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Fuzzy Multi-Objective Linear Programming Problem Using DM's Perspective

Vishnu Pratap Singh, Visvesvaraya National Institute of Technology, Nagpur, India*
https://orcid.org/0000-0002-6846-6187

Madhura Deshmukh, Visvesvaraya National Institute of Technology, Nagpur, India

Kirti Sharma, Visvesvaraya National Institute of Technology, Nagpur, India

ABSTRACT

In this paper, a two-stage method has been proposed for solving fuzzy multi-objective linear programming problem (FMOLPP) with interval type-2 triangular fuzzy numbers (IT2TFNs) as its coefficients. In the first stage of problem solving, the imprecise nature of the problem has been handled. All technological coefficients given by IT2TFNs are first converted to a closed interval and then the objectives are made crisp by reducing a closed interval into a crisp number, and constraints are made crisp by using the concept of acceptability index. The amount by which a specific constraint can be relaxed is decided by the decision maker, and thus, the problem reduces to a crisp multi-objective linear programming problem (MOLPP). In the second stage of problem solving, the multi-objective nature of the problem is handled by using fuzzy mathematical programming approach. In order to explain the methodology, two numerical examples of the proposed methodology in production planning and diet planning problems have also been worked out in this paper.

KEYWORDS

Acceptability Index, Fuzzy Programming Approach, Interval Linear Programming, Interval Type-2 Triangular Fuzzy Numbers, Multi-Objective Linear Programming Problem

1. INTRODUCTION

Optimizing a linear objective subject to certain constraints is one of the most pioneering problems in the field of operations research. The problem finds its application in manufacturing, construction, logistics, energy and many others. Various classes of algorithms based on simplex method (Taha, 2011), ellipsoid method and interior point method have been defined in the literature to solve a linear programming problem (LPP). But in real life situation, precision of data is not always guaranteed and a precise data may also lead to higher information cost. Ordinary methods like simplex method, graphical method, interior point method etc. cannot deal with vagueness and complexity and characteristics like certainty, simplicity and preciseness are not always guaranteed in real life problems. Thus LPP being able to deal with vague and imprecise data greatly contribute to its diffusion and application. So, in order to reduce the information costs and at the same time enhance real life modeling, the use of fuzzy coefficients (Zimmermann, 2011) in LPP was evolved.

Bellman and Zadeh (Bellman, 1970) were first to encounter the problem of decision making in such environment. Various works related to LPP in fuzzy environment have been done in last decades and some of them are (Veeramani, 2014) where Veeramani et. al used trapezoidal fuzzy numbers to

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denote imprecise coefficients and an algorithm based on simplex method has been used to solve the problem. In (Shaocheng, 1994), Tong S proposed methods to solve imprecise LPP with interval numbers and fuzzy numbers as its coefficients. In case of interval numbers as coefficients, the problem is further reduced to two crisp LPPs and then an optimal interval solution is sought. In case of fuzzy numbers as coefficients, α-cuts are used to reduce fuzzy numbers to intervals and the problem is solved as an Interval Linear Programming problem (ILPP). ILPP has also been dealt by Sengupta et.al in (Sengupta A. a., 2001). He interpreted inequality constraints by the use of acceptability index and reduced ILPP to a LPP. A transformation method has been proposed by (Suprajitno, 2021) to solve ILPP. Two solution approaches to inner estimation of optimal solution set in ILPP has been proposed in (HladIk, 2020). An application of ILPP for sustainable selection of marine renewable energy projects is presented in (Akbari, 2021). Various researchers like Moore, Ishibuchi and Tanaka, Sengupta et. al have done extensive research on interval arithmetic, readers may refer (Moore, 1979), (Ishibuchi, 1990) and (Sengupta A. a., 1997). An outcome range problem in ILPP has been discussed in (Mohammadi, 2021). There are various methods by which a fuzzy LPP has been solved. Naseeri et. al (Nasseri, 2005) used simplex method to solve LPP with fuzzy variables. The problem has been solved by Maleki et. al in (Maleki, 2000) and they used the Maleki ranking function for defuzzification process. S. Suneela et. al also used a ranking function to defuzzify a LPP and reduced it to a crisp LPP in (Suneela, 2019). In (Kumar, 2011), a fully fuzzy LPP has been handled by Kumar et. al and the method proposed solves the fuzzy LPP with equality constraints without defuzzification and returns a fuzzy optimal solution. A new method for solving LPP with IT2FN has been discussed in (Javanmard, A solving method for fuzzy linear programming problem with interval type-2 fuzzy numbers, 2019). Linear programming problem with IT2TFN has also been discussed in (Dalman, 2019).

In most of the businesses and real life applications, usually more than one objective is to be optimized in an imprecise environment. This gives rise to FMOLPP. FMOLPP considers an optimization problem which includes optimization of more than one conflicting objectives. The objectives are said to be conflicting in nature when any improvement in one can only be achieved at the expense of other. For example, in case of a firm manufacturing two products A and B, the objective of maximizing profit and minimizing the number of products to be manufactured are conflicting in nature. With an increase in number of products manufactured, the profit is supposed to increase and vice versa. The firm here is in search of an efficient solution (Singh S. K., 2018) and such problems can be solved by using the concepts of MOLPP. The diet planning problem for cardiovascular patients has been handled by using multi-objective LPP in (Eghbali, 2019). Kalyanmoy Deb has discussed the details of MOLPP and the methods to solve such problems in (Deb, 2001). In (Ishibuchi, 1990), (Batamiz, 2020) multi objective programming has been discussed in case of interval objective functions. Fuzzy multi-objective LPP with impreciseness represented by interval coefficients or interval approximation and interval type 2 fuzzy numbers is dealt in (Sharma, 2018), (Ammar, 2020) (Batamiz, 2020) and (Javanmard, Solving the interval type-2 fuzzy linear programming problem by the nearest interval approximation, 2018), (Umarusman, 2021) and (Li, 2019) respectively. Fully fuzzy multi-objective linear programming problem with fuzzy dominant degrees is presented in (Van Hop, 2020) and a geometric programming approach to solve imprecise LPP is proposed in (Islam, 2019). In (Kuwano, 1996), a method for solving FMOLPP has been presented.

