6	0.187	0.170	−1.589	5
x_7	0.192	0.171	−1.642	6
x_8	0.083	0.255	−0.009	1

same. Therefore, the results obtained are varied. Take the comparison of the second couple and the third couple as an example. Their weights are obtained using the proposed method based on their judgment matrices and shown in Table 10.

Table 11 shows that the appropriate house for the second couple is Biguiyuan with excellent property and landscape, and the appropriate one for the third couple is Mudanxincun, which is closer to a school and downtown. The rank of the houses is changed for two different homebuyers; thus, the ranking results are sensitive for the weights.

COMPARATIVE ANALYSIS

We compared Ahmet's method (Ahmet, 2021) and the MCDM model proposed in this study to demonstrate the latter's effectiveness. We changed only the distance measure, using Ahmet's distance to replace the distance measure of the proposed method. Other processes remain unchanged. The results obtained are listed in Table 11. The appropriate house for second couple is x_4 (Biguiyuan) according to Ahmet's method. The result remains the same. Hence, the result of the proposed method is stable and effective.

CONCLUSION

There are many factors to be taken consideration when buying an appropriate house, and these factors create an MCDM problem. The aim of this paper is to help homebuyers to choose an appropriate house using the proposed method that integrates the interval-valued Pythagorean FAHP and interval-valued Pythagorean FTOPSIS. In this approach, first, the evaluation criteria were determined through the consultation of experts and literature review. Second, the weights of criteria were calculated using the interval-valued Pythagorean FAHP method according to the judgment matrices of homebuyers. Third,

Table 10. Local and global weights of the criteria

Sub-Criteria	Weights of Second Couple	Weights of Third Couple
C_{11}	0.038	0.069
C_{12}	0.126	0.359
C_{21}	0.137	0.035
C_{22}	0.126	0.032
C_{23}	0.096	0.021
C_{31}	0.103	0.020
C_{32}	0.078	0.037
C_{33}	0.072	0.012
C_{34}	0.091	0.016
C_{41}	0.015	0.085
C_{42}	0.018	0.035
C_{43}	0.041	0.204
C_{44}	0.059	0.075

Table 11. Sensitivity analysis and comparative analysis

Houses	Second Couple		Third Couple		Rank of Second Couple Based on Ahmet's Method	
	ξ_i	Ranking	ξ_i	Ranking	ξ_i	Ranking
x_1	-1.640	4	-0.689	5	-2.433	4
x_2	-1.880	6	0	1	-2.982	6
x_3	-2.014	8	-1.751	8	-3.679	8
x_4	0	1	-0.238	3	0	1
x_5	-1.755	5	-1.018	7	-2.48	5
x_6	-1.998	7	-0.417	4	-3.081	7
x_7	-1.576	3	-0.745	6	-0.706	3
x_8	-0.831	2	-0.168	2	-0.700	2

a new distance measure between IVPFNs was proposed, and the ranking of houses was evaluated using the interval-valued Pythagorean FTOPSIS method according the new distance. Finally, a case study was executed to verify the feasibility of the proposed approach. Our case study results pointed out that the weights of criteria obtained by using FAHP are not the same according to the judgment matrices of different homebuyers, and thus, the ranking results change. The limitation of this study is that various tools, such as interval-valued fuzzy Pythagorean entropy method and MCGP, can be applied for the house selection problem.

For further research, we suggest that entropy-FTOPSIS should be compared with interval-valued Pythagorean FAHP-FTOPSIS. Alternatively, other types of fuzzy numbers, such as cooperative games (Ye & Li, 2021), and p, q-QOFSs (Seikh & Mandal, 2022a; Seikh & Mandal, 2022b), Fermatean fuzzy number (Seikh & Mandal, 2023), interval-valued spherical fuzzy (IVSF) sets (Mandal & Seikh, 2023), can be employed instead of IVPFSs in the developed FAHP-FTOPSIS method.

