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Impact of Credit Financing on the Ordering Policy for Imperfect Quality Items With Learning and Shortages

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ABSTRACT

The paper develops an order quantity model with trade credit plus shortages under learning effects for deteriorating imperfect quality products. Generally, when the lot has imperfect items, the inspection of a lot is necessary to improve the quality of the lot. In this article, the seller provides a defective lot to his buyer under credit financing scheme, and after that buyer separates the whole lot under the screening process into two categories, one is defective and the other is non-defective items. The buyer sells out defective items at a low price as compared to non-defective items. It is assumed that customers' demand of good quality items is greater than the inspection rate for the whole lot to neglect the shortages situation. After keeping all points together, the buyer optimized his total profit concerning order quantity and shortage. A suitable numerical example and a sensitivity analysis have been provided for the validity of this model. The aim and utility of this paper have been presented in the conclusion section.

KEYWORDS

Defective Items, Deterioration, Learning Concept, Shortages, Trade Credit Policy

INTRODUCTION

The quality of items damages day by day due to deterioration. There are many researchers who provided new strategy to accomplish the quality of items with the support of modern technologies like preservation technology, inspection of lots by human or sensor machinery etc. In this paper, when buyer receives the whole lot, he then inspects it and uses their strategy. In this way, we have studied literature review related to this model.

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Firstly, Porteus (1986) proposed inventory model for defective items. Deterioration and degeneration of an item occurs in situations like physical decay, their evaporation and many such factors. Whitin (1957), Ghare and Schrader (1963) and Goyal (1985) worked on deteriorating items with different strategies. Some renowned researchers discussed about the permissible delay in payment policy and established inventory models with various demand patterns under credit financing policy. We are providing some of them like Aggarwal and Jaggi (1995), Tiwari et al. (2016) and Tiwari et al. (2018) who developed an economic order quantity (EOQ) inventory model by new approach under trade credit policy in one level or two levels for decaying products. An inventory model with inflationary situation under credit period where lot has some defective items was developed by Jaggi et al. (2011). Agarwal et al. (2016) offered an economic order quantity model for profit using data mining concept. Sarkar (2016) considered discount policy in ordered quantity model with shortages. A green production model with partially backlogging situation under trade credit policy was proposed by Tiwari et al. (2018).

Some authors who worked on deteriorating imperfect quality items with different strategy under financing strategy. Jaggi et al. (2013) developed a model for declining things under credit scheme where lots have some defective items. A lot of inventory model with different approaches are developed with learning concept. An inventory model with carbon emission under supply chain management was established by Tiwari et al. (2018). Yadav et al. (2018) enhanced a traditional order quantity model by game theory approach. Wright (1936) derived an inventory model under the learning concept. After that many inventory model proposed in inventory theory. Li and Cheng (1996) discussed an inventory model with break down theory of learning under different approaches. Jaber and Bonney (1998) who presented EOQ model with new approach under learning theory. Sangal et al. (2017) developed an inventory model for declining item with learning effect. Patro et al. (2018) derived an EOQ model for decaying items, where holding and ordering cost are decreasing functions of shipment and defective percentage owes the learning curve. Jayaswal et al. (2019) discussed an EOQ problem with credit scheme under the effect of learning with the assumption that the lot has some imperfect products. In this direction, Jayaswal et al. (2019) discovered an order quantity model with financing scheme and shortages under fuzzy environment and learning effects where defective items present in the lots. The contribution Table 1 has been provided below.

LEARNING CURVE

To reduce the cost and increase the profit, the effect of learning acts as a considerable function. Some authors discussed the impact of the learning shape in the same direction as Wright (1936), Jordon (1958), and Carlson (1973). The quantity of damaged products existing in each lot is assumed by an S-shape logistic learning curve and has been shown in Figure 1.

The carrying cost, cost of ordering and percentage of defective items follow the learning effect which has been proposed by Patro et al. (2018) which are given below:

The impact of learning in the holding cost can be represented mathematically as,

$$C_h\left(n\right) = C_{ho} + \frac{C_{h1}}{n^{\alpha}}, C_{ho}, C_{h1} > 0$$

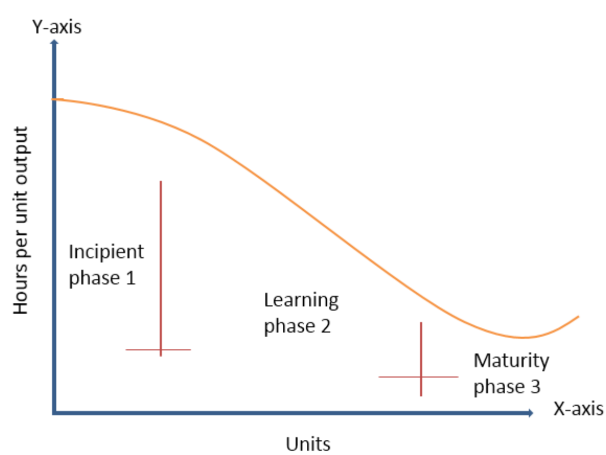
where n represents the number of orders, α is a learning factor, C_{ho} partially fixed holding cost and C_{h1} partially fixed holding cost in each shipment.

