

Dual-Channel Supply Chain Coordination With BOPS and a Revenue-Sharing Contract

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ABSTRACT

The omni-channel strategy buy-online-pickup-in-store (BOPS) is used to cater to customers who want a consistent service experience in different channels. In this paper, the author thinks of BOPS as an effective strategy for encouraging some online customers to switch to offline stores with high online return losses. The author first studies an omni-channel supply chain with centralized and decentralized decision making and explains why online returns hurt the supply chain with respect to the matching rate and the unit return loss. Although different channels can be operated by the same firm or different firms, the author studies how to coordinate the entire chain using a revenue-sharing contract. When online return losses are high, it is effective to adopt BOPS to reduce online return losses; otherwise there is no need to do so. Finally, the author presents numerical experiments, including a special case, and shows that in many cases using an appropriate revenue-sharing contract under the proposed mechanism can increase the profits of the entire supply chain and its members.

KEYWORDS

BOPS, Dual-Channel Supply Chain, Online Customer Returns, Product Matching Rate, Revenue-Sharing Contract

1. INTRODUCTION

Online retailing has developed rapidly in recent years. As customers become accustomed to online shopping, many manufacturers and brick-and-mortar retailers have supplemented their traditional channels with online channels. The online channel has initially been perceived as competition to stores, and early studies have focused on solving channel conflicts. Currently, firms realize the need to integrate multi-channels into their operations to satisfy customers' needs, as customers usually obtain information online before shopping. For example, the brick-and-mortar channel can attract customers by its good service, and the online channel can give customers more conveniences by information availability or home delivery. Is there any way to integrate the advantages of different channels? As a result, "Omni-channel retailing" is proposed. For it can provide customers with a seamless shopping experience through all available shopping channels, it is considered as a new way of firms' development (Bell et al., 2014). The omnichannel environment provides customers more information on their purchasing strategies. In particular, a customer may either buy the product in a brick-and-mortar store and choose a home delivery service; purchase the product online and choose a BOPS strategy if he wants to get the product sooner; or search online for more information about

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the product and find a nearby store from which to buy the product if he is hesitant to shop online. Moreover, if a customer uses BOPS and finds that the product does not match his expectations, he can return the product to stores without the need to post it back to online retailers. In a nutshell, a customer has more choices in an omni-channel environment relative to traditional retailing. From the retailers' perspective, a BOPS strategy can attract more customers to offline stores. When a customer picks up the package from the store, he may know soon whether the product matches his expectations, which lowers the return losses for both the retailer and the customer, or the customer may purchase a different product from the store, which is likely to increase the store's profits. These cross-channel effects can increase the profits of offline stores.

Although an omni-channel environment provides numerous benefits, it also introduces many new challenges. This paper focuses on supply chain coordination. For example, while the buy-online-pickup-in-store option may be offered to customers, the online channel should cooperate with offline stores to coordinate their sale strategies. This cooperation requires not only the integration of a member management system but also that of the entire supply chain, including sales, pricing, and inventories. When a retailer has both online channels and offline channels with centralized decision making, it is straightforward to coordinate the different channels. In contrast, when the channels are controlled by several independent firms with decentralized decision making, each firm strives to maximize its own profits, resulting in a vertical and horizontal competition that hinders omni-channel integration. At the same time, the supply chain must decide what portions of the store's inventory should be allocated to the online and offline channels and how to allocate profits between the two channels.

In this paper, the author considers omni-channel coordination with a BOPS strategy. Some online customers use BOPS to save waiting time, reduce delivery fees or for other reasons. However, the firms that operate the online and offline channels may be independent for many goods are sold by agents. For the BOPS strategy to be used successfully, dual-channel coordination mechanisms are indispensable. In this paper, the author focuses on the following three questions:

- How do online customers' returns affect the profits of the supply chain and its members?
- Should the supply chain controller adopt a BOPS strategy to coordinate the online and offline channels?
- What mechanisms including a BOPS strategy can the author design to allocate profits in the omni-channel supply chain?

The remainder of the paper is organized as follows. Section 2 introduces relevant literatures on omni-channel retailing and supply chain coordination. Section 3 describes the research problem with the BOPS strategy in the omnichannel environment. Section 4 works out the prices and profits of the supply chain under centralized decision-making, decentralized decision-making and coordination with revenue-sharing contracts. And then numerical analysis are presented to propose an optimal management strategy and verify the effectiveness of revenue-sharing contracts. Section 5 concludes.