In this paper, we have proposed a two-stage solution method for solving FMOLPP with IT2TFNs as technological coefficients. The problem is first reduced to an interval linear programming problem by using nearest interval approximation method (Grzegorzewski, 2002) whose constraints are then relaxed according to aspiration level of the decision maker by using acceptability index, as defined in (Sengupta A. a., 2000). The obtained MOLPP is then solved for an efficient solution by using fuzzy mathematical programming approach (Chanas, 1989). To the best of author's knowledge, there is no method defined in the literature to solve FMOLPP with IT2TFN in which the perspective of decision maker is also considered. This paper is organized as follows: In section 2, some definitions and concepts regarding fuzzy set theory and comparison of interval numbers has been reviewed. In section 3, a FMOLPP with IT2TFN as its coefficients has been formulated and its solution methodology has been discussed. The algorithm for solving the problem has been proposed in section 4. A numerical example has been presented in section 5. The last section comprises of concluding remarks.

2. DEFINITIONS

Definition 1. Membership function (Klir, 1996).

The membership function $\mu_{\tilde{A}}\left(x\right)$ is a function mapping the elements of the universal set X to the unit interval $\left[0,1\right]$.

Definition 2. Fuzzy set (Zimmermann, 2011).

If X is a universe of discourse and x is a particular element of X, then a fuzzy set A defined on X can be written as collection of ordered pairs, $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$, where $\mu_{\tilde{A}}\left(x\right)$ is known as membership degree.

In case of fuzzy sets, the grade of membership assigned to each element is a crisp number, but sometimes uncertainty even gets involved in assigning membership degree to elements of universal set. In such cases, the membership grades assigned to each element are fuzzy sets.

Definition 3. Type 2 Fuzzy Sets (Zimmermann, 2011).

The fuzzy sets in which the degree of membership assigned to each element is given by a fuzzy set are known as type-2 fuzzy sets. The membership function of a type-2 fuzzy set is defined as:

$$\mu_{\tilde{A}}\left(x\right) : X \to \mathrm{F}\left[0,1\right] \tag{1}$$

where $\mathrm{F}\left[0,1\right]$ denotes the set of all ordinary fuzzy sets that can be defined in $\left[0,1\right]$. Then $\tilde{A}$ is expressed as:

$$\tilde{A} = \left\{\left(\left(x,u\right), \mu_{\tilde{A}}\left(x,u\right)\right) : \forall x \in X, \forall u \in P_x \subseteq \left[0,1\right]\right\} \tag{2}$$

Definition 4. Interval Type 2 Fuzzy set.

If in the above definition of type-2 fuzzy set, if $\mu_{\tilde{A}}\left(x,u\right) = 1$ for all x and u then it is called interval type 2 fuzzy set. Mathematically:

$$\tilde{A} = \left\{\left(\left(x,u\right),1\right) : \forall x \in X, \forall u \in P_x \subseteq \left[0,1\right]\right\} \tag{3}$$

Thus, when a sub-interval of $[0,1]$ represents the membership grade of an element of universal set, then such a fuzzy set is known as interval type-2 fuzzy set.

The introduction of interval type-2 fuzzy sets solved the problem of representation of uncertainty in membership degree.

Definition 5. Interval Type 2 Triangular Fuzzy Number (Zimmermann, 2011).

An IT2TFN, $\tilde{\tilde{A}}$ is defined on the interval $[\bar{a},\bar{c}]$, its lower membership function (LMF) and upper membership function (UMF), takes the value equal to $\underline{h} \in [0,1]$ at $\underline{b}$ and $\bar{h} \in [0,1]$ at $\bar{b}$, respectively where $\bar{a} \le \underline{a} \le \underline{b} = \bar{b} \le \underline{c} \le \bar{c}$. The shaded area in **Figure 1** is known as foot-print of uncertainty (FOU).

Thus, IT2TFN $\tilde{\tilde{A}}$ is notated as $\tilde{\tilde{A}} = (\underline{A}, \bar{A}) = ((\underline{a}, \underline{b}, \underline{c}), (\bar{a}, \bar{b}, \bar{c}))$.