The impact of learning in the ordering cost can be represented as,

Table 1. Some selected contribution table related to learning effect

Researcher(s)	Impact of learning	Screening	Credit Scheme	Deterioration	Defective products	Shortages
Wright (1936)	✓					
Cunningham (1980)	✓					
Dutton and Thamos (1984)	✓					
Argote et al. (1990)	✓					
Salameh et al. (1993)	✓	✓				
Jaber and Bonney (1996)	✓	✓			✓	
Salameh and Jaber (2000)		✓			✓	
Jaber et al. (2008)	✓	✓			✓	
Khan et al. (2010)	✓	✓			✓	
Anzanello and Fogliatto (2011)	✓					
Jaggi et al. (2011)		✓	✓	✓	✓	
Jaggi et al. (2013)		✓	✓			
Jaggi et al. (2017)		✓	✓	✓	✓	
Patro et al. (2018)	✓	✓				
Nobil et al. (2019)					✓	
Esmaeili and Nasrabadi (2021)			✓	✓		
Barman et al. (2021)				✓		
Jayaswal et al. (2021)	✓		✓			
Jayaswal et al. (2021)	✓	✓	✓	✓	✓	
This Paper	✓	✓	✓	✓	✓	✓

Figure 1. Three Phases of learning curve



$$C_k\left(n\right) = C_{ko} + \frac{C_{k1}}{n^{\beta}}, C_{ko}, C_{k1} > 0$$

where n represents the number of orders, β is a learning factor, C_{ko} partially fixed ordering cost and C_{k1} partially fixed ordering cost in each shipment.

ASSUMPTIONS AND NOTATIONS

Assumption

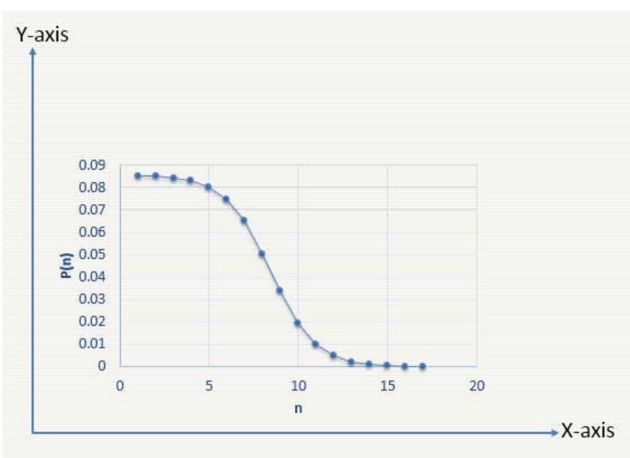
- The replacement permanency is allowed.
- Lead time is zero and shortages are involved.
- Trade credit financing policy is allowed according to Jaggi et al. (2013).
- The rate of demand and screening rate follow the relation: $\lambda > D$ (Jaggi et al., 2013)
- The Finite-time horizon plane is considered.
- The holding and ordering costs are learning affected.
- Inventory Lot contains some faulty items (Salameh and Jaber, 2000).
- Damaged products adopt the pattern of S-shape learning representation suggested by Jaber et al. (2008).
- Once the inspection processes completed, the items with imperfect quality are sold at a discounted price.
- A constant deterioration rate is performed during the cycle.

NOTATIONS

To formulate the mathematical model, the following notations are used in this research.

D Customer's demand rate (units/ year)
M Buyer's credit period time provided from seller side (year)
z Buyer's order quantity (units) which is treated as decision parameter.

