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Intuitionistic Fuzzy Modulus Similarity Measure

Pawan Gora, Deenbandhu Chhotu Ram University of Science and Technology, India*
https://orcid.org/0000-0003-3061-6255

V.P. Tomar, Deenbandhu Chhotu Ram University of Science and Technology, India

ABSTRACT

The concept of intuitionistic fuzzy sets (IFSs) is an expected explanation for finding the appropriate information. It originated from concept of fuzzy set (FS) theory, which extends the classical conception of a fuzzy set. This paper examines a number of widely employed similarity measures then proposes an IFSs modulus similarity measure and a weight similarity measure. Initially, the authors have discussed numerous existing similarity measures, some of which are unable to justify the axioms of being a similarity measure. Furthermore, some numerical examples are presented to compare the existing similarity measures with the proposed similarity measure. The proposed similarity measure is a practical and effective method for determining the qualitative similarity between IFSs, which do not have any paradoxical nature. In addition, the proposed similarity measure has been demonstrated practically in pattern recognition and medical diagnosis problem. Suggestions for future research comprise the conclusions of the paper.

KEYWORDS

Fuzzy Set, Intuitionistic Fuzzy Set, Medical Diagnosis, Pattern Recognition, Similarity Measure, Vague Set, Weight Similarity Measure

ABBREVIATIONS

FSs - Fuzzy sets
IFSs - Intuitionistic fuzzy sets
VSs - vague sets
IVFSs – Interval-valued fuzzy sets
IFMSs - Intuitionistic fuzzy multisets
IFVs - Intuitionistic fuzzy values
RTFTNs - Right-angled transformed fuzzy triangular numbers
PFSs- Pythagorean Fuzzy Sets

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*Corresponding Author

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INTRODUCTION

Information theory is a scientific study for finding the uncleared information. The FSs theory is a significant component of IFSs theory, it makes the study as tractable. IFSs were created Atanassov (1986) with the combination of membership, non-membership and hesitancy index. In general, the available vague, insufficient or inexact information is improved in uncertain ways by the decision maker which can be termed as the measure. With the help of these measures, the authors can find the accurate and reasonable information. This concept is expanded by some various higher order fuzzy sets, including the intuitionistic fuzzy set, hence, the IFSs theory achieved the affluence to arrange the uncertainty. The probabilistic entropy measure is related to the equation of thermodynamical entropy, which is generally quantifiable as randomness, disorder, and uncertainty. In recent years, numerous studies have developed similarity measure & distance measures between intuitionistic fuzzy sets (IFSs) and interval-valued fuzzy sets (IVFSs). Through the literature, Gau and Buehrer (1993) described the concept of vague sets (VSs), it the further generalized extension of fuzzy sets (FSs). The fuzzy set theory is an approach which is used to getting the accurate and desirable information. Later on, Bustince & Burillo (1996) shows that the developed vague set is identical to the IFS; it treated as the further extension of fuzzy sets. IFSs theory has been proved suitable in its assessment for some diverse applications to their related studies. The study of similarity measures is a fascinating and vital approach for finding hidden information. In this contrast, similarity measure provides the comparison between the information carried by IFSs. Consequently, similarity measure is an approach to which detect the degree of similarity between IFSs. It is a tool to be applied to various applications such as pattern recognition, decision making, medical diagnosis, image processing, machine learning, and cluster analysis. Through the concept of similarity measures, many authors introduced the various similarity measures in IFSs. In different studies, it was found that some contradictory cases, which could not provide accurate information. Consequently, as a result of some respective studies, numerous authors overcame the drawback and proposed some novel measures. Therefore, more similarity measures can be derived from distance measures and vice versa. Furthermore, the numerous authors (Arora & Tomar, 2020; Dass et al., 2019; Tomar, 2019; Tomar & Ohlan, 2014) have developed the entropy and discrimination measures related to FSs and IFSs. Initially, IFSs distance measures were conceptualised by Szmidt & Kacprzyk (2000). After that, Wang and Xin (2005) are point out, Szmidt & Kacprzyk (2000) distance measure is not well in some cases. So, they have suggested some new distance measures and their implementation in the pattern recognition problem. IFSs distance measures are tools used to describe the study of differences between IFSs. These are primarily useful for problems involving decision-making, pattern recognition, and medical diagnosis problem.

In a further investigation, Grzegorzewski (2004) created IFSs & IVFSs distances as well as normalized distances on Hausdorff metric distances. Afterwards, Chen (2007) demonstrated the inaccuracy in Grzegorzewski (2004) distance measure by a number of counterexamples. Hung and Yang (2004) developed three similarity measures, which have been extended as IFSs Hausdorff distance. From the present literature review, it observed that various authors have presented several types of similarity measures for IFSs corresponding to the different related study. Dengfeng and Chuntian (2002) suggested a novel similarity measure for IFSs based on their respective membership and non-membership functions. Furthermore, Mitchell (2003) investigated that Dengfeng and Chuntian (2002) developed similarity measure is not well everywhere, it occupying paradoxical scenarios. Consequently, he reconstructed a new similarity measure on the basis of statistical nature. In addition, Liang and Shi (2003) demonstrated that Dengfeng and Chuntian (2002) similarity measure is not authentic in some instances, They overcome on these and developed a new IFSs similarity measure and implemented it for pattern recognition process also. Li and Olson (2007) examined existing similarity measures with their counterintuitive cases and presented a new IFSs similarity measure. On the basis of the above study, Xu (2007) developed the series of generalized similarity measures in IFSs and enforced in decision-making process. Moreover, Xu and Chen (2008) suggested a series of similarity measures as well as distance measures by combining various types of measures, and they also introduced several forms of weight distances. Xu and Yager (2009) gave an IFS similarity measure and showed how it can be used for group decision making and consensus problem analysis. Later, Ye (2011) developed the new IFSs Cosine similarity & Cosine weight similarity measures, and a comparative analysis of several existing similarity measures. The numerous authors use the technique of IFSs similarity measures for solve the problems like decision making, recognising of patterns, and medical diagnoses.

In this present scenario, the authors will discuss several types of IFSs similarity and distance measures. Wei et al. (2011) suggested a similarity measure by using the concept of IFSs. Boran and Akay (2014) introduced a new L_p norm and uncertainty level bi-parametric similarity measure in IFSs nature. Moreover, Papakostas et al.(2013) examined certain fundamental computational and theoretical properties of IFS similarity and distance measures and generalized their relationship between them. On the basis of Cotangent function, Rajarajeswari and Uma (2013) introduced an intuitionistic fuzzy multisets (IFMSs) similarity measure and its applications in medical diagnosis problems. Additionally, Ye and Shi (2013) improved in Ye (2011) Cosine similarity measure and reconstructed it as a VSs similarity measure. Szmidt (2014) provided a concise and illuminating analysis of IFSs similarity and distance measures on membership and non-membership functions. It has been described some existing IFSs similarity and distance measures in two and three terms. On aggregation function without zero divisors Du & Hu (2015) developed an aggregation distance measure for IFSs, and demonstrating its usefulness in decision making problem. In addition, there examined an induced similarity measure from the aggregation of distance measure, it has been implemented in decision making process. Chen et al. (2016) proposed the intuitionistic fuzzy values (IFVs) similarity measure over the centroid points of right-angled transformed fuzzy triangular numbers (RTFTNs) and showed that they have the same similarity properties. A novel approach to IFSs similarity measure has been introduced, based on IFVs similarity measure, with pattern recognition problem application. Furthermore, Ye (2016) developed two cosine similarity measures as well as weight cosine similarity measures for IFSs, it applicable in the context of decision making. Consequently, Garg (2018) developed an improved cosine IFSs similarity measure, by considering intersection of the pair of membership and non-membership functions. The suggested study illustrates the shortcoming of numerous types of IFSs similarity measures, that have been fulfilled by the developed measure. In addition, it has devised a weight similarity measure that have been implemented into decision-making problem. From intensive literature, Hwang et al. (2018) suggested an IFSs similarity measure based on the Jaccard Index and demonstrated its application in clustering challenges. Moreover, Song et al. (2019) defined an IFSs similarity measure by direct operation on their respective functions. It compared with some previous existing similarity measures, their implementation in cluster analysis & medical diagnosis problem. Based on inner product, Singh & Kumar (2020) suggested a novel dice similarity measure for IFSs and improves the limitations of the existing study. The developed study is performed in a variety of their relevant applications includes medical diagnostics and pattern recognition. Additionally, an algorithm for face recognition was developed using the suggested similarity measure, it will be compared to various current approaches using an illustrative example. In addition of these, the numerous studies (S. M. Chen & Chang, 2015; Hwang et al., 2012; Khan et al., 2017; Nguyen, 2016) are obtained to which deal with the patten recognition problem. The intuitionistic fuzzy similarity measures are the most dominating study for finding the best decisions. There are some distinct similarity measures (Beg & Rashid, 2016; Beliakov et al., 2014; H. W. Liu, 2005; J. Liu et al., 2019; Maoying, 2013) to which design the decision making problem in some various ways. The IFSs measures are the tools that have been implemented to describe the information in the various form. Numerous authors described the information through distinct IFSs similarity measures are following as; (Deli, 2016; Deng et al., 2015; Liang & Shi, 2003; Mo et al., 2012; Zhou, 2016). In this study, the authors have proposed an IFSs similarity measure consisting of a basic mathematical expression and the arrangement of predefined functions of membership, non-membership, and hesitation index. The proposed study can be viewed as the consistency and similarity between two IFSs with their prescribed values of respective membership and non-membership functions. It is a non-parametric similarity measure whose generalization involves direct operation on their respective functions. The proposed study satisfies all axiomatic definitions after being compared to numerous current similarity measures. It is a vital source of measuring uncleared information in the form of degree of similarity. Moreover, the authors shall introduce the applicability and validity of the developed measure in their related field through illustrative examples.