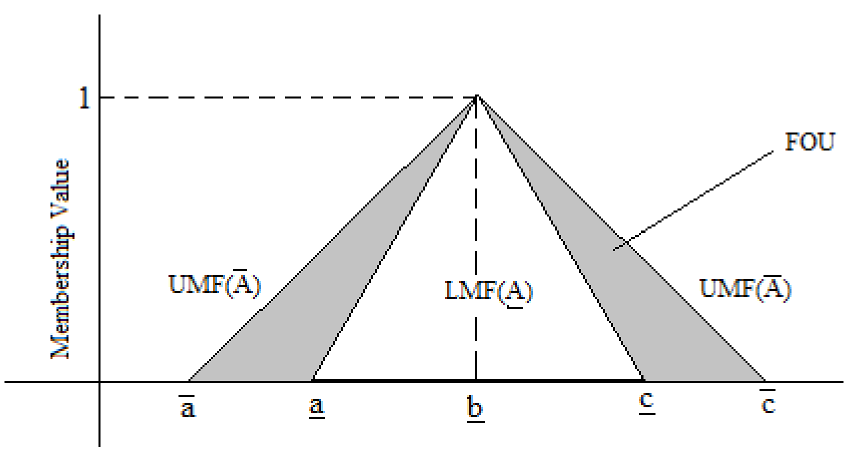
Definition 6. Conversion of Interval Type-2 TFN to a closed interval: Suppose $\tilde{\tilde{A}} = (\underline{A}, \bar{A})$ is an IT2TFN and $\left[\left[\tilde{\underline{A}}_L(\alpha), \tilde{\underline{A}}_R(\alpha)\right], \left[\tilde{\bar{A}}_L(\alpha), \tilde{\bar{A}}_R(\alpha)\right]\right]$ is α cut. Then, the closed interval $C(\tilde{\tilde{A}}) = [\underline{C}, \bar{C}]$ is the nearest interval approximation to $\tilde{\tilde{A}}$ where:

$$\underline{C} = \frac{1}{2}\int_0^1 \left(\tilde{\underline{A}}_L(\alpha) + \tilde{\bar{A}}_L(\alpha)\right) d\alpha$$

$$\bar{C} = \frac{1}{2}\int_0^1 \left(\tilde{\underline{A}}_R(\alpha) + \tilde{\bar{A}}_R(\alpha)\right) d\alpha$$

Remark:

Figure 1. An IT2TFN



An interval $A = [a_L, a_R]$ can be alternatively represented as $A = \langle m(A), w(A) \rangle$ where $m(A) = \frac{a_L + a_R}{2}$ and $w(A) = \frac{a_R - a_L}{2}$.

Definition 7. Sengupta et. al (Sengupta A. a., 2000) defined an acceptability index to order two intervals in terms of value. Let I be the set of all closed intervals on the real line $\mathbb{R}$. Then an acceptability index can be defined as:

$$\mathrm{A} : I \times I \rightarrow [0, \infty)$$

such that:

$$\mathrm{A}(A \leq B) = \frac{m(B) - m(A)}{w(B) + w(A)} \tag{4}$$

Eq. (4) explains the grade of acceptability of "first interval A to be inferior to second interval B". The premise $A < B$ is considered only when $m(A) < m(B)$.

3. PROBLEM FORMULATION AND MODEL DEVELOPMENT

A standard form of FMOLPP with IT2TFNs as its coefficients is given by:

$$Maximize \quad \tilde{\tilde{Z}}(x) = \left[\tilde{\tilde{Z}}_1(x), \tilde{\tilde{Z}}_2(x), \ldots, \tilde{\tilde{Z}}_k(x)\right]$$

subject to:

$$\begin{aligned} \sum \tilde{\tilde{a}}_{ij} x_j &\leq \tilde{\tilde{b}}_i & \quad i &= 1, 2, \ldots, m \\ x_j &\geq 0 & \quad j &= 1, 2, \ldots, n \end{aligned} \tag{5}$$

where $\tilde{\tilde{Z}}_s(x) = \sum_{j=1}^{n} \tilde{\tilde{c}}_{sj} x_j$ for $s = 1, 2, \ldots, k$.

First we use the concept of nearest interval approximation in and convert the FMOLPP with IT2TFNs to a multi-objective interval LPP (MOILPP). After this, the coefficients of the original problem are reduced to an interval number and the new problem is:

$$Maximize \quad Z(x) = \left[Z_1(x), Z_2(x), \ldots, Z_k(x)\right]$$

subject to:

$$\begin{aligned} &\sum\left[a_{Lij}, a_{Rij}\right]x_j \le \left[b_{Li}, b_{Ri}\right] && i = 1,2,\ldots,m \\ &x_j \ge 0 && j = 1,2,\ldots,n \end{aligned} \tag{6}$$

where $Z_s\left(x\right) = \sum_{j=1}^{n}\left[c_{sLj}, c_{sRj}\right]x_j$ for $s = 1,2,\ldots,k$.

In order to get the optimal values of all the objectives, the problem should now be reduced to a LPP, which can be done when a rational decision maker relax the constraints. Let the threshold of decision maker (DM) be given by α, where $\alpha \in \left[0,1\right]$. In case of strict constraints, α is chosen to be 0 and in most relaxed situations, α is chosen to be 1. A spectrum of possibilities within the range of unit interval $\left[0,1\right]$ is allowed for α and it interprets the threshold of DM. The threshold value can be same as well as different for all the constraint in the problem. So, a satisfactory crisp equivalent form of constraints of Eq. (6) is defined as:

$$\sum\left[a_{Lij}, a_{Rij}\right]x_j \le b_i = \begin{cases} \sum a_{Rij}x_j \le b_{Ri} \\ \mathrm{A}\left(\sum\left[a_{Lij}, a_{Rij}\right]x_j \ge b_i\right) \le \alpha \end{cases} \tag{7}$$

The objective functions of Eq. (6) can be defuzzified by using $\frac{1}{2}\sum_{j=1}^{n}\left(c_{sLj} + c_{sRj}\right)x_j$ instead of $\sum_{j=1}^{n}\left[c_{sLj}, c_{sRj}\right]x_j$. Then the MOILPP can be converted to a MOLPP, which is given by:

$$Maximize \quad Z\left(x\right) = \left[Z_1\left(x\right), Z_2\left(x\right), \ldots, Z_k\left(x\right)\right]$$

subject to:

$$\begin{aligned} &\sum_{j=1}^{n} a_{Rij}x_j \le b_{Ri} \\ &\left(b_{Li} + b_{Ri}\right) - \sum_{j=1}^{n}\left(a_{Lij} + a_{Rij}\right)x_j \le \alpha\left(b_{Ri} - b_{Li}\right) + \alpha\left(\sum_{j=1}^{n}\left(a_{Rij} - a_{Lij}\right)x_j\right) \\ &x_j \ge 0 \qquad\qquad j = 1,2,\ldots,n \end{aligned} \tag{8}$$

where $Z_s\left(x\right) = \frac{1}{2}\sum_{j=1}^{n}\left(c_{sLj} + c_{sRj}\right)x_j$ for $s = 1,2,\ldots,k$.