Figure 2. Behavior of defective percentage on shipments

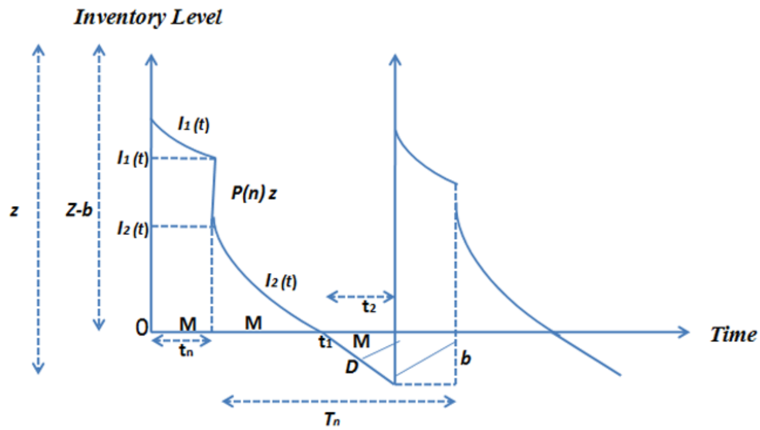


C_k The cost for ordering units (\$/ cycle)
C_p The cost for items cost (\$/ unit)
p The price for selling per unit non defective items (\$/units)
P Percentage of imperfect items in z
$P\left(n\right)$ Percentage defective items present in the lot which is owing learning curve
c_s The price for selling per unit defective items, $c_s < p$ (\$/ units)
C_s The cost for screening units (\$/ units)
θ The rate for deterioration (/year)
C_h The cost for holding units (\$/ unit)/year
λ The rate for inspection, $\lambda > D$ (\$/ unit/year)
t_n Time of Inspection (year)
T_n Buyer's cycle time(year)
b Length of backorder (units)
s Learning rate
S_2 Shortage cost ((\$/ year)
I_e The rate of interest earned (\$/units)
I_p The rate of interest paid (\$/units)
$I_1\left(t\right)$ The level of inventory when $t \in \left[0, t_n\right]$
$I_2\left(t\right)$ The level of inventory when $t \in \left[t_n, T_n\right]$
SR_i Buyer's entire trades income, for several cases where, $i = 1,2,3$ (in \$)
TC_i Buyer's total cost, for several cases, $i = 1,2,3$ (in \$)
$\Psi_i\left(z\right)$ Buyer's total profit, for several cases where, $i = 1,2,3$ (in \$)

FORMULATION OF MODEL

When buyer receives the total lot and later, he inspects the lot with screening rate λ units /time at the buyer's end. After completing inspection, two types of items separated, one is defective and other is non-defective items. In this model, we are assuming that buyer has z units after inspection process, buyer got defective $p\left(n\right)$ units and non-defective $\left(1 - p\left(n\right)\right)z$ during screening $t_n = z / \lambda$ and shortages arise in the time $t_2 = b / D$ where b is the length of backorders. The inventory level is following linear differential equation with boundary condition and modeling (figure 3). After the screening process at time t_n, the defective items are exported as a solitary batch at an affordable price c_s. Let b be the length of the deserted shortages and t_2 the time to build the level of shortages with the rate of demand D , $t_2 = b / D$ through this activity the level of inventory diminishes gradually and fulfill the customer's demand. The deterioration of items reaches to zero, at time t_1. The cycle length is $T_n = t_1 + t_2$. We are assuming that $I_1\left(t\right)$ be the inventory stock $t \in \left[0, t_n\right]$ which is reducing due to demand and deterioration and $I_2\left(t\right)$ be the stock of inventory after inspection of whole lot at t_n in the interval $t \in \left[t_n, t_1\right]$ with the partition of imperfect items which is reducing due to the presence of deterioration and customer's demand. Presently, the existing stock phase is exposed at t $\epsilon \left[0, t_n\right]$ as follows (Figure 3),

Figure 3. Presentation of inventory model under trade credit



$$\frac{dI_1(t)}{dt} + \theta I_1(t) = -D, t \in [0, t_n] \; att = 0, I_1(0) = z. \; I_1(t) = ze^{-\theta t} + \frac{D}{\theta}\left[e^{-\theta t} - 1\right] \tag{1}$$

Further, the level of inventory after complete screening at $[t = t_n]$ was calculated, got defective items and length of backorder are being removed for IEl and is shown below

$$IEl = I_1(t_s) - P(n)z = ze^{-\theta t_s} + \frac{D}{\theta}\left[e^{-\theta t_s} - 1\right] - P(n)z - b = \left(z - zP(n)\right) - Dt_s. \tag{2}$$

Again, $I_2(t)$ in the range of $t \in [t_n, t_1]$ was calculated which is following the linear differential equation under the boundary condition as follows

$$\frac{dI_2(t)}{dt} + I_2(t).\theta = -D, t_n \le t \le t_1 \; I_2(t_s) = IEl = \left(1 - P(n)\right)z - Dt_s, I_2(t_1) = 0.$$

$$I_2(t) = \frac{D}{\theta}\left[e^{\theta(t_n - t)} - 1\right] + \left[\left(1 - P(n)\right)z - Dt_n\right]e^{\theta(t_n - t)} - be^{\theta(t_n - t)} \tag{3}$$

where

$$t_n = \frac{z}{\lambda}. \tag{4}$$

at $I_2(t_1) = 0$, then we can calculate for

$$t_1 = \log\left[\frac{D/\theta}{D/\theta + z - p(n)ze^{\theta t_n} - be^{\theta t_n}}\right]^{1/\theta} \tag{5}$$

Now, the total profit per unit time at buyer's side is,

$\Psi_i\left(z,b\right) =$

Buyer's whole income from all sources – the cost for ordering items - the cost for screening units - the cost for purchasing units – the cost for holding units – the cost for shortage units – the cost for deteriorating units + the rate of interest gained – the rate of Interest paid

All the parameters of profit function have been calculated numerically from the model figure 3 which are given below:

(i) Whole revenue is derived by the sum of income shaped by the customer's demand rate that occurs all over the interval $\left[0,T_n\right]$ and trade of imperfect value items, $SR = z\left(-P\left(n\right)+1\right)p + zc_s P\left(n\right)$

(ii) Set up cost $C_k = C_o + \frac{C_{k_1}}{n^\alpha}$

(iii) Shortage cost $S_2\left(2t_n + t_2\right)b$

(iv) Deterioration cost $c\left(z - DT_n\right)b$

(v) Inspection cost $C_s z$

(vi) Purchasing cost $C_p z$

(vii) Interest gained and interest paid have been calculated for the different cases and the holding cost IHC which is $C_h\left[\int_o^{t_n} I_1\left(t\right)dt + \int_{t_n}^{t_1} I_2\left(t\right)dt\right]$.