2. LITERATURE REVIEW

This paper studies the problem of the coordination of online and offline channels with omni-channel strategies. There have been a few studies on omni-channel retailing. Verhoef et al. (Verhoef et al., 2015) discussed the transition of supply chains from multi-channel to omni-channel mode, explained the potential impact of this transition on supply-chain management and noted three possible future research directions. Hübner et al. (Hübner et al., 2016) presented several structural models of the last kilometer in omni-channel operations and compared their advantages and disadvantages. Gao and Su (Gao & Su, 2016a) analyzed the BOPS strategy and found that it had a significant impact on consumers' purchasing behavior. Their results showed that consumers would always choose the

channel with the highest surplus from which to buy products when the prices of different channels are the same. If a BOPS strategy has the lowest hassle cost and customers find it convenient, more consumers will pick up products from the stores, which may increase the chain's revenues. They also described the chain's optimal stocking decisions and profits when customers have heterogeneous hassle costs associated with the online and offline channels. Gao and Su (Gao & Su, 2016b) also studied information mechanisms in omni-channel retailing when information reduces uncertainty about product value and availability. They considered three types of information mechanisms and found that available offline stocking information of stores and physical or virtual showrooms could change the proportions of consumers choosing different channels, which would affect the stocking decisions of the two channels. In addition, for the three mechanisms to produce different results, retailers should choose only one of them to obtain optimal profits. Bell et al. (Bell et al., 2017) empirically explained why online-first retailers might deploy omni-channel tactics. They figured out that showrooms could increase demand overall and decrease returns in the online channel. Jin et al. (Jin et al., 2018) proposed a model in which one physical store adopting BOPS uses a recommended service area to fulfill orders, and derived optimal decisions on the product price and recommended service radius. Zhang et al. (Zhang et al., 2018) conducted an analytical study about online retailer's optimal pricing and inventory decisions with the omnichannel strategy. Li et al. (Li et al., 2019) investigated the influence of the showrooming effect on firms' pricing and service effort in a dual-channel supply chain. With considering the probability of online return, Liu and Xu (Liu & Xu, 2020) analyzed joint optimization decision on pricing and ordering for retailers before and after opening a BOPS channel. They showed that with or without a BOPS channel, to increase the price and order quantity could be the optimal decision of retailers when the purchase proportion of store channel increased. He et al. (He et al., 2020) established analytical models to explore the impacts of the BOPS strategy on pricing and profit of the dual-channel retailer as well as the environment.

Supply chain coordination is one of the most important components in supply chain management. Issues, such as selfish members of supply chains, decentralized decision-making, and information asymmetry, lower the overall profits of supply chains and cause them to be inefficient (Chen, 2010). One strand of related studies is concerned with strategy design. Yao and Dresner (Yao & Dresner, 2008) compared the impacts of three strategies (an information sharing strategy, a continuous replenishment strategy and a vendor management inventory strategy) on inventory levels. Yan and Zhao (Yan & Zhao, 2011) studied the information asymmetry problem in an inventory sharing supply chain system composed of one manufacturer and two independent retailers. Gallino and Moreno (Gallino & Moreno, 2014) collected a special dataset formed by sales data before and after adopting "online ordering and offline picking" strategy and discovered that this strategy has a significant effect on consumers' decision to switch from online to offline channels. Rofin and Mahanty (Rofin T. M & Mahanty, 2020) explored the impact of information asymmetry of retailer's greening cost on the performance of both the manufacturer and the retailer. Xu and Qiu (Xu & Qiu, 2020) considered a new distribution strategy which online orders were forwarded to the brick-and-mortar store to make the fulfillment in dual-channel supply chains. They found that the new distribution strategy could soften price competition and increase the dual-channel supply chain members' profits under some conditions. The literature on mechanism design is also relevant. Many scholars have designed profit distribution mechanisms, such as quantity discount contract (Wang et al., 2014), wholesale price contract, revenue-sharing contract (Chakraborty et al., 2015), buyback contract (Wu, 2013), etc., between upstream and downstream enterprises of supply chains. Boyaci (Boyaci, 2005) designed an easily operated two-part compensation coordination (TPCC) contract that brings about efficient coordination. Wang and Sun (Wang et al., 2014) studied how to use price discount contracts to coordinate a two-level supply chain. Ouardighi (El Ouardighi & Kim, 2010) studied a two-level supply chain coordination with a wholesale price contract and a revenue-sharing contract and compared the possible outcomes of these contracts. Pu et al. (Pu et al., 2017) considered free riding and sales effort level in a dual-channel supply chain and proposed a cost-sharing contract to coordinate the chain

under decentralized decision-making. Zhang et al. (X.-M. Zhang et al., 2021) proposed an improved advertising costs and revenue sharing contract to coordinate the dual-channel supply chain.

Most of the papers mentioned above concerned definitions, characters, fulfillments and improvements of omni-channel retailing. In contrast, there are a few studies about supply chain management. This paper studies how to coordinate different channels when implementing omni-channel strategies. Unlike previous studies, this paper studies the coordination mechanism design problem in an omni-channel retailing. Although the mechanism design process has not changed, the omni-channel environment raises new research questions, which the author explores in our study.