Now the task is to solve a MOLPP with conflicting objectives in question. In order to solve a MOLPP, we solve each objective for their most desirable values which is the optimal value of each objective, let this be denoted by U_i. While doing this, each objective is held separately without concerning about other conflicting objectives. By doing so, a set of solutions is obtained at which the objectives obtain their optimal values, we denote this set by S. Now, we calculate the least acceptable values every objective can take on this set and let this be denoted by L_i for $i = 1,2,\ldots,k$, i.e. $L_i = \min\left\{Z_i\left(X\right) : X \in S\right\}$. Here, $L_i's$ are the least acceptable values and $U_i's$ are the most preferred values.

To change the fuzzy goal programming model into a crisp LPP, we define different types of linear and non-linear membership functions.

A linear membership function is the most utilized function in decision making process while solving fuzzy mathematical programming problems. A linear approximation is defined by fixing two points, the least and most desirable levels of acceptability of a decision variable. In general fuzzy set theory, such an assumption is not always justified. Thus a justification should be made considering the fuzziness of the data in mind. From this point of view, several linear/non-linear shapes of membership functions are considered.

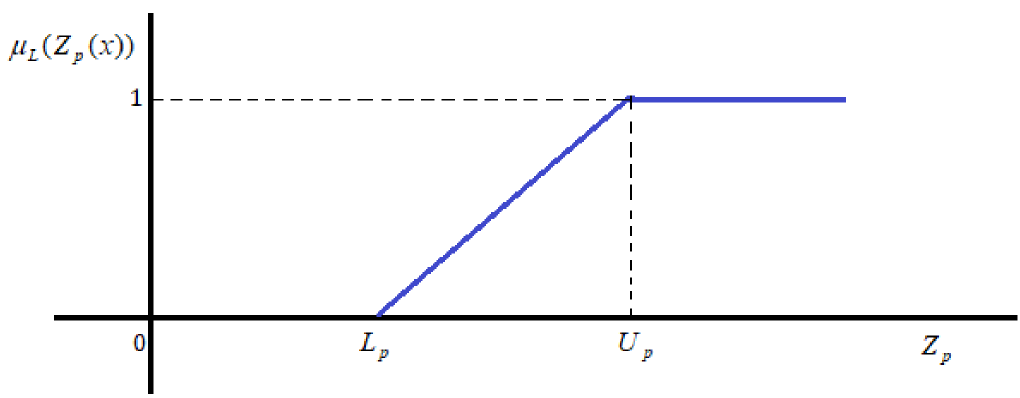
3.1. Linear Membership Function

A linear membership function μ_L can be defined as follows:

$$\mu_L\left(Z_p\left(x\right)\right) = \begin{cases} 0 & if\ Z_p \leq L_p \\ \dfrac{Z_p - L_p}{U_p - L_p} & if\ L_p \leq Z_p \leq U_p \\ 1 & if\ Z_p \geq U_p \end{cases} \tag{9}$$

Figure 2 represents membership function of a linear membership function when the objective is to maximize a function. It can be seen from the membership function that the degree of membership is 1, if the value of the objective function goes beyond the most desirable value in case of a maximization problem and the degree of membership is 0 if the value of the objective function is lesser than the least desirable value.

Figure 2. A Linear Membership Function



3.2. Hyperbolic Membership Function

The hyperbolic membership function is concave over the part when the decision maker is performing better than the goal and he tends to have a small marginal rate of satisfaction and the membership function is convex over the part when the decision maker is worse off the goal and he tends to have a higher marginal rate of satisfaction. The complete function is as follows (Figure 3):

$$\mu_H\left(Z_p\left(x\right)\right) = \begin{cases} 0 & if\ Z_p \le L_p \\ \frac{1}{2} + \frac{1}{2}\tanh\left(Z_p\left(x\right) - \frac{U_p + L_p}{2}\right)\alpha_p & if\ L_p \le Z_p \le U_p \\ 1 & if\ Z_p \ge U_p \end{cases} \quad (10)$$

where $\alpha_p = \frac{6}{U_p - L_p}$.

3.3. Parabolic Membership Function

The parabolic membership function μ_P can be defined as follows (**Figure 4**):

$$\mu_P\left(Z_p\left(x\right)\right) = \begin{cases} 0 & if\ Z_p \le L_p \\ \left(\frac{Z_p - L_p}{U_p - L_p}\right)^2 & if\ L_p \le Z_p \le U_p \\ 1 & if\ Z_p \ge U_p \end{cases} \quad (11)$$

Now, various membership functions can be used according to the satisfaction level of the decision maker. Using Fuzzy mathematical programming approach, the MOLPP can now be reduced to a single objective optimization problem as:

$$Maximize \quad \lambda$$

subject to:

Figure 3. A Hyperbolic Membership Function

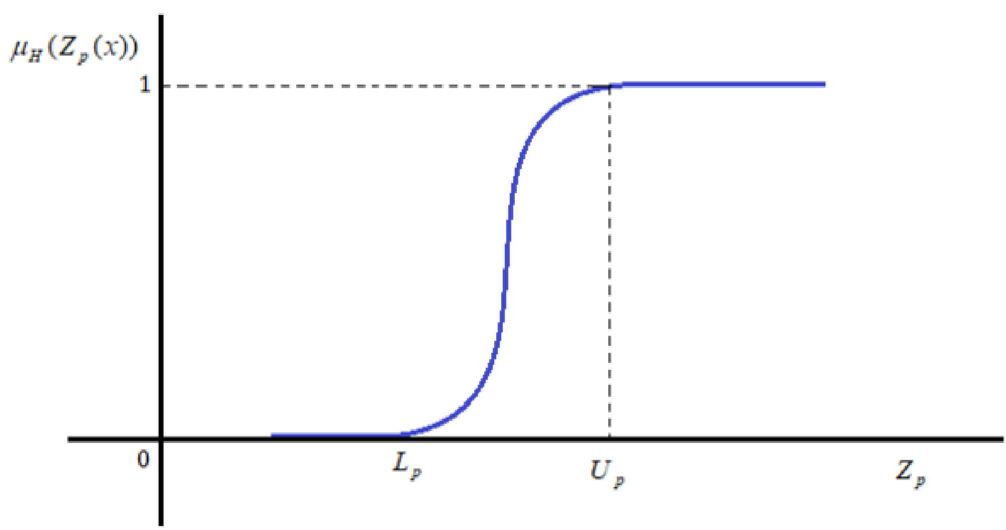
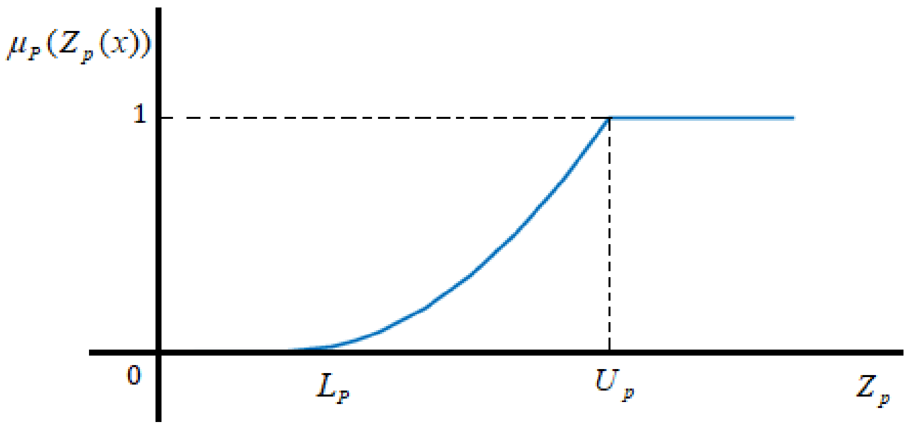


Figure 4. A Parabolic Membership Function



$$\begin{array}{c} \mu\left(Z_s\left(x\right)\right) \geq \lambda \\ \sum_{j=1}^{n} a_{Rij} x_j \leq b_{Ri} \\ \left(b_{Li} + b_{Ri}\right) - \sum_{j=1}^{n}\left(a_{Lij} + a_{Rij}\right) x_j \leq \alpha\left(b_{Ri} - b_{Li}\right) + \alpha\left[\sum_{j=1}^{n}\left(a_{Rij} - a_{Lij}\right) x_j\right] \\ x_j \geq 0 \qquad\qquad j = 1, 2, \ldots, n \end{array} \tag{12}$$

The optimal solution to the single objective optimization problem given by Eq. (12) is given by $\left(\lambda^*, x^*\right)$ which can then be used to find efficient solution of Eq. (5). The efficient solution of Eq. (5) is given by $\left[\tilde{\tilde{Z}}_1\left(x^*\right), \tilde{\tilde{Z}}_2\left(x^*\right), \ldots, \tilde{\tilde{Z}}_k\left(x^*\right)\right]$.

4. ALGORITHM

The whole solution procedure developed in Section 3 can be summarized as:

Step 1: Use nearest interval approximation method to reduce every IT2TFN to a closed interval.
Step 2: Convert MOILPP obtained in Step 1 to a MOLPP by using model Eq. (8).
Step 3: Find optimal solution of each objective function separately and let the optimal solution be denoted by U_s and let the optimal solution set be denoted by S.
Step 4: Find the minimum value of each objective function on the set S; let it be called L_s.
Step 5: Convert the MOLPP model to a single objective optimization problem, as given in Eq. (12).
Step 6: By using various classical methods, the optimal solution to the problem can be sought easily.
Step 7: The solution obtained in **Step 6** can then be used to find efficient solution to objective $Z_s, s = 1, 2, \ldots, k$.

5. NUMERICAL EXAMPLE

In this section, the production planning problem of a manufacturing company and the diet planning problem of a kitchen manager at a private hospital are considered.

Example 1: (Production Planning Problem): Suppose a manufacturing company produces two commodities, say A and B with the objectives of maximizing profit and maximizing the improvement of balance of trade. Each commodity requires some manufacturing time as well as some initial capital investment.

