International Journal of Fuzzy System Applications
Volume 12 • Issue 1

A Multi-Step Interaction Opinion Dynamics Group Decision-Making Method and Its Application in Art Evaluation

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ABSTRACT

Due to the development of intelligent decision-making, social network group decision making (SNGDM) has become increasingly valued. Self-persistence is a significant topic in SNGDM problems, while it is ignored in most existing research. Besides, existing opinion evolution models often ignore high-order interactions because they assume individuals only communicate with their friends or neighbors. With these issues in mind, the authors propose a multi-step interaction opinion dynamics model to manage the consensus in SNGDM problem with self-persistence evolution. Given the decision makers' social network information, the centrality degree, interaction strength, and high-order interactions are combined to construct a social influence network. Inspired by the social influence model, the authors develop an opinion dynamic consensus model which also describes the evolution of self-persistence in a group of decision makers. Finally, an example and detailed simulation experiment are presented to demonstrate the efficiency of the proposed consensus model.

KEYWORDS

Consensus Reaching Process, Group Decision Making, Multi-Step Interaction, Social Network

1 INTRODUCTION

Group decision making (GDM) can be regarded as a powerful tool to select the most desirable alternatives in the situation where a group of decision makers (DMs) participate to achieve a common solution (Bezdek, 1978) and has been applied in various fields (García-Zamora, 2022; Li, Liu & Li, 2021, 2022; Yu & Li 2022; Yu, Fei & Li 2019). The development of the economy and web technologies pushes GDM problem to more complex situations. Consequently, social network group decision making (SNGDM) problem is becoming an important topic and gaining popularity in the decision making field (Gong, Wang, Guo, Gong & Wei, 2020; Wang, Liang & Li, 2022; Liao, & Liu, 2017; Li, Rodríguez & Wei, 2021). But this situation also brings new challenges, such as effective social network information supervision and difficulties of reaching consensus, etc. Reaching consensus is very important in GDM problems, because it can increase the efficiency in the implementation of the obtained decision (Palomares, Estrella, Martínez & Herrera, 2014; Quesada, Palomares & Martínez, 2015; Rodríguez, Labella, Tré & Martínez, 2018; Yu, Li & Fei, 2018; Zuo, Li & Yu, 2020). It is usually very difficult to achieve a common solution accepted by all DMs.

Studies on consensus model of SNGDM are still in the inceptive stage with only a small amount of researches (Ding, Wang, Shang & Herrera, 2019; Liu, Zhou, Ding & Palomares, 2019; Liang, Guo &

DOI: 10.4018/IJFSA.320485 *Corresponding Author

Liu, 2022, Zuo, Li & Yu 2020). Notably, there are mainly two types of model based on social network (Dong, Zhan, Kou, Fujita, Chiclana & Herrera-Viedma, 2018), the one is based on trust relationship (Wu, Chiclana, Fujita, & Herrera-Viedma, 2017; Zhang, Wang, Dong, Chiclana & Herrera-Viedma, 2022; Wu, Zhang, Liu & Cao, 2019) and the other one is based on opinion evolution (Capuano, Chiclana, Fujta, Herrera-Viedma & Loia, 2018; Dong, Ding, Martínez & Herrera, 2017). As for the former one, for instance, Wu, Chang, Cao, & Liang (2019) and Wu, Chiclana, Fujita & Herrera-Viedma (2017) studied the trust propagation approaches and developed some consensus models based on the trust relationship between DMs. Ding, Wang, Shang & Herrera (2019) developed a social network analysis-based conflict relationship investigation process and a conflict degree-based consensus reaching process (CRP) for GDM problems. Wu, Zhang, Liu & Cao (2019) utilized a network partition algorithm based on trust relationship to reduce the complexity of GDM problem and then the weights of independent sub-networks and their individual members are computed by their trust values. Liu, Zhou, Ding & Palomares (2019) proposed a model considering both the preference inconformity and the relationship disharmony to detect and eliminate the conflict for GDM in social network context. Zhang, Iván, Dong & Wang (2019) defined some principles to detect the DMs' non-cooperative behaviors, and then social network analysis was used for the non-cooperative behavior management.

Opinion evolution is capable of playing a key role in SNGDM. Several scholars have recently focused on GDM based on opinion evolution. For instance, Pérez, Mata, Chiclana, Kou & Herrera-Viedma (2016) applied the FJ model to simulate the DMs' discussion process and study the CRP. Dong, Ding, Martínez & Herrera (2017) studied the DeGroot model based on leadership to support the individuals' CRP. Li & Wei (2020) proposed a two stage dynamic influence model for handling the consensus reaching process in GDM with incomplete information, in which the social network is assumed to be static during the decision process. Ureña, Chiclana, Melançon & Herrera-Viedma (2019) proposed a consensus model based on bounded confidence opinion dynamic mechanism. Based on DeGroot model, Zhou, Wu, Altalhi & Herrera (2020) proposed a two-step communication opinion dynamics model, and provided three opinion control strategies for controlling the opinion formation process. Yang, Wang, Ding, Xu & Li (2022) proposed an opinion management-based consensus model to investigate the evolution process of opinions of DMs during the CRP.