The following is the main contribution of the developed article. Several basic definitions pertinent to the proposed study are discussed in Section 2. In Section 3, provides the mathematical expression of numerous existing parametric and nonparametric similarity measures for IFSs. In Section 4, the authors are proposed a non-parametric IFSs similarity measure and a weight similarity measure, as well as their mathematical expression. A Comparative analysis of the proposed IFSs similarity measure with some certain counterintuitive cases is performed in Section 5. The authors will discuss the applications of pattern recognition and medical diagnosis related to the proposed IFSs similarity measure in Section 6, and obtained the conclusion of the article in the last section.

PRELIMINARIES

Definition 1

(L.A.Zadeh, 1965); Let, X = { t_1 , t_2 ,…, t_n } be a non-empty set of the universe of discourse, then defined a non-empty fuzzy set (FS) A on X such that,

$$A = \{ t,\mu_A\left(t\right):\ \mu_A . : X \to [0,1],\ \mathbf{t}\epsilon X\}$$

where, μ_A . , be the membership function for A attains a degree of uncertainty corresponding to all values of **ζ** ϵ X lies between 0 to 1.

Definition 2

(Atanassov, 1986)**;** Let X = { t_1 , t_2 . ,…, t_n } be a non-empty set of the universe of discourse, then defined a non-empty intuitionistic fuzzy set (IFS) A on X such that,

$$A = \{ t,\mu_A\left(t\right),\nu_A\left(t\right),\pi_A\left(t\right):\mu_A,\nu_A,\pi_A : X \to \left[0,1\right], t \int X \ \}$$

where, μ_A , ν_A and π_A be the membership, non-membership functions and hesitancy index respectively, for A attains a degree of uncertainty corresponding to all values of **ζ** ϵ X lies between 0 to 1.

The trio's collection of membership, non-membership functions and hesitancy index for an IFS on X. Then, the collection of all possible IFSs on X are denoted by IFSs(X).

In additional, $\pi_A\left(t\right)=1-\mu_A\left(t\right)-\nu_A\left(t\right)$, are hesitancy index of each $t \in X$ . .

Also, $0 \le \mu_A\left(t\right) + \nu_A\left(t\right) . \le 1,\ \mu_A\left(t\right) + \nu_A\left(t\right) + \pi_A\left(t\right) = 1,\ 0 \le \pi_A\left(t\right) + \le 1.$

Definition 3

Consider the universe of discourse X = { t_1 , t_2 ,…, t_n } for A, B, C ∈ IFSs(X). Then define a map, D: IFSs(X)×IFSs(X) → [0,1] that satisfies some following axioms,

(D1) 0 ≤ $D\left(A,B\right)$ ≤ 1;
(D2) A = B if and only if $D\left(A,B\right)$ = 0;
(D3) $D\left(A,B\right)$ $D\left(B,A\right)$;

Let A, B, C are IFSs on X such that A ⊑ B ⊑ C, then $D\left(A,B\right)$ ≤ $D\left(A,C\right)$, and $D\left(B,C\right)$ ≤ $D\left(A,C\right)$.

Then, $D\left(A,B\right)$ is called the distance measure between the IFSs(X).

Definition 4

Let X be the universe of discourse and A, B, C ϵ IFSs(X). Then define a map, S: IFSs(X)×IFSs(X) → [0,1] that satisfies some following axioms,

(D1) 0 ≤ $S\left(A,B\right)$ ≤ 1;
(D2) A = B if and only if $S\left(A,B\right)$ = 0;
(D3) $S\left(A,B\right)$ = $S\left(B,A\right)$;
(D4) If A, B, C are IFSs on X such that A ⊑ B ⊑ C, then $S\left(A,B\right)$ ≥ $S\left(A,C\right)$, and $S\left(B,C\right)$ ≥ $S\left(A,C\right)$.

Then, $S\left(A,B\right)$ is called the similarity measure between IFSs(X).

Existing IFSs Similarity Measures

In the present section, the authors are introducing numerous existing similarity measures for IFSs. The mathematical expression of the existing study is reviewed as

Chen (1995) proposed the vague set similarity measure:

ng & Kim (1999) developed the similarity measure on IFSs:

$$\mathrm{S}_{HK}\left(A,B\right) = 1 - \frac{\sum_{i=1}^{n}\left|\left(\left(\mu_A\left(t_i\right) - \mu_B\left(t_i\right)\right) + \left(\nu_A\left(t_i\right) - \nu_B\left(t_i\right)\right)\right|}{2n}$$

Li & Xu (2001) introduced the IFSs similarity measure:

$$\mathrm{S}_{LX}\left(A,B\right) = 1 - \frac{\sum_{i=1}^{n}\left\{\begin{matrix}\left|\left(\left(\mu_A\left(t\right) - \nu_A\left(t_i\right)\right) - \left(\mu_B\left(t_i\right) - \nu_B\left(t_i\right)\right)\right| - \\ \left|\left(\left(\mu_A\left(t_i\right) - \mu_B\left(t_i\right)\right) + \left(\nu_A\left(t_i\right) - \nu_B\left(t_i\right)\right)\right|\end{matrix}\right\}}{4n}$$

Li & Zhongxian (2002) originated the vague set (VSs) similarity measure and vague entropy:

$$\mathrm{S}_O\left(A,B\right)=1-\sqrt{\frac{\sum_{i=1}^{n}\left(\left(\mu_A\left(t_i\right)-\mu_B\left(t_i\right)\right)^2+\left(\nu_A\left(t_i\right)-\nu_B\left(t_i\right)\right)^2\right)}{2n}}$$

Dengfeng & Chuntian (2002) proposed IFSs similarity measure:

$$\mathrm{S}_{DC}\left(A,B\right)=1-\sqrt[p]{\frac{\sum_{i=1}^{n}\left|\Phi_A\left(t_i\right)-\Phi_B\left(t_i\right)\right|^p}{n}}$$

where, $\Phi_A\left(t_i\right)=\dfrac{\mu_A\left(t_i\right)+1-\nu_A\left(t_i\right)}{2}$, and $\Phi_B\left(t_i\right)=\dfrac{\mu_B\left(t_i\right)+1-\nu_B\left(t_i\right)}{2}$.