The profit earned by company is taken to be a fuzzy data as it varies from season to season. The profit contribution of commodity A is about $\$1.5$ per piece and for commodity B, the profit contribution is $\$\tilde{\tilde{3}}$ per piece. The commodity A yields a revenue of $\$\tilde{\tilde{5}}$ per piece in foreign countries, whereas commodity B requires imported raw materials of $\$\tilde{\tilde{7}}$ per piece. The revenue is denoted by using fuzzy data since it depends on import and export which usually gets fluctuated because of transportation charges. The profit maximization and maximization of improvement of balance of trade, which is the maximum difference of exports and imports; are two goals of manufacturing company. Now, each commodity also requires some manufacturing time and initial capital investment. The manufacturing time and capital investment are taken to be fuzzy as they keep on altering because of various factors like efficiency of machines, variability of electricity supply and labor hours. The initial capital investment for one piece of commodity A is $\$\tilde{\tilde{4}}$ and for commodity B is about $\$2.5$. The manufacturing time for one unit of commodity A and B are $\tilde{\tilde{1}}$ hour and $\tilde{\tilde{4}}$ hours respectively. The task is to calculate the number of units of each commodity that should be produced with a manufacturing time of about 11 hours and a total initial capital investment of about $\$12$; so that both the goals are achieved.

A model for the explained production planning problem of a manufacturing company in a fuzzy environment is given by Eq. (13) which is then solved by using the proposed approach:

$$Maximize\ \ \tilde{\tilde{Z}}\left(x\right)=\left[\tilde{\tilde{c}}_{11}x_1+\tilde{\tilde{c}}_{12}x_2,\tilde{\tilde{c}}_{21}x_1-\tilde{\tilde{c}}_{22}x_2\right]$$

subject to:

$$\begin{aligned}\tilde{\tilde{a}}_{11}x_1+\tilde{\tilde{a}}_{12}x_2&\le\tilde{\tilde{b}}_1\\ \tilde{\tilde{a}}_{21}x_1+\tilde{\tilde{a}}_{22}x_2&\le\tilde{\tilde{b}}_2\\ x_1,x_2&\ge 0\end{aligned}\tag{13}$$

where:

$$\begin{aligned}\tilde{\tilde{c}}_{11}&=\left(\left(1,1.5,2\right),\left(0.5,1.5,2.5\right)\right)\\ \tilde{\tilde{c}}_{12}&=\left(\left(2,3,4\right),\left(1,3,5\right)\right)\end{aligned}$$

$$\begin{aligned}
\tilde{\tilde{a}}_{11} &= \left(\left(0.5,1,1.5\right),\left(0,1,2\right)\right)\\
\tilde{\tilde{a}}_{12} &= \left(\left(3,4,5\right),\left(2,4,6\right)\right)\\
\tilde{\tilde{b}}_{1} &= \left(\left(9,11,13\right),\left(7,11,15\right)\right)
\end{aligned}$$

$$\begin{aligned}
\tilde{\tilde{c}}_{21} &= \left(\left(4,5,6\right),\left(3,5,7\right)\right)\\
\tilde{\tilde{c}}_{22} &= \left(\left(6,7,8\right),\left(4.5,7,9.5\right)\right)
\end{aligned}$$

$$\begin{aligned}
\tilde{\tilde{a}}_{21} &= \left(\left(3,4,5\right),\left(2,4,6\right)\right)\\
\tilde{\tilde{a}}_{22} &= \left(\left(1.5,2.5,3.5\right),\left(0.5,2.5,4.5\right)\right)\\
\tilde{\tilde{b}}_{2} &= \left(\left(8,12,16\right),\left(4,12,20\right)\right)
\end{aligned}$$

Step 1: First we use nearest interval approximation method to convert every IT2TFN to a closed interval. The corresponding MOILPP becomes:

$$Maximize\ \ Z\left(x\right) = \left[Z_1\left(x\right), Z_2\left(x\right)\right]$$

subject to:

$$\begin{aligned}
\left[0.625,1.375\right]x_1 + \left[3.25,4.75\right]x_2 &\le \left[9.5,12.5\right]\\
\left[3.25,4.75\right]x_1 + \left[1.75,3.25\right]x_2 &\le \left[9,15\right]\\
x_1, x_2 &\ge 0
\end{aligned} \tag{14}$$

where $Z_1\left(x\right) = \left[1.125,1.875\right]x_1 + \left[2.25,3.75\right]x_2$ and $Z_2\left(x\right) = \left[3.25,4.75\right]x_1 - \left[6.125,7.875\right]x_2$.

Step 2: The MOILPP is now converted to a MOLPP, which is given by Eq. (15):

$$Maximize\quad Z\left(x\right) = \left[Z_1\left(x\right), Z_2\left(x\right)\right]$$

subject to:

$$\begin{aligned}
1.375x_1 + 4.75x_2 &\le 12.5\\
4.75x_1 + 3.25x_2 &\le 15\\
-22 + \left(2x_1 + 8x_2\right) &\le \alpha\left(3\right) + \alpha\left(0.75x_1 + 1.5x_2\right)\\
-24 + \left(8x_1 + 5x_2\right) &\le \alpha\left(6\right) + \alpha\left(1.5x_1 + 1.5x_2\right)\\
x_1, x_2 &\ge 0
\end{aligned} \tag{15}$$

where $Z_1\left(x\right) = 1.5x_1 + 3x_2$ and $Z_2\left(x\right) = 5x_1 - 7x_2$.

Step 3: For $\alpha = 1$, the optimal values for objectives Z_1 and Z_2 are 8.963731 and 15.7895 at $(1.692573, 2.141623)$ and $(3.157895, 0)$ respectively.

Step 4: The set S is given by $S = \{(1.692573, 2.141623), (3.157895, 0)\}$. Minimum values of first and second objectives are given by $L_1 = 4.7367585$ and $L_2 = -6.528496$ respectively.