The above research on the opinion dynamic consensus model have enriched the theory and application of SNGDM. The self-persistence is first introduced by Friedkin and Johnsen to represent the degree of an agent's adherence to their initial opinion (Friedkin & Johnsen, 1999). Analyzing self-persistence to assist the CRPs in SNGDM, however, has rarely been previously considered. The real SNGDM cases involve not only the mathematical formulation of the social network analysis but also the DMs' self-persistence psychological behaviors. Although there have been significant developments for consensus model based on opinion evolution, they still suffer from some limitations.

(1) Most existing opinion evolution models ignore high-order interactions because they assume that individuals only communicate with their friends or neighbors and do not consider the opinions of others. Nowadays, this may no longer be true, because online social networks, such as Facebook and WeChat, facilitate the discussion and communication among agents. Therefore, it is necessary to develop an opinion dynamics model that considers the high-order interactions between DMs.
(2) The self-confidence/persistence is regarded as a prominent indicator of influence and plays a key role in opinion evolution. In general, the confidence degrees of DMs are given in advance and are assumed to be unchanged during the CRP (Dong, Ding, Martínez & Herrera 2017; Gupta, 2018; Liu, Dong, Chiclana, Cabrerizo & E. Herrera-Viedma, 2016; Zhou, Wu, Altalhi & Herrera 2020). However, in real decision making cases this may no longer be true. Taking the psychological concept of reflected appraisal into account (Cooley, 1902), where actors' self-appraisals are influenced by the appraisals provided by other actors. And during the discussion process, the DMs' self-persistence degrees should be modified along the discussion process.

Based on the above analysis, we will try to solve the open problem: how to assist the DMs to achieve a consensus by considering the high-order interactions and self-persistence evolution in social network. This paper introduces an opinion dynamics model in which the DMs interact with other DMs using a multi-step interaction. In the proposed model, the self-persistence and opinion evolve simultaneously. The main novelties of this paper are enumerated as follows:

(1) In the proposed framework we proposed two measures for each DM. The first one is the global reputation based on all the DMs' the centrality degrees and interaction strength. The second one is the pairwise social influence which is a measure that indicates the level of influence or trust between two DMs by considering high-order interactions. Based on these two measures, we construct a social influence network and corresponding influence matrix.
(2) The self-confidence degrees of DMs play a prominent role in opinion evolution. Due to the fact that the self-confidence is always difficult to measure in our lives, we allow DMs to voice their mutual evaluations. Then we apply social influence network theory to analyze the changing process of self-persistence and analyze the influence of self-persistence on opinion dynamic in SNGDM problem.

In what follows, some basic knowledge, such as graph and the social influence model are introduced in Section 2. Section 3 introduces a multi-step interaction opinion dynamics model to manage the consensus in SNGDM problem with self-persistence evolution in detail. An application example and simulation experiment are utilized to illustrate the validity of our model in Section 4. Finally, some conclusions are pointed out in Section 5.

2 PRELIMINARIES

In this section, we present some preliminary knowledge which is helpful to understand our methodology, regarding graph and social influence model.

2.1 Graph

In GDM problems, DMs are not independent individuals, generally, they often have some relationships with each other. We introduce some basic concepts about graph which is helpful to study the relationships among DMs.

Definition 2.1. *(Bondy & Murty, 1976) A directed graph is defined by* $G\left(V,E\right)$*, where* $V=\left\{v_1,v_2,\cdots,v_m\right\}$ *is a set of nodes,* E *represents the set of ordered pairs, and the elements of* E *are called edges. We assume that the sets* V *and* E *are finite and* V *is nonempty.*

There are two types of graphs: the directed graph and the undirected graph. In the undirected graph, if there exists edge $\left(v_i,v_j\right)\in E$ then there exists $\left(v_j,v_i\right)\in E$. But in a directed graph, $\left(v_i,v_j\right)\in E$ does not imply that $\left(v_j,v_i\right)\in E$.