3. PROBLEM DESCRIPTION

In the operation management of omni-channel supply chains, retailers usually adopt omni-channel strategies such as buy-online-pick-up-in-store (BOPS) to cater to customers. Encouraging customers to switch from online to offline channels increases not only the earnings of stores but also the conversion rates of customers who may be better served in stores. Moreover, omni-channel strategies can promote the integration of online and offline channels and satisfy consumers' various service demands. However, in reality, the different channels are under the management of independent enterprises sometimes, which has created significant hurdles for implementing omni-channel strategies. In addition, because different channels are using decentralized decision-making, it is difficult to adopt omni-channel strategies and maximally coordinate the entire supply chain.

With the development of new retailing in China, the commodity prices of online and offline channels are gradually converging, so the author assume the price of both channels remain consistent. Then the author discusses and propose a revenue allocation mechanism for coordinating the online and offline channels with BOPS. In a supply chain composed of one manufacturer and one traditional retailer, the manufacturer opens an online channel to sell products directly. To lessen the conflict with the traditional retailer and promote cooperation, the manufacturer sets the same price as the retailer. In the absence of an omni-channel implementation, the manufacturer sets the wholesale price, and the retailer sets the sale price sequentially. With an omni-channel implementation, the manufacturer provides a BOPS strategy to induce some online consumers to buy online and pickup from the offline retailer. After the customers' orders are realized, the manufacturer and the retailer share the revenues from the BOPS sales in a certain proportion.

Based on the description above, the author considers a second-level supply chain. The retailer purchases products from the manufacturer at a wholesale price w and sells them to customers at a price p ; at the same time, the manufacturer directly sells the products to the customers at the same price p , where the production cost of a unit product is c , with $1 > p > c > 0$. Consumers choose to purchase at any channel and their valuation of the product is v , with $v \sim U[0,1]$. The matching proportion of the product in the market is m ($0 < m < 1$), that is, a proportion m of the customers attach a value v to the product after experiencing the product at home, whereas the complementary proportion $1 - m$ attach no value to the product. The latter type of customers will not purchase the product if they chose the offline channel and will return the product if they chose the online channel. The loss of the online channel caused by returning a unit product is k . It can be easily shown that when the retailer set the product price at p , the total demand of the online and offline channels is $D = 1 - p$. Assuming the manufacturer and the retailer set the same price, the author suppose that ϕ ($0 \leq \phi \leq 1$) is the probability that customers prefer the offline channel, and $1 - \phi$ is the probability that they prefer the online channel, for consumers have inherent channel preferences. The supply chain structure is shown in Figure 1.

In addition, the superscript "C" represents a centralized decision-making of the supply chain, the superscript "N" represents a decentralized decision-making of the supply chain, the superscript "A" represents a centralized decision-making of the supply chain with BOPS, and the superscript "B"

Figure 1. Structure of the dual-channel supply chain

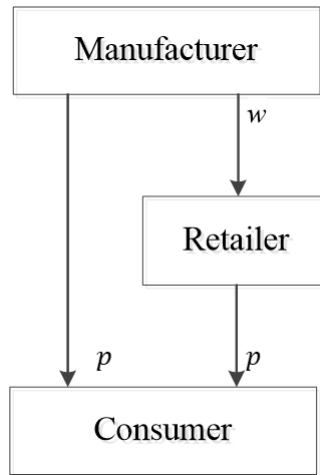
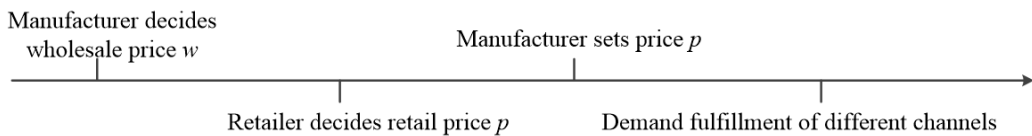


Figure 2. Sequence of events and decisions



represents a decentralized decision-making with BOPS and a revenue-sharing contract. The subscript “r” represents the offline channel or the retailer; the subscript “o” represents the online channel, the subscript “m” represents manufacturer and the subscript “t” represents the returns of the online channel. Table 1 summarizes the model parameters.

Table 1. Model parameters

p	Retail price of one product
c	Production cost of a unit product
v	Customers' valuation of the product
m	Matching rate
ϕ	Probability of customers choosing the offline channel
w	Wholesale price
k	The loss of unit return of the online channel
θ	Switching proportion of online customers when a BOPS strategy is offered
α	Revenue-sharing proportion when a BOPS strategy is offered

According to the above decision-making process and settings, the demands and profit functions of the different channels are:

$$d_r = m\phi(1-p), d_o = m(1-\phi)(1-p), d_t = (1-m)(1-\phi)(1-p)$$

$$\Pi_r = m\phi(1-p)(p-w) \quad (1)$$

$$\Pi_m = m\phi(1-p)(w-c) + (1-\phi)(1-p)[m(p-c) - (1-m)k] \quad (2)$$

4. MODELS AND ANALYSIS

The author is now ready to analyze the equilibrium behaviors with and without coordination. In the absence of coordination, the author assume that the manufacturer and retailer choose not to offer the omni-channel policy and the author consider two cases. With coordination, by contrast, the author assume that the manufacturer and the retailer choose to offer a BOPS strategy and propose a viable revenue-sharing contract.