Mitchell (2003) defined a new similarity measure for IFSs based on Dengfeng & Chuntian (2002) similarity measure,:

$$\mathrm{S}_M\left(A,B\right)=\frac{1}{2}\left(\text{Á}_{\sigma}\left(A,B\right)+\text{Á}_{\omega}\left(A,B\right)\right),$$

where $\text{Á}_{\sigma}\left(A,B\right)=1-\sqrt{\sum_{i=1}^{n}\left|\mu_A\left(t_i\right)-\mu_B\left(t_i\right)\right|^p/n}$,

and $\text{Á}_{\omega}\left(A,B\right)=1-\sqrt{\sum_{i=1}^{n}\left|\nu_A\left(t_i\right)-\nu_B\left(t_i\right)\right|^p/n}$

Liang & Shi (2003) developed three IFSs similarity measures:

$$\mathrm{S}_e^p\left(A,B\right)=1-\sqrt[p]{\frac{\sum_{i=1}^{n}\left(\omega_\mu\left(t_i\right)-\omega_v\left(t_i\right)\right)^p}{n}},$$

where, $\omega_\mu\left(t_i\right)=\left|\mu_A\left(t_i\right)-\mu_B\left(t_i\right)\right|/2$,

$\text{É}_v\left(\mathbf{t}_i\right)=\left|\left(1-\text{¼}_A\left(\mathbf{t}_i\right)\right)-\left(1-\text{¼}_B\left(\mathbf{t}_i\right)\right)\right|/2$.

$$\mathrm{S}_s^p\left(A,B\right)=1-\sqrt[p]{\frac{\sum_{i=1}^{n}\left(\chi_{s1}\left(t_i\right)-\chi_{s2}\left(t_i\right)\right)^p}{n}},$$

where, $\chi_{s1}\left(t_i\right)=\left|n_{A1}\left(t_i\right)-n_{B1}\left(t_i\right)\right|/2$, $\chi_{s2}\left(t_i\right)=\left|n_{A2}\left(t_i\right)-n_{B2}\left(t_i\right)\right|/2$,

$n_{A1}\left(t_i\right)=\left|\mu_A\left(t_i\right)-n_A\left(t_i\right)\right|/2$, $n_{B1}\left(t_i\right)=\left|\mu_B\left(t_i\right)-n_B\left(t_i\right)\right|/2$,

$n_{A2}\left(t_i\right)=\left|1-\upsilon_A\left(t_i\right)-n_A\left(t_i\right)\right|/2$, $n_{B2}\left(t_i\right)=\left|1-\upsilon_B\left(t_i\right)-n_B\left(t_i\right)\right|/2$,

$n_A\left(t_i\right)=\left|1-\upsilon_A\left(t_i\right)-\mu_A\left(t_i\right)\right|/2$, $n_B\left(t_i\right)=\left|1-\upsilon_B\left(t_i\right)-\mu_B\left(t_i\right)\right|/2$.

$$\mathrm{S}_h^p\left(A,B\right)=1-\sqrt[p]{\frac{\sum_{i=1}^{n}\left(\sigma_1\left(t_i\right)+\sigma_2\left(t_i\right)+\sigma_3\left(t_i\right)\right)^p}{3n}},$$

where.., $\sigma_1\left(t_i\right)=\Phi_\mu\left(t_i\right)-\Phi_v\left(t_i\right)$,

$\sigma_2\left(t_i\right)=|\Phi_\mu\left(t_i\right)-\Phi_v\left(t_i\right)|$,

$\sigma_3\left(t_i\right) = \max\left(r_A\left(t_i\right), r_B\left(t_i\right)\right) - \min\left(r_A\left(t_i\right), r_B\left(t_i\right)\right)$,
$r_A\left(t_i\right) = (1 - \mu_A\left(t\right) - \nu_A\left(t_i\right) / 2$ and $r_B\left(t_i\right) = (1 - \mu_B\left(t\right) - \nu_B\left(t_i\right) / 2$.

Hung & Yang (2004) defined three similarity measures on IFSs:

$$\mathrm{S}_{HY}^{a}\left(A,B\right) =- \mathrm{d}_H\left(A,B\right).$$

where $\mathrm{d}_H\left(A,B\right) = \frac{1}{n}\sum_{i=1}^{n} \max\left(\left|(\mu_A\left(t_i\right) - \mu_B\left(t_i\right)\right|, \left|(\nu_A\left(t_i\right) - \nu_B\left(t_i\right)\right|\right)$

$$\mathrm{S}_{HY}^{b}\left(A,B\right) = \frac{e^{\mathrm{d}_H\left(A,B\right)} - e^{-1}}{1 - e^{-1}},\ \mathrm{S}_{HY}^{c}\left(A,B\right) = \frac{1 - \mathrm{d}_H\left(A,B\right)}{1 + \mathrm{d}_H\left(A,B\right)}$$

Liu (2005) developed a similarity measure on IFSs:

$$\mathrm{S}_{L}^{p}\left(A,B\right) = 1 - \sqrt[p]{\sum_{i=1}^{n} \frac{1}{2n} \begin{Bmatrix} \left|\mu_A\left(t_i\right) - \mu_B\left(t_i\right)\right|^p + \\ \left|\nu_A\left(t_i\right) - \nu_B\left(t_i\right)\right|^p + \\ \left|\pi_A\left(t_i\right) - \pi_B\left(t_i\right)\right|^p \end{Bmatrix}} \text{ where } 1 < \mathrm{p} < +\infty$$

Ye (2011) proposed a novel IFSs cosine similarity measure:

$$\mathrm{S}_{Y}\left(A,B\right) = \frac{1}{n}\sum_{i=1}^{n} \frac{\mu_A\left(t_i\right)\mu_B\left(t\right) + \nu_A\left(t_i\right)\nu_B\left(t_i\right)}{\sqrt{\left(\mu_A\left(t_i\right)\right)^2 + \left(\nu_A\left(t_i\right)\right)^2}\sqrt{\left(\mu_B\left(t_i\right)\right)^2 + \left(\nu_B\left(t_i\right)\right)^2}}$$

Ye & Shi (2013) suggested the improved Cosine similarity measure for vague sets (VSs):

$$\mathrm{S}_{SY}\left(A,B\right) = \frac{1}{n}\sum_{i=1}^{n} \frac{\mu_A\left(t_i\right)\mu_B\left(t_i\right) + \nu_A\left(t_i\right)\nu_B\left(t_i\right) + \pi_A\left(t_i\right)\pi_B\left(t_i\right)}{\sqrt{\left(\mu_A\left(t_i\right)\right)^2 + \left(\nu_A\left(t\right)\right)^2 + \left(\pi_A\left(t_i\right)\right)^2}\sqrt{\left(\mu_B\left(t_i\right)\right)^2 + \left(\nu_B\left(t_i\right)\right)^2 + \left(\pi_B\left(t\right)\right)^2}}$$

Boran & Akay (2014) defin the bi-parametric IFSs similarity measure:

$$\mathrm{S}_{t}^{p}\left(A,B\right) = 1 - \sqrt[p]{\sum_{i=1}^{n} \frac{1}{2n\left(1+t\right)^p} \begin{Bmatrix} \left|t\left(\mu_A\left(t_i\right) - \mu_B\left(t_i\right)\right) - \left(\nu_A\left(t_i\right) - \nu_B\left(t_i\right)\right)\right|^p + \\ \left|t\left(\nu_A\left(t_i\right) - \nu_B\left(t_i\right)\right) - \left(\mu_A\left(t_i\right) - \mu_B\left(t_i\right)\right)\right|^p \end{Bmatrix}}$$

Ye (2016) created IFSs similarity measure based on cosine function:

$$\mathrm{S}_{YC}\left(A,B\right) = \frac{1}{n}\sum_{i=1}^{n} \mathrm{Cos}\left[\frac{\pi}{4}\begin{pmatrix} \left|\mu_A\left(t_i\right) - \mu_B\left(t_i\right)\right| + \\ \left|\nu_A\left(t_i\right) - \nu_B\left(t_i\right)\right| + \\ \left|\pi_A\left(t_i\right) - \pi_B\left(t_i\right)\right| \end{pmatrix}\right]$$

l the above similarity measures are distinct as well as unique similarity methods. These are represented with unique mathematical representation.