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REFERENCES

Alrasheedi, M., Mardani, A., Mishra, A. R., Rani, P., & Loganathan, N. (2022). An extended framework to evaluate sustainable suppliers in manufacturing companies using a new Pythagorean fuzzy entropy-SWARA-WASPAS decision-making approach. *Journal of Enterprise Information Management*, *35*(2), 333–357. doi:10.1108/JEIM-07-2020-0263

Atanassov, K. T. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, *20*(1), 87–96. doi:10.1016/S0165-0114(86)80034-3

Atanassov, K. T., & Gargov, G. (1989). Interval valued intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, *31*(3), 343–349. doi:10.1016/0165-0114(89)90205-4

Calik, A. (2021). A novel Pythagorean fuzzy AHP and fuzzy TOPSIS methodology for green supplier selection in the Industry 4.0 era. *Soft Computing*, *25*(3), 2253–2265. doi:10.1007/s00500-020-05294-9

Chen, X.-G., Yu, G.-F., Wu, J., & Yang, Y. (2020). A minimum trust discount coefficient model for incomplete information in group decision making with intuitionistic fuzzy soft set. *International Journal of Fuzzy Systems*, *22*(6), 2025–2040. doi:10.1007/s40815-020-00811-2

Dwivedi, G., Srivastava, R. K., & Srivastava, S. K. (2018). A generalized fuzzy TOPSIS with improved closeness coefficient. *Expert Systems with Applications*, *96*, 185–195. doi:10.1016/j.eswa.2017.11.051

Ejegwa, P. A., Feng, Y., Tang, S., Agbetayo, J. M., & Dai, X. (2023). New Pythagorean fuzzy-based distance operators and their applications in pattern classification and disease diagnostic analysis. *Neural Computing & Applications*, *35*(14), 10083–10095. doi:10.1007/s00521-022-07679-3

Fei, L., & Deng, Y. (2020). Multi-criteria decision making in Pythagorean fuzzy environment. *Applied Intelligence*, *50*(2), 537–561. doi:10.1007/s10489-019-01532-2

Fu, Y.-K. (2019). An integrated approach to catering supplier selection using AHP-ARAS-MCGP methodology. *Journal of Air Transport Management*, *75*, 164–169. doi:10.1016/j.jairtraman.2019.01.011

Habib, A., Akram, M., & Kahraman, C. (2022). Minimum spanning tree hierarchical clustering algorithm: A new Pythagorean fuzzy similarity measure for the analysis of functional brain networks. *Expert Systems with Applications*, *201*, 1–19, 117016. doi:10.1016/j.eswa.2022.117016

Habib, S., Akram, M., & Al-Shamiri, M. M. A. (2023). Comparative analysis of Pythagorean MCDM methods for the risk assessment of childhood cancer. *Computer Modeling in Engineering & Sciences*, *135*(3), 2585–2615. doi:10.32604/cmes.2023.024551

Han, Q., Li, W., Lu, Y., Zheng, M., Quan, W., & Song, Y. (2020). TOPSIS method based on novel entropy and distance measure for linguistic Pythagorean fuzzy sets with their application in multiple attribute decision making. *IEEE Access : Practical Innovations, Open Solutions*, *8*, 14401–14412. doi:10.1109/ACCESS.2019.2963261

Ho, H.-P., Chang, C.-T., & Tan, K. H. (2021). A hybrid fuzzy goal programming for smart phones and rate plan selection. *International Journal of Fuzzy Systems*, *23*(6), 1613–1632. doi:10.1007/s40815-020-00999-3

Ho, L.-H., Lin, Y.-L., & Chen, T.-Y. (2020). A Pearson-like correlation-based TOPSIS method with interval-valued Pythagorean fuzzy uncertainty and its application to multiple criteria decision analysis of stroke rehabilitation treatments. *Neural Computing & Applications*, *32*(12), 8265–8295. doi:10.1007/s00521-019-04304-8

Hwang, C.-L., & Yoon, K. (1981). *Multiple attribute decision making: Methods and applications, a state-of-the-art survey*. Springer. doi:10.1007/978-3-642-48318-9

Karasan, A., Ilbahar, E., Cebi, S., & Kahraman, C. (2018). A new risk assessment approach: Safety and Critical Effect Analysis (SCEA) and its extension with Pythagorean fuzzy sets. *Safety Science*, *108*, 173–187. doi:10.1016/j.ssci.2018.04.031