4.1. Decisions Without Bops

4.1.1. Centralized Decision

When the supply chain is under centralized control or, equivalently, under the management of one decision maker, the administrator will maximize the profits of the entire supply chain. In this setting, the retail price is optimal. Therefore, the profit function of the dual-channel supply chain under centralized decision-making is as follows:

$$\Pi^C = m\phi(1-p)(p-w) + m\phi(1-p)(w-c) + (1-\phi)(1-p)[m(p-c) - (1-m)k] = (1-p)[m(p-c) - (1-\phi)(1-m)k] \quad (3)$$

Solving (3), the optimal retail price under centralized decision-making is as follows:

$$p^C = \frac{m(1+c) + (1-\phi)(1-m)k}{2m} \quad (4)$$

Since the controller has to obtain a positive profit, the author must have $0 < c < p^C < 1$. This constraint can be rewritten as follows:

$$(1-\phi)(1-m)k < m(1-c) \quad (5)$$

Given the equilibrium price p^C , the optimal demand and profit of the supply chain, respectively, are as follows:

$$d^C = \frac{m(1-c) - (1-\phi)(1-m)k}{2} \quad (6)$$

$$\Pi^C = \frac{[m(1-c) - (1-\phi)(1-m)k]^2}{4m} \quad (7)$$

4.1.2. Decentralized Decision

When different channels are operated by independent firms, each firm maximizes its own profits rather than the profits of the entire supply chain. The manufacturer acts as a Stackelberg leader that first decides the wholesale price w , whereas the retailer follows by setting the sale price p of the product. The solution is obtained as follows.

Using the first-order condition $\frac{\partial \Pi_r}{\partial p} = 0$, we obtain $p = \frac{1+w}{2}$. Substituting it into equation (2), we get:

$$\Pi_m = m\phi \left(1 - \frac{1+w}{2}\right) (w-c) + (1-\phi) \left(1 - \frac{1+w}{2}\right) \left[m \left(\frac{1+w}{2} - c \right) - (1-m)k \right]$$

The manufacturer will choose w to maximize Π_m . We can now solve for the equilibrium quantities of the system:

$$w^N = \frac{m(\phi+c) + (1-\phi)(1-m)k}{m(1+\phi)} \quad (8)$$

$$p^N = \frac{m(1+c+2\phi) + (1-\phi)(1-m)k}{2m(1+\phi)} \quad (9)$$

$$\Pi_r^N = \frac{\phi[m(1-c) - (1-\phi)(1-m)k]^2}{4m(1+\phi)^2} \quad (10)$$

$$\Pi_m^N = \frac{[m(1-c) - (1-\phi)(1-m)k]^2}{4m(1+\phi)} \quad (11)$$

With $0 < c < p^N < 1$, we get the same constraint as (5). Comparing the corresponding prices and profits in the centralized and decentralized settings with the constraint (5), we derive the following results. The proof is relegated to the appendix.

Proposition 1: For any m or k satisfying the constraint (5), (i) $p^C < p^N$, (ii) $\Pi^C > \Pi^N$. Here, $\Pi^N = \Pi_r^N + \Pi_m^N$.

According to Proposition 1, the retail price is higher under decentralized decision-making than under centralized decision-making, and the overall profits of the supply chain are higher under centralized decision-making than under decentralized decision-making. This finding is straightforward and indicates that centralized decision-making can render the supply chain globally optimal, whereas decentralized decision-making lowers the profits and efficiency of chain.

Proposition 2: For any m or k satisfying the constraint (5), (i) $\frac{dp^C}{dm} < \frac{dp^N}{dm} < 0$, (ii) $\frac{dp^C}{dk} > \frac{dp^N}{dk} > 0$.

Proposition 3: For any m or k satisfying the constraint (5), (i) $\frac{d\Pi^C}{dm} > \frac{d\Pi^N}{dm} > 0$, (ii) $\frac{d\Pi^C}{dk} < \frac{d\Pi^N}{dk} < 0$.

According to Proposition 2, an increase in the product's matching rate in the market lowers the retail price and wholesale price. An increase in the return loss of the online unit product lowers the retail price and wholesale price. That is, the retail price and wholesale price are negatively (or positively) linked to the matching rate m (or the unit return loss k) under both the centralized and decentralized settings. Proposition 3 shows that the overall profits of the supply chain increase with the product's matching rate in the market and decrease with the unit return loss.