After simplification, we can write it as,

$$C_h\left[\int_0^{t_n} I_1\left(t\right)dt + \int_{1_n}^{t_1} I_2\left(t\right)dt)\right] = C_h\left[\begin{array}{c}\frac{z}{\theta}\left[1-e^{-\theta t_n}\right] + \frac{D}{\theta^2}\left[1-e^{-\theta t_n} - \theta t_n\right] + \\ \frac{1}{\theta}\left(1-e^{\theta\left(t_n - t_1\right)}\right)\left(\left(1-P(n)\right)z - Dt_n\right) - \frac{b}{\theta}\left(1-e^{\theta\left(t_n-t_1\right)}\right)\end{array}\right]$$

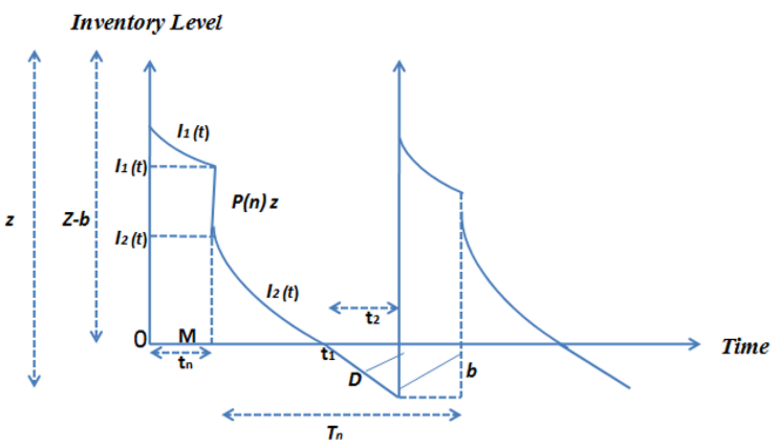
The buyer's interest gained (IE) and interest paid IP is computed according to possible cases which are given below:

Case 1: ($0 \le M \le t_n \le T_n$)

From the figure 3, in this case buyer gets total earned gain due to total revenue at time M which is equal to $pI_e DM^2 / 2$ and the buyer will have to pay in the form of interest paid from to M to T_n the seller which is equal to to $cI_p\left[\int_M^{t_n} I_1\left(t\right)dt + \int_{t_n}^{t_1} I_2\left(t\right)dt\right]$,

$$C_p I_p\left[\begin{array}{c}\frac{z}{\theta}\left[e^{-\theta M} - e^{-\theta t_n}\right] + \frac{D}{\theta^2}\left[e^{-\theta M} - e^{-\theta t_n}\right] + \frac{D}{\theta}\left[M - t_n\right] + \left[\frac{D}{\theta^2} + \frac{\left(1-P\left(n\right)\right)z - Dt_n}{\theta}\right]\left[1-e^{\theta\left(t_n-t_1\right)}\right] \\ + \frac{D}{\theta}\left[t_n - t_1\right] - \frac{b}{\theta}\left(1-e^{\theta\left(t_n-t_1\right)}\right)\end{array}\right].$$

Figure 4. Presentation of inventory model under trade credit for the case 1



$$TC_1 = TC + IP$$

$$TC_1 = C_k + C_s z + C_p z + IHC + S_1 \left(2t_n + t_1\right) b + c\left(z - DT_n\right)$$

$$TC_1 = TC + IP$$

$$TC_1 = C_k + C_s z + C_p z + IHC + S_1 \left(2t_n + t_1\right) b + c\left(z - DT_n\right) \tag{7}$$