Li, D.-F. (2004). Some measures of dissimilarity in intuitionistic fuzzy structures. *Journal of Computer and System Sciences*, *68*(1), 115–122. doi:10.1016/j.jcss.2003.07.006

Li, D.-F. (2009). Relative ratio method for multiple attribute decision making problems. *International Journal of Information Technology & Decision Making*, *8*(2), 289–311. doi:10.1142/S0219622009003405

Li, D.-F. (2010). TOPSIS-based nonlinear-programming methodology for multiattribute decision making with interval-valued intuitionistic fuzzy sets. *IEEE Transactions on Fuzzy Systems*, *18*(2), 299–311. doi:10.1109/TFUZZ.2010.2041009

Li, D.-F. (2011). Closeness coefficient based nonlinear programming method for interval-valued intuitionistic fuzzy multiattribute decision making with incomplete preference information. *Applied Soft Computing*, *11*(4), 3402–3418. doi:10.1016/j.asoc.2011.01.011

Li, D.-F., & Cheng, C.-T. (2002). New similarity measures of intuitionistic fuzzy sets and application to pattern recognitions. *Pattern Recognition Letters*, *23*(1–3), 221–225. doi:10.1016/S0167-8655(01)00110-6

Li, D.-F., & Liu, J.-C. (2015). A parameterized nonlinear programming approach to solve matrix games with payoffs of I-fuzzy numbers. *IEEE Transactions on Fuzzy Systems*, *23*(4), 885–896. doi:10.1109/TFUZZ.2014.2333065

Li, D.-F., & Nan, J.-X. (2011). Extension of the TOPSIS for multi-attribute group decision making under Atanassov IFS environments. *International Journal of Fuzzy System Applications*, *1*(4), 47–61. doi:10.4018/ijfsa.2011100104

Li, D.-F., & Sun, T. (2007). Fuzzy LINMAP method for multiattribute group decision making with linguistic variables and incomplete information. *International Journal of Uncertainty, Fuzziness and Knowledge-based Systems*, *15*(02), 153–173. doi:10.1142/S0218488507004509

Li, D.-F., & Wan, S.-P. (2017). Minimum weighted Minkowski distance power models for intuitionistic fuzzy MADM with incomplete weight information. *International Journal of Information Technology & Decision Making*, *16*(05), 1387–1408. doi:10.1142/S0219622014500321

Liang, K.-R., Li, D.-F., Li, K. W., & Liu, J.-C. (2023). An interval noncooperative-cooperative biform game model based on weighted equal contribution division values. *Information Sciences*, *619*, 172–192. doi:10.1016/j.ins.2022.11.016

Mandal, U., & Seikh, M. R. (2022). Interval-valued Fermatean fuzzy TOPSIS method and its application to sustainable development program. *Congress on Intelligent Systems: Proceedings of CIS 2021* Volume 2*, Lecture Notes on Data Engineering and Communications Technologies Series, 111*, 783–796. doi:10.1007/978-981-16-9113-3_57

Mandal, U., & Seikh, M. R. (2023). Interval-valued spherical fuzzy MABAC method based on Dombi aggregation operators with unknown attribute weights to select plastic waste management process. *Applied Soft Computing*, *145*, 110516. Advance online publication. doi:10.1016/j.asoc.2023.110516

Peng, X., & Yang, Y. (2016). Fundamental properties of interval-valued Pythagorean fuzzy aggregation operators. *International Journal of Intelligent Systems*, *31*(5), 444–487. doi:10.1002/int.21790

Prabhu, S. R., & Ilangkumaran, M. (2019). Selection of 3D printer based on FAHP integrated with GRA-TOPSIS. *International Journal of Materials & Product Technology*, *58*(2–3), 155–177. doi:10.1504/IJMPT.2019.097667

Rahim, M., Garg, H., Amin, F., Perez-Dominguez, L., & Alkhayyat, A. (2023). Improved cosine similarity and distance measures-based TOPSIS method for cubic Fermatean fuzzy sets. *Alexandria Engineering Journal*, *73*, 309–319. doi:10.1016/j.aej.2023.04.057