Furthermore, as the product's matching rate m increases, the reduction in the rate of the retail price is higher in a centralized setting than in a decentralized setting, and so is the growth of the overall profits. With respect to the unit return loss k , the growth rate of the retail price and the reduction rate of the overall profits are higher in a centralized setting than in a decentralized setting. These results suggest that both the product's matching rate and the unit return loss have a greater effect on the chain's price and profits in a centralized setting relative to a decentralized setting.

Proposition 4: For any m or k with the constraint (5), (i) $\frac{dw^N}{dm} < \frac{dp^N}{dm} < 0$, (ii) $\frac{dw^N}{dk} > \frac{dp^N}{dk} > 0$.

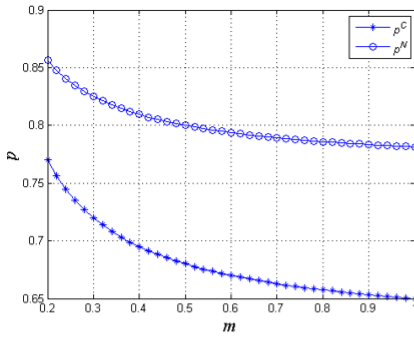
Proposition 5: For any m or k with the constraint (5), (i) $\frac{d\Pi_m^N}{dm} > \frac{d\Pi_r^N}{dm} > 0$, (ii) $\frac{d\Pi_m^N}{dk} < \frac{d\Pi_r^N}{dk} < 0$.

Proposition 4 and 5 show the results obtained in a decentralized setting. Proposition 4 indicates that the retail price and wholesale price are negatively (or positively) linked to the matching rate m (or the unit return loss k). Proposition 4 indicates that the profits of the manufacturer and the retailer increase with the product's matching rate in the market. An increase in the unit return loss lowers the profits of the manufacturer and the retailer. Moreover, the growth rate of the manufacturer's profits is higher than that of the retailer's with respect to the matching rate m , and the reduction rate of the manufacturer's profits is higher than that of the retailer's with respect to the unit return loss k . These results suggest that both the product's matching rate and the unit return loss have a greater effect on the manufacturer than on the retailer. This is obvious because the manufacturer acts as a Stackelberg leader in this game of price setting. If the product can match more customers, the manufacturer can obtain higher profits by setting a higher wholesale price, and vice versa. If the unit return loss in the online channel increases, the retailer will set a higher price and thereby decrease customer demand in both channels. This explains why the manufacturer and the retailer want to improve the matching rate and lower the unit return loss. The omni-channel BOPS strategy can attract more customers to the stores and cut down returns while providing a better customer experience.

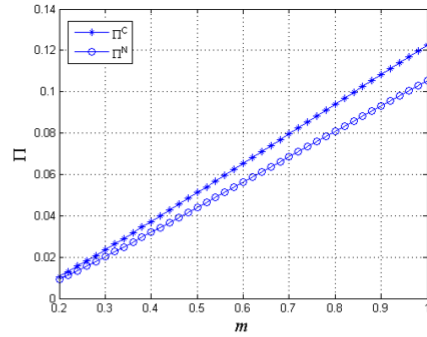
4.1.3 Numerical Example

In this section, the author presents numerical examples to verify the previous propositions. Let $\phi = 0.6, c = 0.3, m = 0.2 \sim 1(0.7), k = 0.15(0 \sim 0.4)$, where m and k are variables. The author

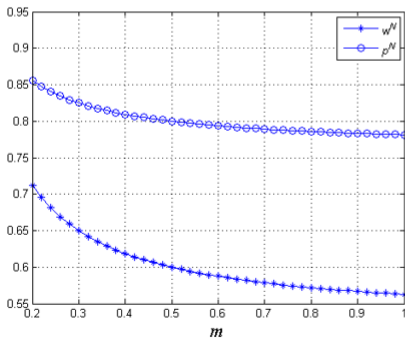
Figure 3. Results of different cases (when the matching rate is changed and the unit return loss is fixed)



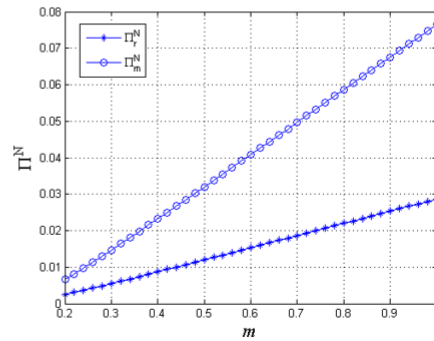
(a) Retail prices vs. Matching rate



(b) Profits of the entire supply chain vs. Matching rate



(c) Wholesale and retail price vs. Matching rate under decentralized decision



(d) Profits of the manufacturer and retailer vs. Matching rate under decentralized decision

shows the impact of the product's matching rate and unit return loss on the wholesale price w , the retail price p , the demands and profits of the manufacturer and the retailer under both centralized and decentralized settings. The results are shown in Figures 3 and 4.