Proposed Intuitionistic Fuzzy Modulus Similarity Measure

Let X = { t_1 , t_2 ,…, t_n } be the universe of discourse then defined the IFSs A = { $t, \mu_A(t), \nu_A(t), \pi_A(t)$ } and B = { t, ¼$_B$ (t), ½$_B$ (t), À$_B$ (t) } in X. The IFSs are the sets of elements with a significant degree of membership and non-membership. In this section, the authors are suggesting an IFS similarity measure and present their axioms of validation & authenticity. Then, the proposed similarity measure for IFSs A and B is followed as:

$$S_{GV}\left(A,B\right) = 1 - \log_2\left[1+\frac{1}{3n}\sum_{i=1}^{n}\left(\begin{array}{c}\left|\sqrt{\mu_A(t_i)}-\sqrt{\mu_B(t_i)}\right|+\\ \left|\sqrt{\nu_A(t_i)}-\sqrt{\nu_B(t_i)}\right|+\\ \left|\sqrt{\pi_A(t_i)}-\sqrt{\pi_B(t_i)}\right|\end{array}\right)\right]$$

In addition, the authors are examined the concept to determine the consistency between two IFSs. The similarity measure has a high tendency to obtain the best accuracy between two IFSs.

Theorem 1 The proposed similarity measure, $S_{GV}\left(A,B\right)$ satisfies the axioms of similarity measure between A, B ϵ IFSs(X).

Proof. Suppose the given IFSs A, B ϵ IFSs(X), such that

A = { $t_i, \mu_A(t_i), \nu_A(t_i), \pi_A(t_i)$ } and B = { $t_i, \mu_B(t_i), \nu_B(t_i), \pi_B(t_i)$ }

(SP1): Since, $0 \le \mu_A(t_i) \le 1$, $0 \le \nu_A(t_i) \le 1$, $0 \le \pi_A(t_i) \le 1$ and $0 \le \mu_B(t_i) \le 1$, $0 \le \nu_B(t_i) \le 1$, $0 \le \pi_B(t_i) \le 1$.

Therefore, $0 \le \left|\sqrt{\mu_A(t_i)} - \sqrt{\mu_B(t_i)}\right| \le 1, 0 \le \left|\sqrt{\nu_A(t_i)} - \sqrt{\nu_B(t_i)}\right| \le 1, 0 \le \left|\sqrt{\pi_A(t_i)} - \sqrt{\pi_B(t_i)}\right|$ ≤ 1, the author has:

$$\Longrightarrow 0 \le \left|\sqrt{\mu_A(t_i)}-\sqrt{\mu_B(t_i)}\right| + \left|\sqrt{\nu_A(t_i)}-\sqrt{\nu_B(t_i)}\right| + \left|\sqrt{\pi_A(t_i)}-\sqrt{\pi_B(t_i)}\right| \le 3$$

$$\Longrightarrow 0 \le \frac{1}{3}\left[\begin{array}{c}\left|\sqrt{\mu_A(t_i)}-\sqrt{\mu_B(t_i)}\right|+\\ \left|\sqrt{\nu_A(t_i)}-\sqrt{\nu_B(t_i)}\right|+\\ \left|\sqrt{\pi_A(t_i)}-\sqrt{\pi_B(t_i)}\right|\end{array}\right] \le 1$$

$$\Longrightarrow 1 \le 1 + \frac{1}{3}\left[\begin{array}{c}\left|\sqrt{\mu_A(t_i)}-\sqrt{\mu_B(t_i)}\right|+\\ \left|\sqrt{\nu_A(t_i)}-\sqrt{\nu_B(t_i)}\right|+\\ \left|\sqrt{\pi_A(t_i)}-\sqrt{\pi_B(t_i)}\right|\end{array}\right] \le 2$$

$$\Longrightarrow 0 \leq \log_2 \left(1 + \frac{1}{3} \left[\begin{array}{c} \left| \sqrt{\mu_A(t_i)} - \sqrt{\mu_B(t_i)} \right| + \\ \left| \sqrt{\nu_A(t_i)} - \sqrt{\nu_B(t_i)} \right| + \\ \left| \sqrt{\pi_A(t_i)} - \sqrt{\pi_B(t_i)} \right| \end{array} \right] \right) \leq log2$$

Given expression is also possible,

$$0 \leq \log_2 \left(1 + \frac{1}{3} \left[\begin{array}{c} \left| \sqrt{\mu_A(t_i)} - \sqrt{\mu_B(t_i)} \right| + \\ \left| \sqrt{\nu_A(t_i)} - \sqrt{\nu_B(t_i)} \right| + \\ \left| \sqrt{\pi_A(t_i)} - \sqrt{\pi_B(t_i)} \right| \end{array} \right] \right) \leq 1$$

$$-1 \leq - \log_2 \left(1 + \frac{1}{3} \left[\begin{array}{c} \left| \sqrt{\mu_A(t_i)} - \sqrt{\mu_B(t_i)} \right| + \\ \left| \sqrt{\nu_A(t_i)} - \sqrt{\nu_B(t_i)} \right| + \\ \left| \sqrt{\pi_A(t_i)} - \sqrt{\pi_B(t_i)} \right| \end{array} \right] \right) \leq 0$$

$$\Longrightarrow 0 \leq 1 - \log_2 \left(1 + \frac{1}{3} \sum_{i=1}^{n} \left[\begin{array}{c} \left| \sqrt{\mu_A(t_i)} - \sqrt{\mu_B(t_i)} \right| + \\ \left| \sqrt{\nu_A(t_i)} - \sqrt{\nu_B(t_i)} \right| + \\ \left| \sqrt{\pi_A(t_i)} - \sqrt{\pi_B(t_i)} \right| \end{array} \right] \right) \leq n$$

$$\Longrightarrow 0 \leq 1 - \log_2 \left(1 + \frac{1}{3n} \sum_{i=1}^{n} \left[\begin{array}{c} \left| \sqrt{\mu_A(t_i)} - \sqrt{\mu_B(t_i)} \right| + \\ \left| \sqrt{\nu_A(t_i)} - \sqrt{\nu_B(t_i)} \right| + \\ \left| \sqrt{\pi_A(t_i)} - \sqrt{\pi_B(t_i)} \right| \end{array} \right] \right) \leq 1$$

Therefore, $0 \leq S_{GV}(A,B) \leq 1$.

(SP2): Let A, B $\in$ IFSs, such that A = B $\Leftrightarrow$ $S_{GV}(A,B) = 1$.

Considered as, if A = B, then $\mu_A(t_i) = \mu_B(t_i)$, ½$_A(\mathrm{t}_i)$ = ½$_B(\mathrm{t}_i)$, and À$_A(\mathrm{t}_i)$ = À$_B(\mathrm{t}_i)$ for all i

Thus, $S_{GV}(A,B) = 1$.
Conversely, let, $S_{GV}(A,B) = 1$
Now, show that, A = B
Since, $S_{GV}(A,B) = 1$

$$\Longrightarrow 1 - \log_2\left(1+\frac{1}{3n}\sum_{i=1}^{n}\left[\begin{array}{c}\left|\sqrt{\mu_A\left(t_i\right)}-\sqrt{\mu_B\left(t_i\right)}\right|+\\ \left|\sqrt{\nu_A\left(t_i\right)}-\sqrt{\nu_B\left(t_i\right)}\right|+\\ \left|\sqrt{\pi_A\left(t_i\right)}-\sqrt{\pi_B\left(t_i\right)}\right|\end{array}\right]\right) = 1, \text{ for all i.}$$

$$\Longrightarrow \log_2\left(1+\frac{1}{3n}\sum_{i=1}^{n}\left[\begin{array}{c}\left|\sqrt{\mu_A\left(t_i\right)}-\sqrt{\mu_B\left(t_i\right)}\right|+\\ \left|\sqrt{\nu_A\left(t_i\right)}-\sqrt{\nu_B\left(t_i\right)}\right|+\\ \left|\sqrt{\pi_A\left(t_i\right)}-\sqrt{\pi_B\left(t_i\right)}\right|\end{array}\right]\right) = 0$$

$$\Longrightarrow \log_2\left(1+\frac{1}{3n}\sum_{i=1}^{n}\left[\begin{array}{c}\left|\sqrt{\mu_A\left(t_i\right)}-\sqrt{\mu_B\left(t_i\right)}\right|+\\ \left|\sqrt{\nu_A\left(t_i\right)}-\sqrt{\nu_B\left(t_i\right)}\right|+\\ \left|\sqrt{\pi_A\left(t_i\right)}-\sqrt{\pi_B\left(t_i\right)}\right|\end{array}\right]\right) = log_2\left(1\right)$$

$$\Longrightarrow \left|\sqrt{\mu_A\left(t_i\right)}-\sqrt{\mu_B\left(t_i\right)}\right|+\left|\sqrt{\nu_A\left(t_i\right)}-\sqrt{\nu_B\left(t_i\right)}\right|+\left|\sqrt{\pi_A\left(t_i\right)}-\sqrt{\pi_B\left(t_i\right)}\right| = 0, \text{ for all i.}$$