Step 5: Using fuzzy mathematical programming approach and taking $\alpha = 1$, the problem can be reduced to a crisp LPP as follows:

$$Maximize \quad \lambda$$

subject to:

$$\begin{aligned} \frac{1.5x_1 + 3x_2 - 4.7367585}{4.2269725} &\geq \lambda \\ \frac{5x_1 - 7x_2 + 6.528496}{22.317996} &\geq \lambda \\ 1.375x_1 + 4.75x_2 &\leq 12.5 \\ 4.75x_1 + 3.25x_2 &\leq 15 \\ 1.625x_1 + 7.25x_2 &\leq 23.5 \\ 7.25x_1 + 4.25x_2 &\leq 27 \\ x_1, x_2 &\geq 0 \end{aligned} \tag{16}$$

Step 6: The optimal solution to the above problem is $\lambda = 0.5000$ at $x_1 = 2.425238$ and $x_2 = 1.070806$.

Step 7: The efficient solution of given problem is:

$$\tilde{\tilde{Z}}_1 = ((4.5668, 6.85027, 9.1337), (2.2834, 6.85027, 11.4171))$$

$$\tilde{\tilde{Z}}_2 = ((3.72611, 4.630548, 5.98498), (2.457087, 4.630548, 6.80400))$$

Applying models given by Eq. (9) – Eq. (11) and solving by LINGO, the solution to the problem is summarized in **Table 1** for $\alpha = 0.5$.

Example 2: (Diet planning problem): Suppose, the kitchen manager at ABC hospital can buy three types of food packets namely, X, Y and Z. The objective is to find the number of units of each type of food packets that should be bought so as to minimize the total cost and the total cholesterol consumption of the patients. Each type of food packet has some amount of proteins, carbohydrates, fats and cholesterol associated with it.

The kitchen manager has to make the diet plan according to the dietary instructions given by the doctors. The dietary instructions are that each patient must get at least about one gm of protein, about

Table 1. Solution for Production Planning Problem

	Linear Function	Hyperbolic Function	Parabolic Function
Solution	$(2.425238, 1.070806)$	$(2.425238, 1.070806)$	$(2.425242, 1.070800)$
Z_1	$\left(\begin{array}{l}(4.5668, 6.85027, 9.1337);\\(2.2834, 6.85027, 11.4171)\end{array}\right)$	$\left(\begin{array}{l}(4.5668, 6.85027, 9.1337);\\(2.2834, 6.85027, 11.4171)\end{array}\right)$	$\left(\begin{array}{l}(4.56684, 6.85026, 9.13368);\\(2.28342, 6.85026, 11.41710)\end{array}\right)$
Z_2	$\left(\begin{array}{l}(3.72611, 4.630548, 5.98498);\\(2.457087, 4.630548, 6.80400)\end{array}\right)$	$\left(\begin{array}{l}(3.72611, 4.630548, 5.98498);\\(2.457087, 4.630548, 6.80400)\end{array}\right)$	$\left(\begin{array}{l}(3.276168, 4.63061, 5.985052);\\(2.457126, 4.63061, 6.804094)\end{array}\right)$
λ	0.5000	0.50000	0.25000

one gm of fat and around 3 gms of carbohydrates. Additional instructions are that in no case the carbohydrate content should exceed about 6 gms per patient. The dietary instructions as well as dietary information of food packets are not precise because of variability of product. The availability of protein, fats and carbohydrates in grams per kg of food packets to type A, B and C, is given in Table 2.

Table 2. Dietary information of Food packets

	Proteins	Fats	Carbohydrates	Cholestrol	Cost
Packet A	10	2	0	2	30
Packet B	2	1	15	10	5
Packet C	2	0	10	5	4

The task is to calculate the number of packets of each type of food that should be bought to prepare a diet according to the dietary instructions for 100 patients, so that both the goals achieved:

$$Minimize\ \ \tilde{\tilde{Z}}(x) = \left[\tilde{\tilde{c}}_{11}x_1 + \tilde{\tilde{c}}_{12}x_2 + \tilde{\tilde{c}}_{13}x_3, \tilde{\tilde{c}}_{21}x_1 + \tilde{\tilde{c}}_{22}x_2 + \tilde{\tilde{c}}_{23}x_3\right]$$

subject to:

$$\begin{aligned}
\tilde{\tilde{a}}_{11}x_1 + \tilde{\tilde{a}}_{12}x_2 + \tilde{\tilde{a}}_{13}x_3 &\geq \tilde{\tilde{b}}_1\\
\tilde{\tilde{a}}_{21}x_1 + \tilde{\tilde{a}}_{22}x_2 + \tilde{\tilde{a}}_{23}x_3 &\geq \tilde{\tilde{b}}_2\\
\tilde{\tilde{a}}_{31}x_1 + \tilde{\tilde{a}}_{32}x_2 + \tilde{\tilde{a}}_{33}x_3 &\geq \tilde{\tilde{b}}_3\\
\tilde{\tilde{a}}_{41}x_1 + \tilde{\tilde{a}}_{42}x_2 + \tilde{\tilde{a}}_{43}x_3 &\leq \tilde{\tilde{b}}_4\\
x_1, x_2, x_3, x_4 &\geq 0
\end{aligned}$$

where:

$$\tilde{\tilde{c}}_{11} = \left(\left(25, 30, 35\right), \left(20, 30, 40\right)\right)$$
$$\tilde{\tilde{c}}_{12} = \left(\left(3, 5, 7\right), \left(2, 5, 8\right)\right)$$
$$\tilde{\tilde{c}}_{13} = \left(\left(3, 4, 5\right), \left(2, 4, 6\right)\right)$$