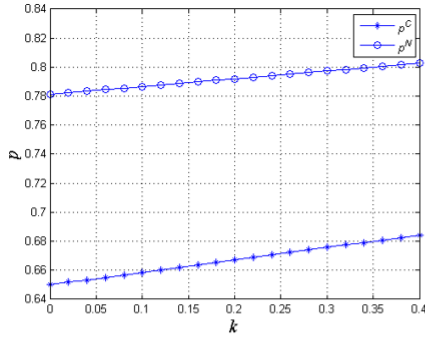
Figure 3 shows the changes in the prices and profits with a certain unit return loss and a variable matching rate, and Figure 4 shows these changes with a certain matching rate and variable unit return loss. These figures verify our results. As explained above, when the matching rate is low or the unit return loss is high, the retailer has to raise prices to make up for the return losses, which lower the profits of both the manufacturer and the retailer. Because online channels make customers uncertain about the product, the author suggest that the online retailer cooperates with the stores through omni-channel policies to decrease return costs. The coordination mechanism should therefore consider the matching rate and customers' switching behavior.

4.2. Omni-Channel Coordination With BOPS

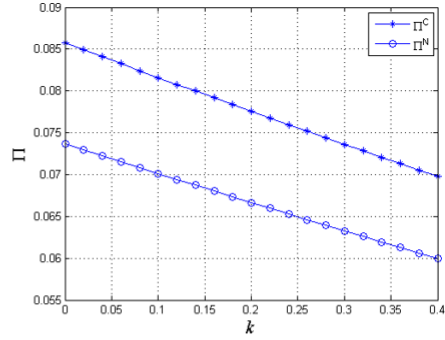
4.2.1. Centralized Decision With BOPS

As before, when the two channels are managed by the same decision maker, the profit of the whole supply chain is:

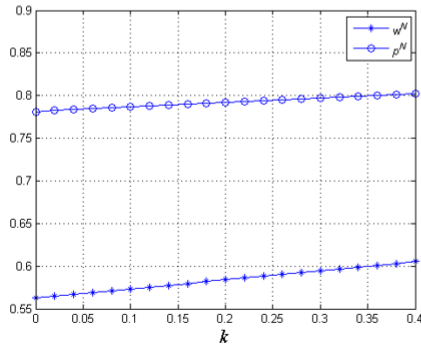
Figure 4. Results of different cases (when the unit return loss is changed and the matching rate is fixed)



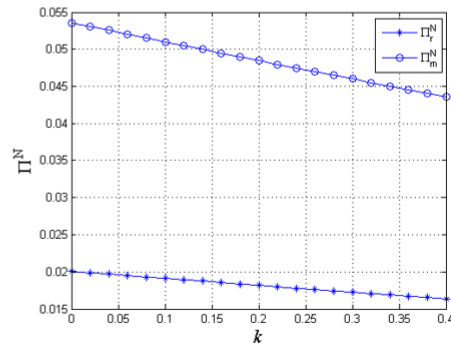
(a) Retail prices vs. Unit return loss



(b) Profits of the entire supply chain vs. Unit return loss



(c) Wholesale and retail price vs. Unit return loss under decentralized decision



(d) Profits of the manufacturer and retailer vs. Unit return loss under decentralized decision

$$\begin{aligned}\Pi^A &= m(\phi + \theta)(1 - p)(p - w) + m(\phi + \theta)(1 - p)(w - c) + (1 - \phi - \theta)(1 - p)[m(p - c) - (1 - m)k] \\ &= (1 - p)[m(p - c) - (1 - \phi - \theta)(1 - m)k]\end{aligned}$$

and then we can figure out that the price and the sales respectively:

$$p^A = \frac{m(1 + c) + (1 - \phi - \theta)(1 - m)k}{2m}, d^A = \frac{m(1 - c) - (1 - \phi - \theta)(1 - m)k}{2}$$

Finally, the profit with BOPS under centralized decision is calculated as:

$$\Pi^A = \frac{[m(1 - c) - (1 - \phi - \theta)(1 - m)k]^2}{4m} \quad (12)$$

Compared with equations 4, 6 and 7, when the parameters are the same, it can be found that the unit price decreases but the sales increase, while the overall profit increases. Therefore, it can

be seen that the adoption of BOPS strategy in online channels has a positive impact on the profit improvement of the whole supply chain.

4.2.2. Decentralized Decision With BOPS and a Revenue-Sharing Contract

For customers who are uncertain about the non-digital attributes of the products in online channels, online returns cause significant losses to the supply chain. In some cases, different channels are controlled by different firms, thereby decreasing the efficiencies of the supply chain and its members. The development of omni-channel retailing introduces new ways to coordinate between and thereby benefit different channels. Therefore, the author studies the popular BOPS strategy under decentralized decision-making, and then design a contract that achieves coordination in the omni-channel environment.