It is only possible when, $\mu_A\left(t_i\right)=\mu_B\left(t_i\right)$, $\nu_A\left(t_i\right)=\nu_B\left(t_i\right)$, and $\pi_A\left(t_i\right) = \pi_B\left(t_i\right)$ for all i
Therefore, A = B.
Hence, A = B ⇔ $S_{GV}\left(A,B\right) = 1$.

(SP3): It can be illustrated as the present expression is commutative.

Hence, $S_{GV}\left(A,B\right) = S_{GV}\left(B,A\right)$.

(SP4): Let A, B, C are IFSs on X such that A ⊑ B ⊑ C. Thus, we get, $0 \leq \mu_A\left(t_i\right) \leq \mu_B\left(t_i\right) \leq \mu_C\left(t_i\right) \leq 1$ and $0 \leq \nu_C\left(t_i\right) \leq \nu_B\left(t_i\right) \leq \nu_A\left(t_i\right) \leq 1$, for all $t_i \in X$.

Now, show that,

$S\left(A,C\right) \leq S\left(A,B\right)$, and $S\left(A,C\right) \leq S\left(B,C\right)$.

For this,

$$\left[\begin{array}{c}\left|\sqrt{\mu_A\left(t_i\right)}-\sqrt{\mu_C\left(t_i\right)}\right|+\\ \left|\sqrt{\nu_A\left(t_i\right)}-\sqrt{\nu_C\left(t_i\right)}\right|+\\ \left|\sqrt{\pi_A\left(t_i\right)}-\sqrt{\pi_C\left(t_i\right)}\right|\end{array}\right] \geq \left[\begin{array}{c}\left|\sqrt{\mu_A\left(t_i\right)}-\sqrt{\mu_B\left(t_i\right)}\right|+\\ \left|\sqrt{\nu_A\left(t_i\right)}-\sqrt{\nu_B\left(t_i\right)}\right|+\\ \left|\sqrt{\pi_A\left(t_i\right)}-\sqrt{\pi_B\left(t_i\right)}\right|\end{array}\right], \text{ for all i.}$$

$$\Longrightarrow \log_2\left(1+\frac{1}{3n}\sum_{i=1}^{n}\left[\begin{array}{c}\left|\sqrt{\mu_A(t_i)}-\sqrt{\mu_C(t_i)}\right|+\\ \left|\sqrt{\nu_A(t_i)}-\sqrt{\nu_C(t_i)}\right|+\\ \left|\sqrt{\pi_A(t_i)}-\sqrt{\pi_C(t_i)}\right|\end{array}\right]\right)$$

$$\geq \log_2\left(1+\frac{1}{3n}\sum_{i=1}^{n}\left[\begin{array}{c}\left|\sqrt{\mu_A(t_i)}-\sqrt{\mu_B(t_i)}\right|+\\ \left|\sqrt{\nu_A(t_i)}-\sqrt{\nu_B(t_i)}\right|+\\ \left|\sqrt{\pi_A(t_i)}-\sqrt{\pi_B(t_i)}\right|\end{array}\right]\right)$$

$$\Longrightarrow 1-\log_2\left(1+\frac{1}{3n}\sum_{i=1}^{n}\left[\begin{array}{c}\left|\sqrt{\mu_A(t_i)}-\sqrt{\mu_C(t_i)}\right|+\\ \left|\sqrt{\nu_A(t_i)}-\sqrt{\nu_C(t_i)}\right|+\\ \left|\sqrt{\pi_A(t_i)}-\sqrt{\pi_C(t_i)}\right|\end{array}\right]\right)$$

$$\leq 1-\log_2\left(1+\frac{1}{3n}\sum_{i=1}^{n}\left[\begin{array}{c}\left|\sqrt{\mu_A(t_i)}-\sqrt{\mu_B(t_i)}\right|+\\ \left|\sqrt{\nu_A(t_i)}-\sqrt{\nu_B(t_i)}\right|+\\ \left|\sqrt{\pi_A(t_i)}-\sqrt{\pi_B(t_i)}\right|\end{array}\right]\right)$$

So, $S_{GV}(A,C) \leq S_{GV}(A, B)$.

Similarly, $S_{GV}(A,C) \leq S_{GV}(B, C)$.

Therefore, $S_{GV}(A,C) \leq S_{GV}(A, B)$ and $S_{GV}(A, C) \leq S_{GV}(B,C)$.

Hence, the present expression satisfies all axioms of the IFSs similarity measure, so it is a valid IFSs similarity measure.

Consider w_i is weight function corresponding to each t_i; therefore, it is defined as the IFSs weight similarity measure. Suppose that the weight function for $t_i \epsilon X$, and $\sum_{i=1}^{n} w_i = 1$, i = 1, 2, …n

$$S_{WGV}(A,B) = 1-\log_2\left[1+\frac{1}{3}\sum_{i=1}^{n} w_i\left(\begin{array}{c}\left|\sqrt{\mu_A(t_i)}-\sqrt{\mu_B(t_i)}\right|+\\ \left|\sqrt{\nu_A(t_i)}-\sqrt{\nu_B(t_i)}\right|+\\ \left|\sqrt{\pi_A(t_i)}-\sqrt{\pi_B(t_i)}\right|\end{array}\right)\right]$$

where w_i is the weight function corresponding to t_i element of IFS, A and $0 \leq t_i \leq 1$, $0 \leq w_i \leq 1$, $\sum_{i=1}^{n} w_i = 1$.

Theorem 2

The proposed weight similarity measure, $S_{WGV}(A,B)$ satisfies the axioms of weight similarity measure between A, B ϵ IFSs(X).

Proof. Postulates (SP1) to (SP4) of proposed similarity measure are necessary to prove for weight similarity measure.

(SP1):

Since $0 \leq \left|\sqrt{\mu_A(t_i)}-\sqrt{\mu_B(t_i)}\right|+\left|\sqrt{\nu_A(t_i)}-\sqrt{\nu_B(t_i)}\right|+\left|\sqrt{\pi_A(t_i)}-\sqrt{\pi_B(t_i)}\right|\leq 3$