Definition 2.2. *(Bondy & Murty, 1976) In the directed graph* $G\left(V,E\right)$*, a sequence of edges* $\left(v_i,v_{k_1}\right),\left(v_{k_1},v_{k_2}\right),\cdots,\left(v_{k_s},v_j\right)$ *is called a directed path from* v_i *to* v_j*, denoted as* $v_i\rightarrow v_j$*. The number of the edges in the directed path is called the length of the path and is denoted as* $len\left(v_i\rightarrow v_j\right)$*.*

Definition 2.3. *(Bondy & Murty, 1976) An adjacent matrix* $S=\left(s_{ij}\right)_{m\times m}$ *is defined as the zero-one matrix, where* $s_{ij}=1$ *indicates that there is an edge from* v_i *to* v_j*;* $s_{ij}=0$ *indicates that there is no edge from* v_i *to* v_j*.*

Definition 2.4. (Wasserman & Faust, 1994) *Let* $V = \left\{ v_1, v_2, \cdots, v_m \right\}$ *be a set of nodes and* $S = (s_{ij})_{m\times m}$ *be the adjacent matrix corresponding to a directed graph* $G\left(V, E\right)$, *then*

(1) $\deg^+(v_r) = \sum_{l=1,l\neq r}^{m} s_{rl}$ *is called the out-degree centrality of the node* v_r.

(2) $\deg^-(v_r) = \sum_{l=1,l\neq r}^{m} s_{lr}$ *is called the in-degree centrality of the node* v_r.

An adjacent matrix is used to describe whether there is a connection between nodes and it is a binary relation. However, in some situations, we not only want to know whether the connection between nodes exists or not, but also the connection strength between them. So a weighted adjacent matrix is proposed for this purpose.

Definition 2.5. *(Wasserman & Faust, 1994) For a directed graph* $G\left(V, E\right)$ *with* $V = \left\{ v_1, v_2, \cdots, v_m \right\}$, *we use the membership function* $\mu_R : V \times V \to \left[0,1\right]$, $s_{ij} = \mu_R\left(v_i, v_j\right)$ *to define the weighted adjacent matrix* $S = (s_{ij})_{m\times m}$, *where* s_{ij} *means the social connection strength from* v_i *to* v_j.

2.2 Social Influence Model

Actors' emotions and thoughts are liable to change resulting from actors' interaction. In real GDM situations, actors' opinions will evolve due to the social influence. Specifically, let $\left\{ e_1, e_2, \cdots, e_m \right\}$ be a set of actors and $y^{(0)} = (y_1^{(0)}, y_2^{(0)}, \cdots, y_m^{(0)})^T$ be a vector representing the actors' initial opinions on an issue. A matrix $W = (\omega_{ij})_{m\times m}$ is utilized to depict the influence relationship among the actors, where ω_{ij} represents the direct influence of actor e_j to e_i and satisfies $\omega_{ij} \geq 0$ and $\sum_{j=1}^{m} \omega_{ij} = 1$ for all $i, j \in \left\{1, 2, \cdots, m\right\}$. Let $y^{(t)} = (y_1^{(t)}, y_2^{(t)}, \cdots, y_m^{(t)})^T$ be the vector of actors' opinions of time t. Different social influence models (Degroot, 1974; Krause, 2000; Dittmer, 2001; Jia, Friedkin, & Bullo, 2015; Pérez, Mata, Chiclana, Kou, & Herrera-Viedma 2016; Basu & Sly, 2017) were proposed to model the influence among the actors. The DeGroot model is considered as a seminal model (Degroot, 1974) and can be described as

$$y^{(t)} = (y_1^{(t)}, y_2^{(t)}, \cdots, y_m^{(t)})^T = Wy^{(t-1)}. \tag{1}$$

Definition 2.6. *(Dong, Ding, Martínez, & Herrera, 2017) All actors can reach a consensus if for any* $y^{(0)} = (y_1^{(0)}, y_2^{(0)}, \cdots, y_m^{(0)})^T \in \mathbb{R}^m$ *there exists* $c \in R$ *such that* $\lim_{t\to\infty} y_i^{(t)} = c$, $i = 1, 2, \cdots, m$.

In the DeGroot model, the consensus condition has been proposed as shown in Lemmas 2.1.

Lemma 2.1. (Dong, Ding, Martínez, & Herrera, 2017) In DeGroot model, all actors can reach a consensus if and only if there exists at least one globally reachable node.

The FJ model (Friedkin & Johnsen, 1999) is a variation of the classical DeGroot model. The opinion evolution of actor d_i can be described by the following expressions:

$$y_i^{(t+1)} = \left(1 - \omega_{ii}\right) \sum_{j=1}^{n} \omega_{ij} y_j^{(t)} + \omega_{ii} y_i^{(0)}. \tag{2}$$

From Eq. (2), the opinion evolution process can be compactly written as

$$y^{(t+1)} = AWy^{(t)} + \left(I - A\right)y^{(0)}, t = 0,1,\cdots \tag{3}$$

Where A is an $m \times m$ diagonal matrix, $a_{ii} = 1 - \omega_{ii}$ and I is the $m \times m$ identity matrix.