When the manufacturer adopts a BOPS strategy, some online consumers who choose that strategy will pick up the product at a store after ordering it online. The author assumes that $\theta(0 < \theta < 1, \phi + \theta \leq 1)$ represents the proportion of this type of customers. When these customers pick up their ordered products at the store with no returns, the manufacturer should share a fraction of the revenues with the offline retailer to maintain the cooperation. The author therefore proposes a revenue-sharing contract with a fraction α . This means that the retailer obtains a fraction α of the total revenues and the manufacturer obtain the complementary fraction $1 - \alpha$. Thus, the profits of the retailer and the manufacturer are as follows:

$$\Pi_r^B = m\phi(1-p)(p-w) + m\theta(1-p)(\alpha p - w) \quad (13)$$

$$\Pi_m^B = m(\phi + \theta)(1-p)(w-c) + m\theta(1-p)(1-\alpha)p + (1-\phi-\theta)(1-p)[m(p-c) - (1-m)k] \quad (14)$$

Similar to the previous analysis, we solve $\frac{\partial \Pi_r^B}{\partial p} = 0$ and get $p = \frac{\phi + \theta\alpha + (\phi + \theta)w}{2(\phi + \theta\alpha)}$.

Substituting it into (14) we obtain the following:

$$\begin{aligned} \Pi_m^B = & m(\phi + \theta) \left[1 - \frac{\phi + \theta\alpha + (\phi + \theta)w}{2(\phi + \theta\alpha)} \right] (w - c) \\ & + m\theta \left[1 - \frac{\phi + \theta\alpha + (\phi + \theta)w}{2(\phi + \theta\alpha)} \right] (1 - \alpha) \frac{\phi + \theta\alpha + (\phi + \theta)w}{2(\phi + \theta\alpha)} \\ & + (1 - \phi - \theta)(1 - p) \left[m \left[\frac{\phi + \theta\alpha + (\phi + \theta)w}{2(\phi + \theta\alpha)} - c \right] - (1 - m)k \right] \end{aligned}$$

Solving $\Pi_m^B = 0$, we derive the following results:

$$w^B = \frac{(\phi + \theta\alpha)[m(\phi + \theta\alpha + c) + (1 - \phi - \theta)(1 - m)k]}{m(\phi + \theta)(1 + \phi + \theta\alpha)} \quad (15)$$

$$p^B = \frac{m(1 + c + 2\phi + 2\theta\alpha) + (1 - \phi - \theta)(1 - m)k}{2m(1 + \phi + \theta\alpha)} \quad (16)$$

$$\Pi_r^B = \frac{(\phi + \theta\alpha)[m(1-c) - (1-\phi-\theta)(1-m)k]^2}{4m(1+\phi+\theta\alpha)^2} \quad (17)$$

$$\Pi_m^B = \frac{[m(1-c) - (1-\phi-\theta)(1-m)k]^2}{4m(1+\phi+\theta\alpha)} \quad (18)$$

Since the prices and profits have to satisfy $0 < c < p^B < 1$ and $\Pi_r^B, \Pi_m^B \geq 0$, we can obtain a new constraint as follows:

$$\begin{cases} (1-\phi-\theta)(1-m)k < m(1-c) \\ \phi + \theta\alpha \geq 0 \end{cases} \quad (19)$$

To maximize the profits of the entire supply chain, the author has to find the optimal revenue-sharing fraction. As the total profits under centralized decision-making are globally optimal, the author uses them as the benchmark for the optimal α^* . Hence, the author obtains the following propositions (the proofs are in the appendix).

Proposition 6: When a revenue-sharing contract is used to coordinate the supply chain with the omni-channel strategy BOPS, it can achieve coordination and maximize the profits of the entire supply chain if and only if there exists α^* that satisfies the following conditions:

$$\begin{cases} \frac{\sqrt{1+2\phi+2\theta\alpha^*}}{1+\phi+\theta\alpha^*} = \frac{m(1-c) - (1-\phi)(1-m)k}{m(1-c) - (1-\phi-\theta)(1-m)k} \\ \phi + \theta\alpha^* \geq 0 \\ (1-\phi)(1-m)k < m(1-c) \\ (1-\phi-\theta)(1-m)k < m(1-c) \end{cases} \quad (20)$$

Proposition 6 shows the conditions that all the parameters should satisfy to coordinate the entire supply chain. Under these conditions, the profits of the entire supply chain are maximal. This proposition gives the global controller a solution for allocating profits effectively between the manufacturer and the retailer. It implies that omni-channel developments provide supply chains a new way to increase the chain's effectiveness.

Proposition 7: With the optimal fraction α^* , we have $\Pi_r^{B*} \geq \Pi_r^{N*}$ when $\phi^3 + \theta\alpha^*(1+\phi^2) \geq 0$ and $\Pi_r^{B*} < \Pi_r^{N*}$ when $\phi^3 + \theta\alpha^*(1+\phi^2) < 0$.