Seikh, M. R., & Mandal, U. (2022a). Multiple attribute decision-making based on 3, 4-quasirung fuzzy sets. *Granular Computing*, *7*(4), 965–978. doi:10.1007/s41066-021-00308-9

Seikh, M. R., & Mandal, U. (2022b). Multiple attribute group decision making based on quasirung orthopair fuzzy sets: Application to electric vehicle charging station site selection problem. *Engineering Applications of Artificial Intelligence*, *115*, 105299. Advance online publication. doi:10.1016/j.engappai.2022.105299

Seikh, M. R., & Mandal, U. (2023). Interval-valued Fermatean fuzzy Dombi aggregation operators and SWARA based PROMETHEE II method to bio-medical waste management. *Expert Systems with Applications*, *226*, 120082. doi:10.1016/j.eswa.2023.120082

Sun, Z.-H., Pan, L., Wang, Y.-Y., & Zhang, D.-H. (2013). The purchase house choice research based on the analytic hierarchy process (AHP). *Proceedings* of *the 19th International Conference on Industrial Engineering and Engineering Management,* 897–902. doi:10.1007/978-3-642-38391-5_95

Szmidt, E., & Kacprzyk, J. (2000). Distances between intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, *114*(3), 505–518. doi:10.1016/S0165-0114(98)00244-9

Wang, C. (2013). Family house-purchase decision model based on analytic hierarchy process. *Applied Mechanics and Materials*, *423–426*, 2973–2976. . doi:10.4028/www.scientific.net/AMM.423-426.2973

Yager, R. R. (2013). Pythagorean fuzzy subsets. In *Proceedings of the 2013 Joint IFSA World Congress & NAFIPS Annual Meeting (IFSA/NAFIPS)* (pp. 57–61). IEEE. doi:10.1109/IFSA-NAFIPS.2013.6608375

Yager, R. R. (2014). Pythagorean membership grades in multicriteria decision making. *IEEE Transactions on Fuzzy Systems*, *22*(4), 958–965. doi:10.1109/TFUZZ.2013.2278989

Ye, Y.-F., & Li, D.-F. (2021). A direct approach to compute triangular fuzzy Banzhaf values of cooperative games with coalitions' values represented by triangular fuzzy numbers. *IEEE Transactions on Fuzzy Systems*, *29*(6), 1567–1575. doi:10.1109/TFUZZ.2020.2981006

Yu, G.-F., Li, D.-F., & Fei, W. (2018). A novel method for heterogeneous multi-attribute group decision making with preference deviation. *Computers & Industrial Engineering*, *124*, 58–64. doi:10.1016/j.cie.2018.07.013

Yu, G.-F., Li, D.-F., Liang, D.-C., & Li, G.-X. (2021). An intuitionistic fuzzy multi-objective goal programming approach to portfolio selection. *International Journal of Information Technology & Decision Making*, *20*(5), 1477–1497. doi:10.1142/S0219622021500395

Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, *8*(3), 338–353. doi:10.1016/S0019-9958(65)90241-X

Zhang, X. (2016). Multicriteria Pythagorean fuzzy decision analysis: A hierarchical QUALIFLEX approach with the closeness index-based ranking methods. *Information Sciences*, *330*, 104–124. doi:10.1016/j.ins.2015.10.012

Zhang, X., & Xu, Z. (2014). Extension of TOPSIS to multiple criteria decision making with Pythagorean fuzzy sets. *International Journal of Intelligent Systems*, *29*(12), 1061–1078. doi:10.1002/int.21676

Zulqarnain, R. M., Siddique, I., Ali, R., Jarad, F., & Iampan, A. (2023). Einstein weighted geometric operator for Pythagorean Fuzzy hypersoft with its application in material selection. *Computer Modeling in Engineering & Sciences*, *135*(3), 2557–2583. doi:10.32604/cmes.2023.023040

Zulqarnain, R. M., Siddique, I., Iampan, A., & Baleanu, D. (2022). Aggregation operators for interval-valued Pythagorean fuzzy soft set with their application to solve multi-attribute group decision making problem. *Computer Modeling in Engineering & Sciences*, *131*(3), 1717–1750. doi:10.32604/cmes.2022.019408