Assume the influence process can reach an equilibrium, then $\lim_{t \to \infty} y^{(t)} = y^{(\infty)}$ and the original equation becomes

$$y^{(\infty)} = Vy^{(0)}, \tag{4}$$

where V is called control matrix and can be calculated by

$$V = (I - AW)^{-1}\left(I - A\right). \tag{5}$$

3 MODELING OPINION DYNAMICS IN A SOCIAL INFLUENCE NETWORK

In this paper, we focus on a GDM problem in social network context, where there are a lot of interactions among individuals. The social network can be described by a directed graph $G\left(D, E\right)$, where $D = \left\{d_1, d_2, \cdots, d_m\right\}$ is a set of m DMs and E represents the set of edges, which represents the relationship between them. Let $x = \left\{x_1, x_2, \cdots, x_n\right\}$ be the considered alternatives, $C = \left\{c_1, c_2, \cdots, c_p\right\}$ be a predefined set of attributes and the weight vector of these attributes be denoted as $\omega = \left\{\omega_1, \omega_2, \cdots, \omega_p\right\}$, where $\sum_{k=1}^{p} \omega_k = 1$. $A_k\left(0\right) = (a_{ij}^k\left(0\right))_{n \times p}$ is the decision matrix provided by d_k, where a_{ij}^k represents the attribute values for x_i with respect to attribute c_j. The DMs provide their initial mutual evaluation matrix $Y\left(0\right)$ and initial opinions $A_k\left(0\right)$.

By consideration of the self-persistence and opinion evolution, we aim to obtain the consensus opinion. The main notations used in this paper are as follows.

$Y\left(t\right) = (y_{ij}\left(t\right))_{m \times m}$ is the mutual evaluation matrix of time t, where $y_{ij}\left(t\right)$ is the DM d_i's evaluation on DM d_j of time t.

$A\left(t\right) = (a_{ij}\left(t\right))_{n \times p}$ is the opinion of the DMs at time t, where $a_{ij}\left(t\right)$ is the DM d_i's opinion on alternative x_j with respect to attribute c_j of time t.

$A^c = (a_{ij}^c)_{n \times p}$ is the final consensus opinions for all the alternatives, where a_{ij}^c is the consensus opinion corresponding to the alternative x_j with respect to attribute c_j.

3.1 Influence Network Considering Multi-Step Communication

Multi-step communication or interaction indicates that the agents trust not only their own neighbors but also the neighbors of their neighbors. In other words, the agents consider the opinions of those they can reach in several steps, which is referred to as multi-step communication.

Definition 3.1. *For a directed graph $G\left(D, E\right)$, let $U_k\left(d_i\right)$ be the k -step communication set in which each node can be reached by d_i through a path with length k, i.e., $U_k\left(d_i\right) = \{d_j \mid len\left(d_i \to d_j\right) = k\}$, and must satisfy $d_i \notin U_k\left(d_i\right)$ and $U_l\left(d_i\right) \cap U_k\left(d_i\right) = \varnothing$ for $\forall l \neq k$.*

Let $U\left(d_i\right) = U_1\left(d_i\right) \cup U_2\left(d_i\right) \cup \cdots U_k\left(d_i\right)$ be DM d_i's communication set. In the opinion dynamic model, DMs modify their opinions by referring to the opinions of their communication sets at each step. How should d_i assign weights to the DMs in $U_k\left(d_i\right)$? A novel method based on node centrality and interaction strength will be developed to address this problem. The higher the local centrality, the more central the node is in the social network. Generally, there are two kinds of centrality measurements for a DM d_i, namely in-centrality $\frac{\deg_i^-}{n-1}$ and out-centrality $\frac{\deg_i^+}{n-1}$ (Zhou, Wu, Altalhi, & Herrera, 2020). The comprehensive centrality can be represented by

$$CEN_i = \frac{\deg_i^+ + \deg_i^-}{2\left(n-1\right)}. \tag{6}$$

The comprehensive centrality measures the DM's fame value, which provides a better reference for the weight allocation. The interaction strength between DMs e_h and e_l in social network can be measured by their interaction frequency f_{hl}. The normalized interaction strength between e_l and e_h can be represented by (Li &Lai, 2014):

$$IS_{hl} = \frac{f_{hl} - f_h^{min}}{f_h^{max_h^{min}}}, \tag{7}$$

where f_h^{min} and f_h^{max} are the minimum and maximum interaction frequency from e_h to other DMs, respectively. Note that IS_{hl} does not have to be equal to IS_{lh}. We combine the centrality degree and interaction strength to characterize the global reputation of DM d_i,

$$GR_i = \sum\nolimits_{j \in U_1\left(d_i\right)} CEN_j \times IS_{ji}. \tag{8}$$