$\Longrightarrow 0 \leq w_i\left[\left|\sqrt{\mu_A(t_i)}-\sqrt{\mu_B(t_i)}\right|+\left|\sqrt{\nu_A(t_i)}-\sqrt{\nu_B(t_i)}\right|+\left|\sqrt{\pi_A(t_i)}-\sqrt{\pi_B(t_i)}\right|\right]\leq 3w_i$

$$\Longrightarrow 0 \leq \sum_{i=1}^{n} w_i\begin{bmatrix}\left|\sqrt{\mu_A(t_i)}-\sqrt{\mu_B(t_i)}\right|+\\ \left|\sqrt{\nu_A(t_i)}-\sqrt{\nu_B(t_i)}\right|+\\ \left|\sqrt{\pi_A(t_i)}-\sqrt{\pi_B(t_i)}\right|\end{bmatrix}\leq 3\sum_{i=1}^{n} w_i$$

$$\Longrightarrow 0 \leq \frac{1}{3}\sum_{i=1}^{n} w_i\begin{bmatrix}\left|\sqrt{\mu_A(t_i)}-\sqrt{\mu_B(t_i)}\right|+\\ \left|\sqrt{\nu_A(t_i)}-\sqrt{\nu_B(t_i)}\right|+\\ \left|\sqrt{\pi_A(t_i)}-\sqrt{\pi_B(t_i)}\right|\end{bmatrix}\leq 1$$

$$\text{Now, } 1 \leq 1 + \frac{1}{3}\sum_{i=1}^{n} w_i\begin{bmatrix}\left|\sqrt{\mu_A(t_i)}-\sqrt{\mu_B(t_i)}\right|+\\ \left|\sqrt{\nu_A(t_i)}-\sqrt{\nu_B(t_i)}\right|+\\ \left|\sqrt{\pi_A(t_i)}-\sqrt{\pi_B(t_i)}\right|\end{bmatrix}\leq 2$$

$$\Longrightarrow 0 \leq \log_2\left(1+\frac{1}{3}\sum_{i=1}^{n} w_i\begin{bmatrix}\left|\sqrt{\mu_A(t_i)}-\sqrt{\mu_B(t_i)}\right|+\\ \left|\sqrt{\nu_A(t_i)}-\sqrt{\nu_B(t_i)}\right|+\\ \left|\sqrt{\pi_A(t_i)}-\sqrt{\pi_B(t_i)}\right|\end{bmatrix}\right)\leq 1$$

$$\Longrightarrow -1 \leq -\log_2\left(1+\frac{1}{3}\sum_{i=1}^{n} w_i\begin{bmatrix}\left|\sqrt{\mu_A(t_i)}-\sqrt{\mu_B(t_i)}\right|+\\ \left|\sqrt{\nu_A(t_i)}-\sqrt{\nu_B(t_i)}\right|+\\ \left|\sqrt{\pi_A(t_i)}-\sqrt{\pi_B(t_i)}\right|\end{bmatrix}\right)\leq 0$$

$$\Longrightarrow 0 \leq 1-\log_2\left(1+\frac{1}{3}\sum_{i=1}^{n} w_i\begin{bmatrix}\left|\sqrt{\mu_A(t_i)}-\sqrt{\mu_B(t_i)}\right|+\\ \left|\sqrt{\nu_A(t_i)}-\sqrt{\nu_B(t_i)}\right|+\\ \left|\sqrt{\pi_A(t_i)}-\sqrt{\pi_B(t_i)}\right|\end{bmatrix}\right)\leq 1$$

Therefore, $0 \leq S_{WGV}(A,B) \leq 1$.

(SP2) let, if A = B ⇔ $\mathrm{S}_{WGV}\left(A,B\right) = 1$.

First, Suppose that A, B ϵ IFSs(X), such that A = B.

$$\Longrightarrow \mu_A\left(t_i\right)=\mu_B\left(t_i\right),\ \text{½}_A\left(\mathrm{t}_i\right)=\text{½}_B\left(\mathrm{t}_i\right), \text{ and } \text{À}_A\left(\mathrm{t}_i\right)= \text{À}_B\left(\mathrm{t}_i\right), \text{ for all i}$$

Thus, $\mathrm{S}_{WGV}\left(A,B\right) = 1$.
Conversely, suppose, $\mathrm{S}_{WGV}\left(A,B\right) = 1$

$$\Longrightarrow 1 - \log_2\left(1+\frac{1}{3}\sum_{i=1}^{n} w_i\left[\begin{array}{c}\left|\sqrt{\mu_A\left(t_i\right)}-\sqrt{\mu_B\left(t_i\right)}\right|+\\ \left|\sqrt{\nu_A\left(t_i\right)}-\sqrt{\nu_B\left(t_i\right)}\right|+\\ \left|\sqrt{\pi_A\left(t_i\right)}-\sqrt{\pi_B\left(t_i\right)}\right|\end{array}\right]\right)=1$$

$$\Longrightarrow \log_2\left(1+\frac{1}{3}\sum_{i=1}^{n} w_i\left[\begin{array}{c}\left|\sqrt{\mu_A\left(t_i\right)}-\sqrt{\mu_B\left(t_i\right)}\right|+\\ \left|\sqrt{\nu_A\left(t_i\right)}-\sqrt{\nu_B\left(t_i\right)}\right|+\\ \left|\sqrt{\pi_A\left(t_i\right)}-\sqrt{\pi_B\left(t_i\right)}\right|\end{array}\right]\right)=0$$

$$\Longrightarrow w_i\left[\begin{array}{c}\left|\sqrt{\mu_A\left(t_i\right)}-\sqrt{\mu_B\left(t_i\right)}\right|+\\ \left|\sqrt{\nu_A\left(t_i\right)}-\sqrt{\nu_B\left(t_i\right)}\right|+\\ \left|\sqrt{\pi_A\left(t_i\right)}-\sqrt{\pi_B\left(t_i\right)}\right|\end{array}\right]=0$$

The present expression is true, when $\mu_A\left(t_i\right) = \mu_B\left(t_i\right),\ \text{½}_A\left(\mathrm{t}_i\right) = \text{½}_B\left(\mathrm{t}_i\right)$, and $\text{À}_A\left(\mathrm{t}_i\right) = \text{À}_B\left(\mathrm{t}_i\right)$
For all i.
Thus, A = B.

(SP3) It can be illustrated as the present expression is commutative,

Therefore, $\mathrm{S}_{WGV}\left(A,B\right) = \mathrm{S}_{WGV}\left(B,A\right)$.

(SP4) Let A, B, C are IFSs on X such that A ⊑ B ⊑ C,

then, $0 \le \text{¼}_A\left(\mathrm{t}_i\right) \le \text{¼}_B\left(\mathrm{t}_i\right) \le \text{¼}_C\left(\mathrm{t}_i\right) \le 1$ and $0 \le \text{½}_C\left(\mathrm{t}_i\right) \le \text{½}_B\left(\mathrm{t}_i\right) \le \text{½}_A\left(\mathrm{t}_i\right) \le 1$, for each $\mathrm{t}_i \in X$.

$$w_i\left[\begin{array}{c}\left|\sqrt{\mu_A\left(t_i\right)}-\sqrt{\mu_C\left(t_i\right)}\right|+\\ \left|\sqrt{\nu_A\left(t_i\right)}-\sqrt{\nu_C\left(t_i\right)}\right|+\\ \left|\sqrt{\pi_A\left(t_i\right)}-\sqrt{\pi_C\left(t_i\right)}\right|\end{array}\right] \ge w_i\left[\begin{array}{c}\left|\sqrt{\mu_A\left(t_i\right)}-\sqrt{\mu_B\left(t_i\right)}\right|+\\ \left|\sqrt{\nu_A\left(t_i\right)}-\sqrt{\nu_B\left(t_i\right)}\right|+\\ \left|\sqrt{\pi_A\left(t_i\right)}-\sqrt{\pi_B\left(t_i\right)}\right|\end{array}\right], \text{ for all i.}$$

$$\Longrightarrow \log_2\left(1+\frac{1}{3}\sum_{i=1}^{n} w_i \left[\begin{array}{c}\left|\sqrt{\mu_A\left(t_i\right)}-\sqrt{\mu_C\left(t_i\right)}\right|+\\ \left|\sqrt{\nu_A\left(t_i\right)}-\sqrt{\nu_C\left(t_i\right)}\right|+\\ \left|\sqrt{\pi_A\left(t_i\right)}-\sqrt{\pi_C\left(t_i\right)}\right|\end{array}\right]\right)$$

$$\geq \log_2\left(1\ \frac{1}{3}\sum_{i=1}^{n} w_i \left[\begin{array}{c}\left|\sqrt{\mu_A\left(t_i\right)}-\sqrt{\mu_B\left(t_i\right)}\right|+\\ \left|\sqrt{\nu_A\left(t_i\right)}-\sqrt{\nu_B\left(t_i\right)}\right|+\\ \left|\sqrt{\pi_A\left(t_i\right)}-\sqrt{\pi_B\left(t_i\right)}\right|\end{array}\right]\right)$$

$$\Longrightarrow 1 - \log_2\left(1+\frac{1}{3}\sum_{i=1}^{n} w_i \left[\begin{array}{c}\left|\sqrt{\mu_A\left(t_i\right)}-\sqrt{\mu_C\left(t_i\right)}\right|+\\ \left|\sqrt{\nu_A\left(t_i\right)}-\sqrt{\nu_C\left(t_i\right)}\right|+\\ \left|\sqrt{\pi_A\left(t_i\right)}-\sqrt{\pi_C\left(t_i\right)}\right|\end{array}\right]\right)$$

$$\leq 1 - \log_2\left(1+\frac{1}{3}\sum_{i=1}^{n} w_i \left[\begin{array}{c}\left|\sqrt{\mu_A\left(t_i\right)}-\sqrt{\mu_B\left(t_i\right)}\right|+\\ \left|\sqrt{\nu_A\left(t_i\right)}-\sqrt{\nu_B\left(t_i\right)}\right|+\\ \left|\sqrt{\pi_A\left(t_i\right)}-\sqrt{\pi_B\left(t_i\right)}\right|\end{array}\right]\right)$$

Therefore, S_{WGV} (A, C) $\leq$ S_{WGV} (A, B).