From eq. (8) and eq. (9), the total profit function is

$$\Psi_1\left(z,b\right) = \frac{SR_1 - TC_1}{T_n}$$

$$\Psi_1\left(z,b\right) \frac{\begin{array}{l} p\left(1-P\left(n\right)\right)z + c_s P\left(n\right)z + \frac{1}{2} p I_e D M^2 \\ -C_k - C_p z - C_s z - C_h \begin{bmatrix} \frac{z}{\theta}\left[1-e^{\theta t_n}\right] - \frac{D}{\theta^2}\left[e^{-\theta t_n} + \theta t_n - 1\right] \\ + \frac{1}{\theta}\left[1-e^{\theta\left(t_n - t_1\right)}\right]\left[\left(1-P\left(n\right)\right) - D t_n\right] \\ -\frac{D}{\theta^2}\left[\left(e^{\theta\left(t_n - T_n\right)}\right) + \left(t_1 - t_n\right)\theta - 1\right] - \frac{b}{\theta}\left(1-e^{\theta\left(t_n - t_1\right)}\right) \end{bmatrix} - S_1 b\left(2t_n + t_1\right) - c\left(z - DT_n\right) \\ -C_p I_p \begin{bmatrix} \frac{z}{\theta}\left[e^{-\theta M} - e^{-\theta t_n}\right] + \frac{D}{\theta^2}\left[e^{-\theta M} - e^{-\theta t_n}\right] + \\ \frac{D}{\theta}\left[M - t_n\right] + \left[\frac{D}{\theta^2} + \frac{\left(1-P\left(n\right)\right)z - D t_n}{\theta}\right]\left[1-e^{\theta\left(t_n - t_1\right)}\right] + \frac{b}{\theta}\left(1-e^{\theta\left(t_n - t_1\right)}\right) \end{bmatrix} \end{array}}{T_n} \tag{8}$$

Case 2: $0 \le t_n \le M \le T_n$

Figure 5, explains that the buyer gets total earned gain due to total revenue at time *M* and non-defective items at time $\left(M - t_1\right)$ and shortages which is $pI_e DM^2 / 2 + c_s P\left(n\right) I_e z\left(M - t_n\right) + b\left(M - t_n\right)$ and the buyer will have to pay in the form of interest paid to the seller which is equal to

$$cI_p\left[\int_M^{t_1} I_2\left(t\right)dt\right],\ C_p I_p\left[\left[\frac{D}{\theta^2} + \frac{\left(1 - P\left(n\right)\right)z - Dt_1}{\theta}\right]\left[1 - e^{\theta\left(M - t_1\right)}\right] + \frac{D}{\theta}\left[M - t_1\right] - \frac{b}{\theta}\left(1 - e^{\theta\left(M - t_1\right)}\right)\right].$$

The entire revenue and entire cost both are given as:

$$\begin{aligned} TC_2 &= TC + IP \\ &= C_k + C_s z + C_p z + IHC + DC + SC \\ &+ C_p I_p\left[\left[\frac{D}{\theta^2} + \frac{\left(1 - P\left(n\right)\right)z - Dt_1}{\theta}\right]\left[1 - e^{\theta\left(M - t_1\right)}\right] + \frac{D}{\theta}\left[M - t_1\right] - \frac{b}{\theta}\left(1 - e^{\theta\left(M - t_1\right)}\right)\right]. \end{aligned}$$

The buyer' total profit is given below,

$$\Psi_2\left(z,b\right) = \frac{SR_2 - TC_2}{T_n}$$

$$\Psi_2\left(z,b\right)\frac{\begin{array}{l} p\left(1 - P\left(n\right)\right)z + c_s P\left(n\right)z + \frac{1}{2}pI_e DM^2 + c_s P\left(n\right)I_e z\left(M - t_n\right) + C_k - C_p z \\ -C_s z - c\left(z - DT_n\right) - S_1 b\left(2t_n + t_1\right) \\ -C_h\left[\begin{array}{l} \frac{z}{\theta}\left[1 - e^{\theta t_n}\right] - \frac{D}{\theta^2}\left[e^{\theta t_n} + \theta t_n - 1\right] \\ + \frac{1}{\theta}\left[1 - e^{\theta\left(t_n - T_n\right)}\right]\left[\left(1 - P\left(n\right)\right) - Dt_n\right] \\ - \frac{D}{\theta^2}\left[\left(e^{\theta\left(t_n - T_n\right)}\right) + \left(T_n - t_n\right)\theta - 1\right] - \frac{b}{\theta}\left(1 - e^{\theta\left(t_n - t_1\right)}\right) \end{array}\right] \\ C_p I_p\left[\begin{array}{l} \frac{D}{\theta^2} + \frac{\left(1 - P\left(n\right)\right)z - Dt_1}{\theta}\left[1 - e^{\theta\left(M - t_1\right)}\right] \\ + \frac{D}{\theta}\left[M - t_1\right] - \frac{b}{\theta}\left(1 - e^{\theta\left(M - t_1\right)}\right) \end{array}\right] \end{array}}{T_n} \tag{9}$$

Case 3**:** $0 \le t_n \le T_n \le M$

From Figure 6, in this case buyer gets total earned gain due to total revenue at time M and non-defective items at time $\left(M - t_1\right)$ which is

$$pI_e Dt_1^2 / 2 + c_s P\left(n\right) I_e z\left(M - t_n\right) + pI_e Dt_1\left(M - t_1\right) + bpIe\left(M - t_n\right)$$