Let $C = (c_{ij})_{m \times m}$ be the k -step interaction matrix which is a row-stochastic matrix with $c_{ii} = 0$ for $i = 1, 2, \cdots, m.$ Parameter $\rho (0 < \rho \le 1)$ represents a direct (one-step) communication path has the force of ρ probability of effectiveness. A k -step communication path has probability ρ^k of being effective. The interaction matrix C is calculated as follows,

$$c_{ij} = \begin{cases} \dfrac{GR_j \times \rho^k}{con_i} & d_j \in U_k\left(d_i\right) \\ 0 & otherwise \end{cases}, \tag{9}$$

Where $con_i = \sum\nolimits_{d_l \in U_1\left(d_i\right)} GR_l \times \rho^1 + \sum\nolimits_{d_l \in U_2\left(d_i\right)} GR_l \times \rho^2 +, \cdots, + \sum\nolimits_{d_l \in U_k\left(d_i\right)} GR_l \times \rho^k..$

The social influence network can be represented by a directed graph $\bar{G}\left(D, \bar{E}\right)$ where D is a set of DMs connected by a set of directed edges $\bar{E}$ that interconnect the DMs in pairs with a set of weights attached to it, that is matrix $C = (c_{ij})_{m\times m}$ and is referred to as the weight adjacency matrix of graph $\bar{G}$ .The influence matrix can be written as

$$W = AC + I - A. \tag{10}$$

where I is $m \times m$ identity matrix, A is a diagonal matrix with $a_{ii} = 1 - \omega_{ii}$ where ω_{ii} is the self-persistence degree of d_i to his/her initial opinion.

For example, we consider the social network of six agents with the information of normalized interaction strength between agents as shown in Fig.1.

Let $\rho = 0.5$ and $k = 3$. $U_1\left(d_i\right), U_2\left(d_i\right)$ and $U_3\left(d_i\right)$ are list in Table 1.

According to Eq. (6), we have $cen_1 = 0.3$., $cen_2 = 0.3$, $cen_3 = 0.2$, $cen_4 = 0.2$, $cen_5 = 0.4$, $cen_6 = 0.3$ .According to Eq. (8), we can obtain $GR_1 = 0.18$, $GR_2 = 0.21$, $GR_3 = 0.18$, $GR_4 = 0.24$, $GR_5 = 0.61$, $GR_6 = 0.36$. According to Eq. (9), we can obtain the interaction matrix

$$C = \begin{pmatrix} 0 & 0.2246 & 0.0963 & 0.2567 & 0.3262 & 0.0963 \\ 0.1417 & 0 & 0.1417 & 0.0945 & 0.4803 & 0.1417 \\ 0 & 0 & 0 & 0 & 0.4586 & 0.5414 \\ 0 & 0 & 0 & 0 & 0.7722 & 0.2278 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Figure 1. The experts' social network

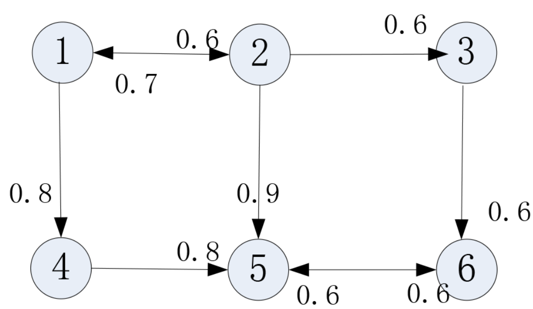


Table 1. Trust set for each DM in Fig. 1

	d_1	d_2	d_3	d_4	d_5	d_6
$U_1\left(d_i\right)$	$\{d_2, d_4\}$	$\{d_1, d_3, d_5\}$	$\{d_6\}$	$\{d_5\}$	$\{d_6\}$	$\{d_5\}$
$U_2\left(d_i\right)$	$\{d_3, d_5\}$	$\{d_4, d_6\}$	$\{d_5\}$	$\{d_6\}$	$\varnothing$	$\varnothing$
$U_3\left(d_i\right)$	$\{d_6\}$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$