Similarly, S_{WGV} (A, C) $\leq$ S_{WGV} (B, C).

Thus, S_{WGV} (A, C) $\leq$ S_{WGV} (A, B) and S_{WGV} (A, C) $\leq$ S_{WGV} (B, C).

Hence, the present expression satisfies all axioms of the IFSs similarity measure in terms of weight similarity measure, so the expression is a valid IFSs weight similarity measure.

Numerical Evaluation Comparison

This section will determine the degree of similarity for various existing similarity measures. Now, in order to contrast the effectiveness of the proposed study to that of the existing measures, there are six distinct sets has been taken of IFSs A and B for an illustration, as shown in Table 1. On the basis of these similarity measures, the authors shall demonstrate the advantages and disadvantages of existing similarity measures. Table 1 shows the findings related to the proposed measures and the existing measures. Comparing existing studies to the proposed study reveals the superiority and dominance of the proposed study. In addition, demonstrates the reasonability of the proposed measure by some numerical examples. The various similarity measures cannot yield good results in a few cases. Consequently, these can't provide the authentic information, which creates the controversy for the decision-maker. Some of existing similarity measures having the potent nature of analogy. From Table 1, the authors conclude that the following measures S_L^p, S_{SY}, S_T^p, S_{CCL}, S_{YC} & S_{GV} (p=1) produce good results without any counterintuitive instances, however the remaining measures do not offer good. Moreover, S_C, S_{DC} and S_Y similarity measures are violating the necessary condition of being a similarity measure in some cases, these provides the undesirable value of similarity between some cases. It created a controversy for the present existing similarity measure and did not satisfy the axioms 2. In most of the existing similarity measures found the identical values of similarity between two IFSs A and B. For S_{HY}^{a}, S_{HY}^{b}, and S_{HY}^{c}, these are created a worse situation, since all cases have the same degree of similarity except cases 3 & 4. It is determined that the values of similarity measure S_{Y} corresponding the sets as A = { t,1,0,0 }, B = { t,0,0,1 } and C = { t,0.5,0.5,0 } are undefined. The study's findings, the proposed similarity measure is a moderated approach, it created no counterintuitive cases. On other hands, it seems that there are no counterintuitive cases are obtained and it provides the desirable values of similarity. In all cases, the values of the proposed similarity measure are well defined.

The proposed similarity measure is an aggregable and different approach among all the existing similarity measures. Table 1, classified the degree of similarity of various IFSs similarity measures with the analysis of their respective cases. The proposed similarity measure satisfied the all axioms of similarity & make a judgement of valuable study.

SOME APPLICATION OF PROPOSED SIMILARITY MEASURE

Pattern Recognition

In addition, the proposed study can be implemented as a pattern recognition problem. Moreover, the pattern recognition is a fascinating approach, it revealed the actual pattern among the number of known patterns with the help of similarity degree between them. The maximum value of similarity corresponding to any of the IFSs will provide the reasonable decision of the determining problem. This part will consider the pattern recognition problem for IFSs defined by Dengfeng & Chuntian (2002). The authors have utilized the proposed measure to classify the pattern recognition problem and providing an appropriate result.

Suppose the study relates to the pattern recognition problem between patterns and samples. Therefore, it is competent to solve the relevant situation and provide valuable results. The authors are concluded that there are r-patterns which are represented IFSs are as follows:

P_{k} = { $t_{i},\mu_{A_{k}}\left(t_{i}\right),\nu_{A_{k}}\left(t_{i}\right),\mu_{A_{k}},\nu_{A_{k}}:X\rightarrow\left[0,1\right],t_{i}\int X$ }, where $\mathrm{A}_{k}\in\mathrm{IFSs}\left(\mathrm{X}\right)$, i= 1, 2, …, n.

In this way, the present sample of the study will explain the pattern recognition problem is represented as.

B = { $t_{i},\mu_{B}\left(t_{i}\right),\nu_{B}\left(t_{i}\right),\mu_{B},\nu_{B}:X\rightarrow\left[0,1\right],t_{i}\int X$ }.

The pattern recognition problem can be determined by the most significant and desirable value of degree of similarity. Maximum value of similarity between A_{k} and B communicate the pattern recognition process. In other words, the correct pattern is determined from the maximum possible value of degree of similarity between known and unknown patterns. However, the equivalent classification will be unable to decide the correct pattern.

$$\mathrm{j}=\arg\max_{k=1,2,..,r}\left\{S\left(\mathrm{P}_{k},B\right)\right\}$$

Numerical 1. Suppose the grade labels G_{1}, G_{2} and G_{3} there corresponding some patterns P_{1}, P_{2} and P_{3} respectively. On the universe of discourse X = { $\mathrm{t}_{1},\mathrm{t}_{2},\mathrm{t}_{3}$ }, it will be represented as the composition of IFSs;

Table 1.The comparison between the degree of similarity of various existing similarity measures with the proposed study

Cases → **Sources ↓**	C_1	C_2	C_3	C_4	C_5	C_6
	K= $\zeta,0.3,0.3$ **L=** $\zeta,0.3,0.3$	**K=** $\zeta,0.3,0.4$ **L=** $\zeta,0.4,0.3$	**K=** $\zeta,1,0$ **L=** $\zeta,0,0$	**K=** $\zeta,0.5,0.5$ **L=** $\zeta,0,0$	**K=** $\zeta,0.4,0.2$ **L=** $\zeta,0.5,0.3$	**K=** $\zeta,0.4,0.2$ **L=** $\zeta,0.5,0.2$
S_{C}	1	0.900	0.500	1	1	0.950
S_{HK}	0.900	0.900	0.500	0.500	0.900	0.950
S_{LX}	0.950	0.900	0.500	0.750	0.950	0.950
S_{O}	0.900	0.900	0.300	0.500	0.900	0.930
S_{DC}	1	0.900	0.500	1	1	0.950
S_{M}	0.900	0.900	0.500	0.500	0.900	0.950
S_{e}^{p}	0.900	0.900	0.500	0.500	0.900	0.950
S_{s}^{p}	0.950	0.900	0.500	0.750	0.950	0.950
S_{h}^{p}	0.930	0.930	0.500	0.670	0.930	0.950
S_{HY}^{a}	0.900	0.900	0	0.500	0.900	0.900
S_{HY}^{b}	0.850	0.850	0	0.380	0.850	0.850
S_{HY}^{c}	0.820	0.820	0	0.330	0.820	0.820
S_{L}^{p}	0.830	0.900	0	0.130	0.830	0.900
S_{Y}	1	0.960	NA	NA	0.990	0.990
S_{SY}	0.910	0.970	0	0	0.920	0.970
S_{t}^{p}	0.970	0.900	0.500	0.830	0.970	0.950
S_{CCL}	0.970	0.900	0.500	0.830	0.970	0.940
S_{YC}	0.950	0.990	0	0	0.400	0.990
Proposed similarity measure, $\mathrm{S}_{GV}\left(A,B\right)$	0.770	0.921	0.263	0.148	0.540	0.925

$P_1 = \{ t_1, 1,0,0, t_2, 0.8,0,0.2, t_3, 0.7,0.1,0,2 \}$

$P_2 = \{ t_1, 0.8,0.1,0,1, t_2, 1,0,0, t_3, 0.9,0,0.1 \}$

$P_3 = \{ t_1, 0.6,0.2,0.2, t_2, 0.8,0,0.2, t_3, 1,0,0 \}$

It will consider as the sample B to be recognized is:

$B = \{ t_1, 0.5,0.3,0.2, t_1, 0.6,0.2,0.2, t_1, 0.8,0.1,0.1 \}$

The authors will determine the degree of similarity between the patterns P_k (k = 1,2,3) to B, by utilizing proposed IFSs similarity measure. There may be more than one value of similarity that is both unique and the same, but the right evaluation will be based on the value of similarity that is the highest. Finally, the calculated values of the proposed similarity measure have been determined from the relevant data, and their respective assessments are as follows.