Figure 5. Presentation of inventory model under trade credit for the case 2

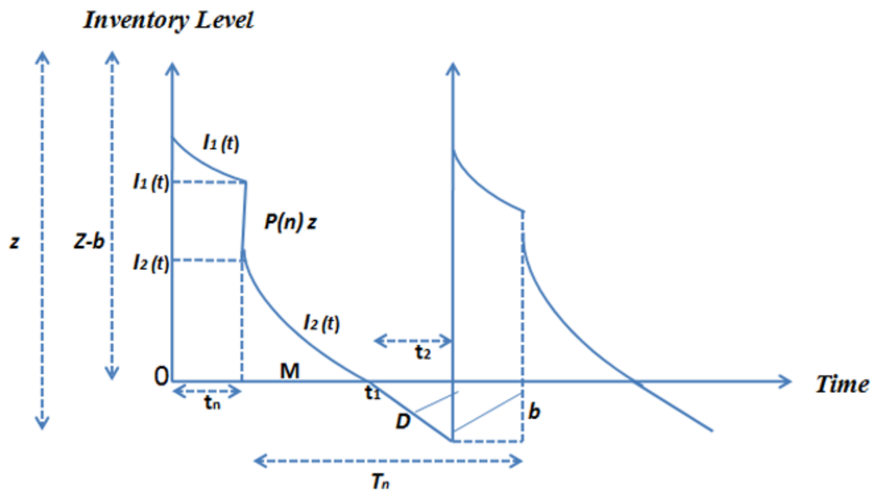
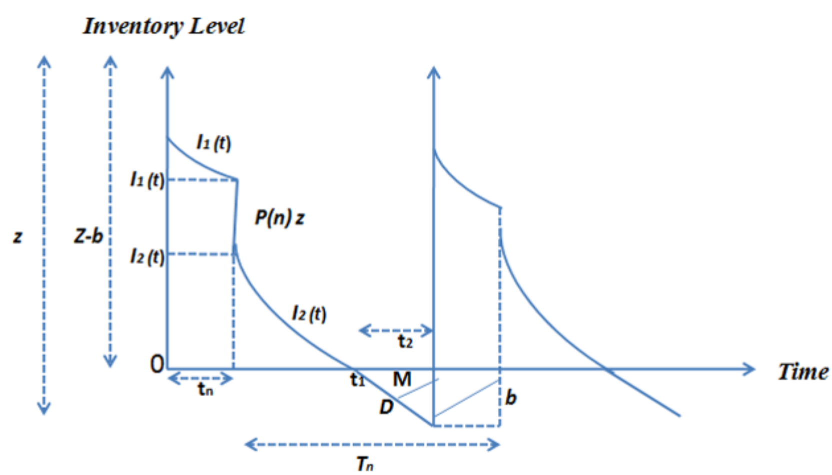


Figure 6. Presentation of inventory model under trade credit for the case 3



and does not give interest to the seller due to no boundary of trade credit.

$$TC_3 = TC + IP$$

$$= C_k + C_s z + C_p z + IHC + DC + SC$$

The buyer's total profit is given under below,

$$\Psi_3\left(z,b\right)=\frac{SR_3-TC_3}{T_n}$$

$$\Psi_3\left(z,b\right)\frac{\begin{matrix}\left(pz-pzP\left(n\right)\right)+c_s zP\left(n\right)+\frac{1}{2}pI_e Dt_1^{\,2}+c_s P\left(n\right)I_e z\left(M-t_n\right)+pI_e Dt_1\left(M-t_1\right)\\ -C_k-C_p z-c\left(z-DT_n\right)-S_1\left(2t_n+t_1\right)b-C_s z\\ -C_h\left[\begin{matrix}\frac{z}{\theta}\left[1-e^{\theta t_n}\right]-\frac{D}{\theta^2}\left[e^{\theta t_n}+\theta t_n-1\right]\\ +\frac{1}{\theta}\left[1-e^{\theta\left(t_n-T_n\right)}\right]\left[\left(1-P\left(n\right)\right)-St_n\right]\\ -\frac{D}{\theta^2}\left[\left(e^{\theta\left(t_n-T_n\right)}\right)+\left(T_n-t_n\right)\theta-1\right]-\frac{b}{\theta}\left(1-e^{\theta\left(t_n-t_1\right)}\right)\end{matrix}\right]\end{matrix}}{T_n} \tag{10}$$

Here, $\Psi\left(z,b\right)$ is the buyer's total profit, which is defined below with concern cases,

$$\Psi\left(z,b\right)=\begin{cases}\Psi_1\left(z,b\right)0\le M\le t_n\le T_n case-1\\ \Psi_2\left(z,b\right)0\le t_n\le M\le T_n case-2\\ \Psi_3\left(z,b\right)0\le t_n\le T_n\le M case-3\end{cases} \tag{11}$$

SOLUTION PROCESS

For optimal values of z and b, if we take $\frac{\partial\Psi\left(z,b\right)}{\partial z}=0,$ and $\frac{\partial\Psi\left(z,b\right)}{\partial b}=0,$ to all cases and

The buyer's total profit will show the property of concavity if

$$\left(\frac{\partial^2\Psi\left(z,b\right)}{\partial z\partial b}\right)^2-\left(\frac{\partial^2\Psi\left(z,b\right)}{\partial z^2}\right)\left(\frac{\partial^2\Psi\left(z,b\right)}{\partial b^2}\right)\le 0 \text{ and } \left(\frac{\partial^2\Psi\left(z,b\right)}{\partial z^2}\right)\le 0,\ \left(\frac{\partial^2\Psi\left(z,b\right)}{\partial b^2}\right)\le 0,$$

Graphically, it is shown in figure 7.