3.2 Modeling Self-Persistence and Opinion Dynamic in Social Influence Network

In GDM process, DMs usually negotiate and interact with each other for achieving an agreement. In this section our aim is to model how the DMs' opinions evolve during the interaction process. DMs may differ in some of the opinions and it is expected that they talk to each other to clarify, defend and modify their opinions. Taking the psychological concept of reflected appraisal into account (Cooley, 1902), the DMs' self-persistence will be modified along the discussion process. During each discussion, the DMs' mutual evaluation matrix will be updated as follows

$$Y\left(t+1\right)=AWY\left(t\right)+\left(I-A\right)Y\left(0\right),t=0,1,\cdots \tag{11}$$

where $Y\left(0\right)$ is initial mutual evaluation matrix, $Y\left(t\right)=(y_{ij}\left(t\right))_{m\times m}$ and $y_{ij}\left(t\right)$ is the updated evaluation value of d_i to d_j at time t and $y_{ii}\left(t\right)$ is the self-evaluation of DM d_i of time t. Group members elevate or dampen their self-persistence degrees by averaging their own and others' appraisals. A time-dependent self-persistence can be written as

$$z_{ii}\left(t\right)=\frac{1}{m}\sum_{j=1}^{m}y_{ji}\left(t\right),t=1,2,\cdots \tag{12}$$

The proposed model assumes that the interaction matrix C is constant. If the influence process can reach an equilibrium, i.e., $\lim_{t\to\infty}Y\left(t\right)=Y\left(\infty\right)$, then the original equation becomes

$$Y\left(\infty\right)=VY\left(0\right), \tag{13}$$

where V can be calculated by

$$V=(I-AW)^{-1}\left(I-A\right). \tag{14}$$

Let $z\left(t\right)=(z_{11}\left(t\right),z_{22}\left(t\right),\cdots,z_{mm}\left(t\right))^T$ and $Z\left(t\right)=diag\left(z\left(t\right)\right)$, then a time-dependent influence matrix can be written as

$$W\left(t\right)=\left(I-Z\left(t\right)\right)C+Z\left(t\right). \tag{15}$$

During the discussion process, DMs' opinions and self-persistence evolve simultaneously. Then model can be represented as

$$X\left(t+1\right)=W\left(t\right)X\left(t\right). \tag{16}$$

Theorem 3.1. If the influence network has at least one globally reachable node, then the DMs can reach a consensus by the model as defined in Eq. (16).

Proof. The mutual evaluation matrix will reach reach an equilibrium, i.e., $Y\left(\infty\right)=VY\left(0\right)$, as we explained in Section 2.2. Thus we know $z\left(t\right)$ converge to z^* for any initial mutual evaluation matrix $Y(0)$,i.e., $\lim_{t\to\infty} z(t) = z^*$. In other words, the influence matrix $W(t)$ converge to $W^* = diag(z^*) + (I_k - diag(z^*))C$. We suppose that $W(t)$ reach an equilibrium at time t^*, then after time t^*, the model becomes the DeGroot model. For the reason that there is at least one globally reachable node in the influence network, according to Lemma 2.1, the proof of theorem3.1 is completed.

3.3 The Main Steps of the Proposed Model

To summarize, the main steps in the consensus process are described in Algorithm 1 as follows:

Algorithm 1.

Input: DMs' initial decision matrix $A_k(0)$, $k = 1,2,...,m$, the weight vector of these attributes $\omega = \{\omega_1, \omega_2, \cdots, \omega_p\}$, DMs' initial mutual evaluation matrix $Y(0)$, the consensus threshold δ and $t = 0$..

Output: The final consensus opinions X^*, and the number of discussion rounds t.

Step 1. According to Eqs. (6), (7), (8) and (9), we can obtain the comprehensive centrality CEN_i, the normalized interaction strength IS_{hl}, the global reputation GR_i, and the interaction matrix C respectively.

Step 2. Calculate the influence matrix W by the Eq. (10).

Step 3. Update the DMs' mutual evaluation matrix $Y(t)$ according to Eq. (11). Then, a time-dependent self-persistence $z_{ii}(t)$ can be calculated by Eq. (12).

Step 4. A time-dependent influence matrix $W(t)$ can be calculated by Eq. (15). Update the DMs' opinions $X(t)$ by Eq. (13).

Step 5. If $|X(t) - X(t-1)| < \delta$, let $X^* = X(t)$, or let $t = t + 1$ and go back to step 3.

End

4 ILLUSTRATIVE EXAMPLE AND SIMULATION EXPERIMENT

This section provides an example to show the effectiveness of our proposed model, and design a simulation experiment to explore the effects of the proposed model on the final consensus solution.