$S_{GV}\left(P_1, B\right) = 0.7047$

$S_{GV}\left(P_2, B\right) = 0.6645$

$S_{GV}\left(P_3, B\right) = 0.7818.$

On the basis of calculated values of similarity, it observed that the pattern P_3 to B having the maximum value of similarity by their respective grade label P_3, thus the correct pattern is P_3. Hence, the proposed measure shows their valid result and proves that the proposed similarity measure is strongly suitable for the pattern recognition process.

Analogue to the study of similarity measure, it can also be classified as weight similarity measure. Because it behaves similarly to the similarity measure, the proposed weight similarity measure is obtained to discuss the similarity. Now, using the proposed weighted similarity measure, the authors will determine the values of each alternative based on its ideal alternative. Ye (2011) assume some weight corresponding to t_1, t_2 and t_3 are 0.5, 0.3 and 0.2 respectively. Finally, the calculated values of weight similarity degree of weight similarity measure has been determined on the relevant data, and their respective assessments are as follows.

$S_{WGV}\left(P_1, B\right) = 0.5771$

$S_{WGV}\left(P_2, B\right) = 0.6948$

$S_{WGV}\left(P_3, B\right) = 0.8651.$

Since it has the same behaviour as general study, thus the correct pattern is P_3. It is easy to explain that the proposed IFSs weight similarity measure having the identical result as of the existing. Therefore, it demonstrates that the proposed weight similarity measure is an excellent tool from the previous existing weight measures. Hence, the proposed IFSs weight similarity measure is an effective and transparent approach for pattern recognition.

Medical Diagnosis

In this section, the authors will introduce the medical diagnosis problem, with the help of the proposed IFSs similarity measure by using an example. The IFSs describing the problem are defined based on the prescribed values of membership, non-membership and the hesitation index that are pertinent to patients, symptoms, and diseases. Moreover, utilising the facts of Iqbal and Rizwan (2019), the authors are developing the IFSs medical diagnosis problem is as follows.

Numerical 2. The classification of the problem follows as, consider P, S and D are some IFSs sets denotes patients, symptoms, and diseases, respectively.

$P = \{ P_1, P_2, P_3, P_4 \}$,

$S = \{ t_1$ (Temperature), t_2 (Cough), t_3 (Throat pain), t_4 (Headache), t_5 (Body pain)},

$D = \{ D_1$ (Viral fever), D_2 (Tuberculosis), D_3 (Typhoid), D_4 (Throat disease)}.

Then, defined the IFSs on the grade values of diseases and symptoms, which is expressed as:

$$D_1 = \left\{ t_1, 0.8, 0.1, 0.1, t_2, 0.2, 0.7, 0.1, t_3, 0.3, 0.5, 0.2, t_4, 0.5, 0.3, 0.2, t_5, 0.5, 0.4, 0.1 \right\},$$
$$D_2 = \left\{ t_1, 0.2, 0.7, 0.1, t_2, 0.9, 0, 0.1, t_3, 0.7, 0.2, 0.1, t_4, 0.6, 0.3, 0.1, t_5, 0.7, 0.2, 0.1 \right\},$$
$$D_3 = \left\{ t_1, 0.5, 0.3, 0.2, t_2, 0.3, 0.5, 0.2, t_3, 0.2, 0.7, 0.1, t_4, 0.2, 0.6, 0.2, t_5, 0.4, 0.4, 0.2 \right\},$$
$$D_4 = \left\{ t_1, 0.1, 0.7, 0.2, t_2, 0.3, 0.6, 0.1, t_3, 0.8, 0.1, 0.1, t_4, 0.1, 0.8, 0.1, t_5, 0.1, 0.8, 0.1 \right\}.$$

Now, define the IFSs on grade values of patients and symptoms, it expressed as:

$$P_1 = \left\{ \begin{matrix} t_1, 65, 0.15, 0.2, t_2, 0.35, 0.45, 0.2, t_3, 0.15, 0.7, 0.15, \\ t_4, 0.55, 0.35, 0.1, t_5, 0.25, 0.25, 0.5 \end{matrix} \right\},$$
$$P_2 = \left\{ \begin{matrix} t_1, 0.35, 0.45, 0.2, t_2, 0.65, 0.2, 0.15, t_3, 0.55, 0.3, 0.15, \\ t_4, 0.45, 0.5, 0.05, t_5, 0.75, 0.15, 0.1 \end{matrix} \right\},$$
$$P_3 = \left\{ \begin{matrix} t_1, 0.15, 0.65, 0.2, t_2, 0.25, 0.3, 0.45, t_3, 0.75, 0.05, 0.2, \\ t_4, 0.25, 0.65, 0.1, t_5, 0.35, 0.55, 0.1 \end{matrix} \right\},$$
$$P_4 = \left\{ \begin{matrix} t_1, 0.45, 0.4, 0.15, t_2, 0.35, 0.4, 0.25, t_3, 0.15, 0.65, 0.2, \\ t_4, 0.55, 0.35, 0.1, t_5, 0.45, 0.50, 0.05 \end{matrix} \right\}.$$

The authors will determine the degree of similarity between prescribed values of patients and diseases by using the proposed IFSs similarity measure and it provide a proper diagnosis for the patients. If the degree of similarity between IFSs is maximum then it will be treated as correct disease of the respective patient. The classified values of the degree of similarity between patient to disease is as follows

The authors have determined that the calculated similarity values decide the proper diagnosis. From the above Table 2, which shows that patient P_1 suffering from D_3 (Typhoid) disease, patient P_2 suffering from D_2 (Tuberculosis) disease, patient P_3 suffering from D_4 (Tuberculosis) disease and patient P_4 suffering from D_3 (Typhoid) disease. Therefore, it obtain that the proposed study

Table 2. The value of similarity between patients and diseases

Diseases → Patient ↓	D_1	D_2	D_3	D_4
P_1	0.8403	0.6692	**0.8766**	0.6897
P_2	0.7495	**0.8353**	0.7617	0.7175
P_3	0.7015	0.7346	0.7638	**0.8466**
P_4	0.8351	0.7205	**0.8609**	0.6990

coincides with Iqbal & Rizwan (2019) proposed study. Now, by the above consideration, the authors can justify that the proposed IFSs similarity measure contributed to adequate diagnosis for a patient. Hence, the authors can conclude any medical diagnosis relevant problems with the help of the proposed similarity measure.

CONCLUSION

In the present study, the authors have proposed a new intuitionistic fuzzy modulus similarity measure and the weight similarity measure for IFSs. Through the survey of literature review, it determined that the various similarity measures have created certain unexpected conditions. Nevertheless, the proposed study follows a respectable methodology and provides no paradoxical evidence. Comparative analysis and validation of the developed study with some existing measures has been presented along with counterintuitive cases and it is an excellent, reliable approach for determining the better exploration of IFSs. The proposed study is a pertinent method in the domain of pattern recognition and medical diagnosis. Furthermore, the authors used numerical examples to implement the proposed study in medical diagnosis processes and pattern recognition problems. In the future, the scope of the present study can be applied to distinct higher-order fuzzy sets such as interval-valued fuzzy sets (IVFSs) and Pythagorean fuzzy sets (PFSs) in various measures for face recognition and cluster analysis, etc.

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CONFLICT OF INTEREST

The authors declared no conflict of interest.

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