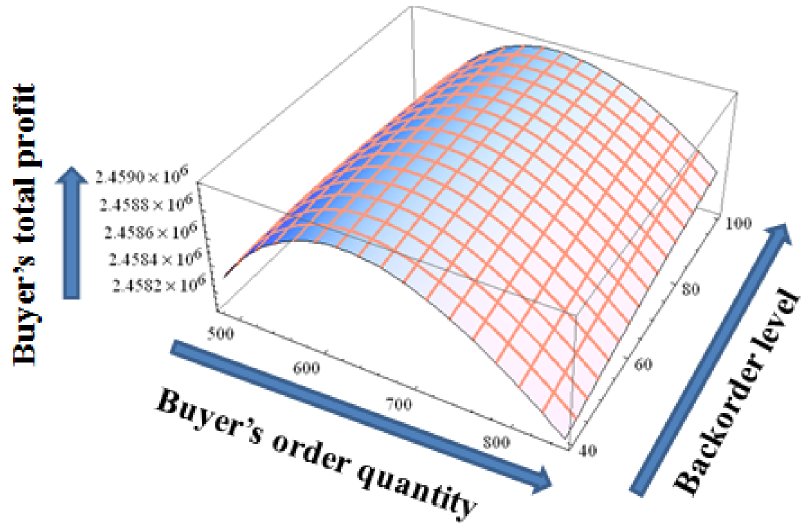
SOLUTION PROCESS FOR ALL CASES

For optimal worth, we get $\frac{\partial\Psi\left(z,b\right)}{\partial z}=0,$ and $\frac{\partial\Psi\left(z,b\right)}{\partial b}=0,$

$$z=z^* \text{ and } b=b^* \tag{12}$$

This is envisioned with Mathematica tool, and we got the optimal worth corresponding to these optimal values, from eq (10)

Figure 7. Concavity of the profit function



$$\Psi\left(z,^* b^*\right) = \frac{SR_1 - TC_1}{T_n} \tag{13}$$

ALGORITHM

In this section an algorithm is used for the best possible of trade credit period, lot size and back orders (Shin et al., 2016).
All model parameters

$\left[I_e, IHC, I_p, D, I_p, C_p, C_s, C_{k,}, n, \alpha, \beta, \theta, P\left(n\right), M, p, c_s, \lambda, S_1, c, b, a, g, e, n\right]$ used in equation (13).

Step 2: In this part, we supposed that the optimal buyer's lot size $z^* = z_s\left(say\right)$ and $b^* = b_s\left(say\right)$ from equation (12) and substituting these in equation (13) after that we calculated buyer's lot size using Mathematica software and find out the inspection time $t_n = z / \lambda$ and buyer's total cycle length $T_n = t_1 + t_2$ from equation (4) and equation(5). If it holds the case 1 ($0 \le M \le t_n \le T_n$), then calculate the buyer's total profit from the equation (8) otherwise move to next step.
Step 3: If case 1 does not hold then, we find out the buyer's lot size with help of Mathematica software and suppose that $z^* = z_t\left(say\right)$ is the lot size optimized value and $b^* = b_z\left(say\right)$ is the optimal backorder from eq. (12). The lot size and backorder optimized values both are then substituted in equation (13) and further buyer's profit was calculated. After that, we calculated t_n and T_n with help of the equations (4) and (5). After getting the values of t_n, T_n and *M* and if it holds the case 2

($0 \le t_n \le M \le T_n$), then calculate the buyer's total profit from the equation (9) otherwise move to next step.

Step 4: Working task is same as mentioned in step 1 and step 2, we find out the buyer's lot size with help of Mathematica software and suppose that $z^* = z_t\left(say\right)$ and $b^* = b_l\left(say\right)$ using (12) and (13). After that we calculated t_n and T_n with help of the equations (4) and (5). After getting the values of t_n, T_n and *M* and if it holds the case 3 ($0 \le t_n \le T_n \le M$), then calculate the buyer's total profit from the equation (10).

Step 5: In this step, we compare all the possible cases related to buyer's total profit with suitable inventory parameters. After that we decided with the help of algorithm which case is better for this model.

NUMERICAL EXAMPLE

The input parameters have been taken from Patro et al. (2018) and case 2 has been considered best case for the calculation of the buyer's total profit according to the algorithm.

From the algorithm, we got buyer's lot size, $z^* = 653\left(units\right)$ per year and $b^* = 96\left(units\right)$.