4.1 Illustrative Example

We consider a group composed of six experts to reaching a consensus in art evaluation. The experts are with the network described in Section 3.1 that is given in Fig. 1. Set $k = 3$ and $\rho = 0.5$. To avoid over raising or belittling, let the mutual evaluation value be $y_{ij}(0) \in [0.2, 0.9]$. The experts' initial mutual evaluation matrix and their initial opinions are randomly generated form $[0.2, 0.9]$ and $[0,1]$, respectively. Suppose that

$$X(0)=\begin{pmatrix} 0.26 & 0.34 & 0.36 & 0.46 & 0.55 & 0.44 \\ 0.24 & 0.45 & 0.55 & 0.45 & 0.35 & 0.45 \\ 0.35 & 0.25 & 0.56 & 0.34 & 0.56 & 0.34 \\ 0.23 & 0.44 & 0.64 & 0.37 & 0.35 & 0.53 \\ 0.34 & 0.33 & 0.45 & 0.56 & 0.57 & 0.24 \\ 0.35 & 0.34 & 0.38 & 0.45 & 0.46 & 0.36 \end{pmatrix}.$$

and

$$Y(0)=\begin{pmatrix} 0.6 & 0.4 & 0.6 & 0.6 & 0.5 & 0.4 \\ 0.4 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.4 & 0.5 & 0.6 & 0.4 & 0.6 & 0.4 \\ 0.3 & 0.4 & 0.4 & 0.7 & 0.5 & 0.3 \\ 0.2 & 0.3 & 0.4 & 0.6 & 0.7 & 0.4 \\ 0.3 & 0.4 & 0.4 & 0.5 & 0.8 & 0.6 \end{pmatrix}.$$

According to Eq. (10) and the obtained interaction matrix C in Section 3.1, we can obtain the initial influence matrix

$$W(0)=\begin{pmatrix} 0.6 & 0.0898 & 0.0385 & 0.1027 & 0.1305 & 0.0385 \\ 0.0709 & 0.5 & 0.0709 & 0.0472 & 0.2402 & 0.0709 \\ 0 & 0 & 0.6 & 0 & 0.1835 & 0.2165 \\ 0 & 0 & 0 & 0.7 & 0.2316 & 0.0684 \\ 0 & 0 & 0 & 0 & 0.7 & 0.3 \\ 0 & 0 & 0 & 0 & 0.4 & 0.6 \end{pmatrix}$$

Based on Eq. (11), the evolution of DMs' self-persistence degrees is shown in Fig. 2.

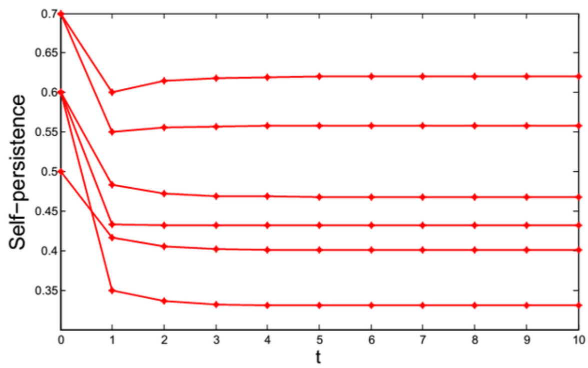
Let $con=\sum_{i=1}^{5}\sum_{k=i+1}^{6}\sum_{j=1}^{6}\left|x_{ij}-x_{kj}\right|$. We assume that the consensus has been reached if $con\leq 0.001$. By the model (16), we obtain the final consensus decision matrix

$$X^{c}=\begin{pmatrix} 0.3442 & 0.3342 & 0.4203 & 0.5133 & 0.5233 & 0.2909 \\ 0.3442 & 0.3342 & 0.4203 & 0.5133 & 0.5233 & 0.2909 \\ 0.3442 & 0.3342 & 0.4203 & 0.5133 & 0.5233 & 0.2909 \\ 0.3442 & 0.3342 & 0.4203 & 0.5133 & 0.5233 & 0.2909 \\ 0.3442 & 0.3342 & 0.4203 & 0.5133 & 0.5233 & 0.2909 \\ 0.3442 & 0.3342 & 0.4203 & 0.5133 & 0.5233 & 0.2909 \end{pmatrix}.$$

4.2 Comparison and Simulation Experiments

To show the effectiveness of the proposed model, we use the consensus model based on DeGroot model to solve the decision making problem above. Here we take the opinion evolution of the opinions

Figure 2. The DMs' self-persistence evolution



on alternative x_1 for example, shown in Fig. 3. The comparison and analysis will be given together after the simulation experiment below.