$$\tilde{\tilde{a}}_{21} = \left(\left(1, 2, 3\right), \left(0, 2, 4\right)\right)$$
$$\tilde{\tilde{a}}_{22} = \left(\left(0.5, 1, 1.5\right), \left(0, 1, 2\right)\right)$$
$$\tilde{\tilde{a}}_{23} = \left(\left(0, 0, 0\right), \left(0, 0, 0\right)\right)$$
$$\tilde{\tilde{b}}_{2} = \left(\left(90, 100, 110\right), \left(80, 100, 120\right)\right)$$

$$\tilde{\tilde{c}}_{21} = \left(\left(1, 2, 3\right), \left(0, 2, 4\right)\right)$$
$$\tilde{\tilde{c}}_{22} = \left(\left(8, 10, 12\right), \left(6, 10, 14\right)\right)$$
$$\tilde{\tilde{c}}_{23} = \left(\left(3, 5, 7\right), \left(2, 5, 8\right)\right)$$

$$\tilde{\tilde{a}}_{31} = \left(\left(0, 0, 0\right), \left(0, 0, 0\right)\right)$$
$$\tilde{\tilde{a}}_{32} = \left(\left(10, 15, 20\right), \left(5, 15, 25\right)\right)$$
$$\tilde{\tilde{a}}_{33} = \left(\left(8, 10, 12\right), \left(6, 10, 14\right)\right)$$
$$\tilde{\tilde{b}}_{3} = \left(\left(250, 300, 350\right), \left(200, 300, 400\right)\right)$$

$$\tilde{\tilde{a}}_{11} = \left(\left(8, 10, 12\right), \left(6, 10, 14\right)\right)$$
$$\tilde{\tilde{a}}_{12} = \left(\left(1, 2, 3\right), \left(0, 2, 4\right)\right)$$
$$\tilde{\tilde{a}}_{13} = \left(\left(1, 2, 3\right), \left(0, 2, 4\right)\right)$$
$$\tilde{\tilde{b}}_{1} = \left(\left(90, 100, 110\right), \left(80, 100, 120\right)\right)$$

$$\tilde{\tilde{a}}_{41} = \left(\left(0, 0, 0\right), \left(0, 0, 0\right)\right)$$
$$\tilde{\tilde{a}}_{42} = \left(\left(10, 15, 20\right), \left(5, 15, 25\right)\right)$$
$$\tilde{\tilde{a}}_{43} = \left(\left(8, 10, 12\right), \left(6, 10, 14\right)\right)$$
$$\tilde{\tilde{b}}_{4} = \left(\left(500, 600, 700\right), \left(400, 600, 800\right)\right)$$

Applying models given by Eq. (9) – Eq. (11) and solving by LINGO, the solution to the diet planning problem is summarized in **Table 3** for $\alpha = 0.5$.

6. CONCLUSION

In this work, a method for solving FMOLPP with IT2TFNs as technological coefficients has been presented. The problem of impreciseness of information can be handled by using fuzzy numbers, in which a degree of membership gets associated with every element of the universal set. In this work, technological coefficients are represented by using IT2TFNs. The ability to present the impreciseness of membership degree is one of the best benefits of IT2TFNs since they make the modeling of real life problems easier. The solution procedure for the optimization problem has been divided into two stages. In the first stage of problem solving, the given problem is first reduced to MOILPP and then

Table 3. Solution for Diet planning problem

	Linear Function	Hyperbolic Function	Parabolic Function
Solution	$(63.0537, 21.8924, 1.9069)$	$(62.7414, 22.5170, 1.0803)$	$(63.0537, 21.8925, 1.9069)$
Z_1	$\begin{pmatrix}(1647.74, 2014.42, 2369.66);\\(1314.39, 2014.42, 2708.73)\end{pmatrix}$	$\begin{pmatrix}(1639.33, 1999.15, 2358.97);\\(1302.02, 1999.15, 2696.27)\end{pmatrix}$	$\begin{pmatrix}(1647.74, 2008.70, 2369.66);\\(1308.67, 2008.70, 2708.73)\end{pmatrix}$
Z_2	$\begin{pmatrix}(243.91, 354.56, 465.22);\\(135.16, 354.56, 573.96)\end{pmatrix}$	$\begin{pmatrix}(246.12, 356.055, 465.99);\\(137.26, 356.055, 574.84)\end{pmatrix}$	$\begin{pmatrix}(239.64, 347.44, 455.24);\\(132.31, 347.44, 562.56)\end{pmatrix}$
λ	0.6924	0.48235	0.479497

by using the perspective of decision maker, the constraints are relaxed and the problem is further reduced to a MOLPP. In the second stage of the problem, an efficient solution to the problem is sought by using fuzzy mathematical programming approach. In the second stage, various membership functions are used which represent the aspiration level of the decision makers. These membership functions provide flexibility to the decision maker so as to choose the function which better fits the problem and provide more satisfaction. Two examples representing the Diet planning problem and Production planning problem are modeled and solved by using the proposed approach. In both the numerical example presented, the aspiration level of the decision maker follows the order Linear $\geq$ Hyperbolic $\geq$ Parabolic.

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Vishnu Pratap Singh has completed M.Sc. and Ph.D. from Department of Mathematics, IIT Kharagpur. Currently, they are working as an assistant professor in the Department of Mathematics, Visvesvaraya National Institute of Technology, Nagpur.

Madhura Deshmukh was born in Amravati. Madhura completed her BSc from Amravati University and MSc from VNIT, Nagpur. Her areas of interest are Applied Mathematics, Optimization and Fuzzy Theory. Madhura also has an interest in Operations Research and desires to gain more knowledge in the field to solve real world problems. Currently seeking to enroll in a PhD program and TA positions in the USA where she is currently based. Her personal hobbies include teaching mathematics, singing, traveling, and learning new tools and technologies.

Kirti Sharma holds an M.Sc in Mathematics.