Substituting the optimal values of $z^* = 653\left(units\right)$ and $b^* = 96\left(units\right)$ in Equation (11), Eq. (4) and Eq. (5), we got the retailer's profit as, $\Psi_2\left(z^*\right) = 2458980\$$ and inspection time is $t_n = \frac{z}{\lambda} = 0.0037\text{year}$ and the buyer's total cycle length is $T_n = t_1 + t_2 = 0.0100 year$.

When learning concept is not applied in this model then, buyer's lot size, backorder size and total profit which are $z^* = 680 b^* = 95 and \Psi_2\left(z^*, b^*\right) = 2458900\$$ respectively.

SENSITIVITY ANALYSIS

The sensitivity of this model on the various parameters have been shown with the help of tables.

Further, change in rate of interest rate will be analyzed with the help of figure 8.

OBSERVATION

(i) It is analyzed with the Table 2, when the value of M increases, the buyer's profit and cycle length increase whereas lot size decreases and backorder become constant.

(ii) From the table 3, it is detected that when learning rate increases, buyer's total profit and backorder increases whereas the lot size decreases.

Table 2. Effect of Trade credit on the buyer's lot size, cycle length, backorders, inspection time and profit

Credit period M (year)	**Time of Inspection** t_n **(year)**	**Buyer's cycle length** T_n **(Year)**	**Buyer's lot size (Units)**	**Backorder** b (Units)	**Buyer's total profit** $\Psi_2\left(z\right)$ **($)**
0.0054	0.0037	0.0100	653	96	2458980
0.0082	0.0035	0.0090	629	96	2459930
0.0109	0.0034	0.0119	600	90	2460990

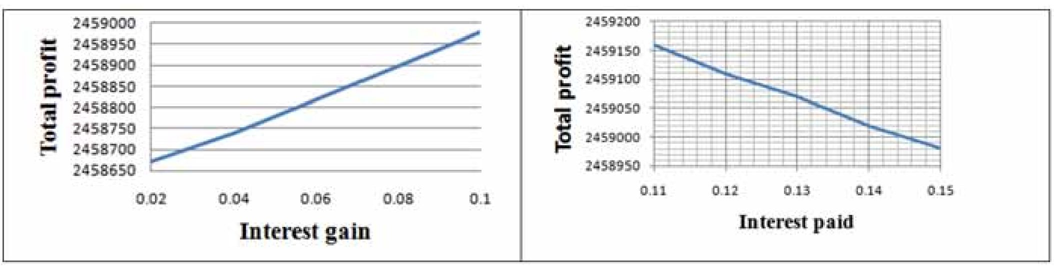
Table 3. Impact of changing learning rate s on order quantity, backorder and total profit

Credit period M (Year)	Number of shipments n	Learning rate s	Buyer's lot size z (Units)	Backorder b (Units)	Buyer's total profit $\Psi_2(z)$ ($)
0.005	4	0.79	653	96	2458930
0.005	4	1.30	645	99	2459440
0.005	4	1.40	642	100	2459640

Table 4. Impact of changing number of shipments on percentage defective, order quantity, backorder and total profit at buyer's side

Shipments number (n)	Percentage of defective $P(n)$	Trade credit period M (year)	Buyer's lot size z (units)	Backorder b (Units	Buyer's total profit $\Psi_2(z)$ ($)
1	0.08524	0.005	651	101	2458460
2	0.08497	0.005	652	98	2458710
3	0.08436	0.005	653	96	2458850
4	0.08305	0.005	653	95	2458980

Figure 8. Impacts of interest gain and interest paid on profit function



(iii) It is seen from the Table 4, if the number of shipments increases, initially the level of backorder and order of quantity increase up to the 4$^{\text{th}}$ shipment but there after it decreases and the profit of buyer increases due to the learning effect in $C_h(n), C_k(n)$ and $p(n)$.

(iv) From Figure 8, if I_e increases, then buyer's profit increases but I_p increases then profit decreases.

DISCUSSION

In this section, we have analyzed from the algorithm that case 2 is suitable for the proposed inventory model. Case 1 and case 3 are not providing suitable solutions for this inventory model. This is so as in case 1, the buyer gets very less trade credit period and have to face more economical problem when credit items are not sold in the credit time period, finally buyer is not interested to select case 1. In case 3, buyer gets more time, but seller does not agree for long time period for payment. In the end case 2 is beneficial for both players. The Case 2 has been considered for sensitivity analysis.

CONCLUSIONS

In this paper, an EOQ model has been proposed with trade credit financing and leaning effect where shortages are allowed. The learning concept provided positive effect on the total profit. This paper tried to develop a mathematical formula to find out the maximum total profit with respect to order quantity and shortages, where holding and ordering cost are the function of shipment and defective percentage follows the learning concept. Finally, the presence of trade credit financing, the buyer got positive impact on the total profit. The present paper can be extended with the concept of carbon emissions as well as preservation technology.

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