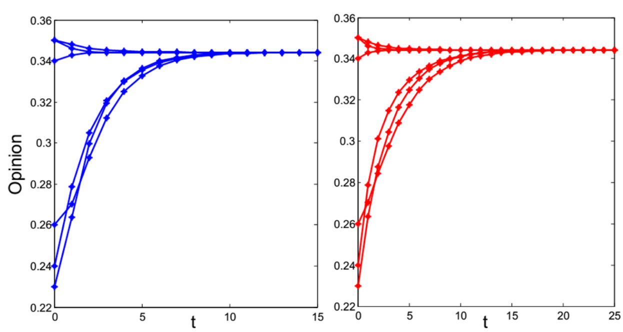
To better illustrate the influence of the DMs' self-persistence evolution on the opinion formation, in this section, we use the ER random network (Ersős and Rényi, 1960) for this opinion dynamics simulation experiment. In the simulation experiment, the initial DMs' mutual evaluation matrix and their initial opinions are randomly generated from $\left[0.3, 0.9\right]$ and $\left[0,1\right]$, respectively. The influence network is randomly generated ER random graph where each pair of nodes connect with the probability of $p(0 < p < 1)$ and the weight of each edge is randomly generated from $\left[0.2, 0.8\right]$. The parameter t denotes the rounds to reach a consensus. Let $con = \sum_{i=1}^{m-1}\sum_{k=i+1}^{m}\sum_{j=1}^{n}\left|x_{ij} - x_{kj}\right|$. The parameter eps is the consensus threshold value representing the consensus has been reached if $con \leq eps$.

In the simulation experiment, we set different input parameters m, eps and p, and run the simulation method 500 times by using the proposed model and the DeGroot model, respectively, to obtain the average values of t, which are described as Fig. 4.

Based on this, the proposed consensus model and the consensus based on DeGroot model will be compared. As result of the illustrative example and simulation experiment the following points must be highlighted:

(1) The proposed model performs effectively by adapting the self-persistence in the discussion process as can be seen in Figs. 2, 3 and 4.

Figure 3. The DMs' opinion evolution

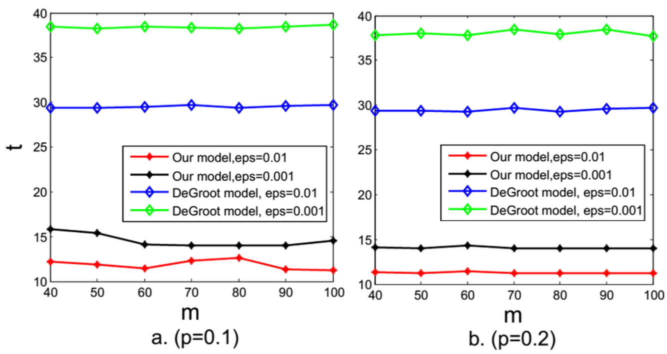


a. Proposed model

b. DeGroot model

(2) By comparing with the consensus model based on DeGroot model for the illustrative example, we find that the proposed model needs less rounds than the DeGroot for reaching a consensus, which can be found in Fig. 3. It indicates that self-persistence evolution can better model human decision behavior in the CRP and can effectively promote consensus.
(3) According to the Fig. 4, for different parameters, the necessary number of rounds to achieve the required consensus degree with consensus model based on DeGroot model is greater than our model, therefore the latter reduces the time cost effectively.

Figure 4. The DMs' opinion evolution



5 CONCLUSION

CRP is significantly associated with the degree of decision satisfaction among the DMs. This study investigates the CRPs of SNGDM. We propose a multi-step communication dynamic consensus model in which DMs' self-persistence degrees evolution during the CRP. The main contributions are presented as follows. In the proposed framework we proposed two measures for each DM, namely global reputation and pairwise social influence. Based on the influence network information, by considering centrality degree, interaction strength and high-order interactions, we construct a social influence network and corresponding influence matrix. The self-confidence degrees of DMs play a prominent role in opinion evolution. Due to the fact that the self-confidence is always difficult to measure in our lives, we allow DMs to voice their evaluations. Then we apply social influence networks theory to analyze the changing process of self-confidence and analyze the influence of self-confidence on SNGDM. Finally, an example and simulation analyses are provided to illustrate the feasibility and effectiveness of our approach. Particularly, the results show the proposed consensus model is robust and effective, and can significantly decrease the time cost by comparing with the consensus model based on DeGroot model.

As future research work, we will study how the social network analysis can be used in the study of non-cooperative behavior to assist the decision makers for achieving a consensus and how to manage experts' non-cooperative behavior that can make difficult to reach a consensus.

ACKNOWLEDGMENT

The research described in this paper is supported by National Natural Science Foundation of China under Grants 71971190 and 71702087, Shanxi Province Philosophy and Social Science Planning Project under Grants 2021YJ145 and Scientific and Technological Innovation Programs of Higher Education Institutions in Shanxi (2021L406).

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