Proposition 7 shows that the profits of the retailer under coordinated decision-making with a BOPS strategy are not always higher than those under decentralized decision-making. In some fortunate settings, the coordination benefits the retailer. This is because a BOPS strategy induces some customers who are used to shopping online to switch to offline stores. The growth of traffic in the offline stores may increase their sales, and the stores can also obtain more profits from the

revenue-sharing mechanism. This is why traditional retailers tend to favor the current transition to omni-channel retailing. In other, less fortunate settings, by contrast, retailers cannot obtain higher profits because of unfavorable revenue allocations. In reality, the author needs to prevent these situations from materializing.

Proposition 8: If there exists a threshold $\hat{\alpha}$, we have $\Pi_m^B \geq \Pi_m^N$ when $\alpha \leq \hat{\alpha}$ and $\Pi_m^B < \Pi_m^N$ when $\alpha > \hat{\alpha}$. Here, $\hat{\alpha}$ is the solution of $\sqrt{\frac{1+\phi}{1+\phi+\theta\hat{\alpha}}} = \frac{m(1-c)-(1-\phi)(1-m)k}{m(1-c)-(1-\phi-\theta)(1-m)k}$ with constraints in (5) and (19).

Proposition 8 shows that when the manufacturer's share in the profits increases, it will increase its revenue. However, when the sharing ratio is above a threshold, it actually reduces the profits of the manufacturer. Therefore, when $\alpha^* \leq \hat{\alpha}$, the optimal revenue-sharing contract benefits both the manufacturer and the retailer, and when $\alpha^* > \hat{\alpha}$ it only benefits the retailer. Therefore, it is necessary for decision-makers to set the profit distribution proportion according to actual situations. An appropriate proportion can achieve a win-win situation for manufacturers and retailers.

4.2.3. Numerical Example

To obtain additional results on a supply chain with a BOPS strategy and a revenue-sharing contract, the author presents below some numerical examples followed by an analysis.

First, the author assumes $\phi = 0.5, c = 0.3, m = 0.6 \sim 0.75, k = \{0.2, 0.3, 0.4\}, \theta = 0.3$ (the range of m is to set to ensure that customer demands in different channels are non-negative) to calculate the fraction α , and demonstrate the impact of m on the profits of the entire supply chain and its members before and after coordination is implemented. All the parameters satisfy the constraints and the members make their decisions. The results are shown in Figures 5 to 7.

Figure 5. Sharing fraction vs. Matching rate

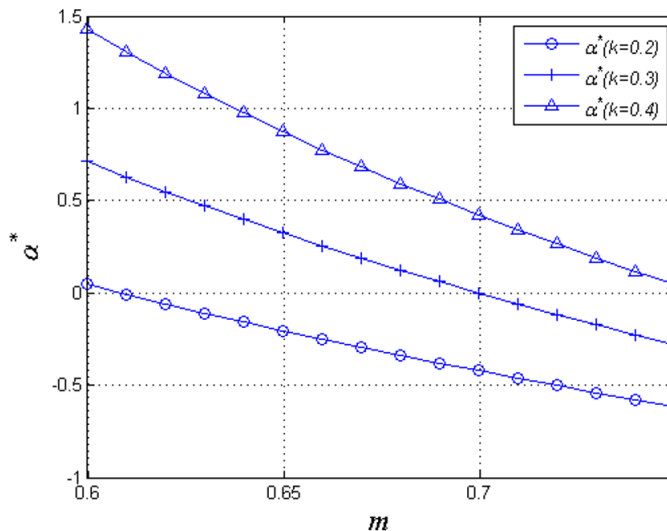


Figure 6. Profits of the entire supply chain vs. Matching rate

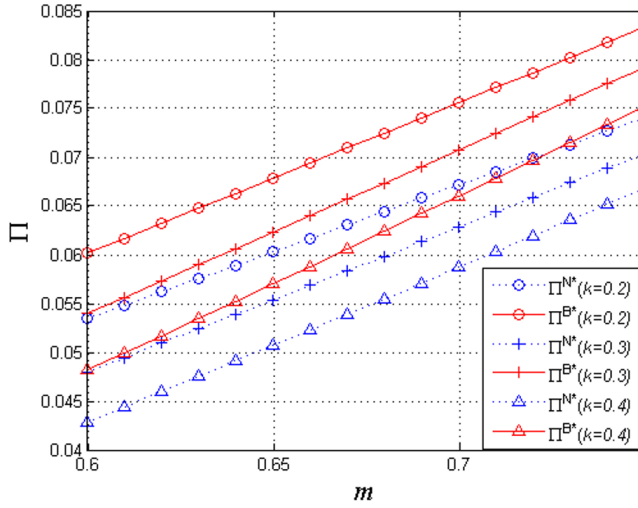


Figure 7. Retail and wholesale prices vs. Matching rate

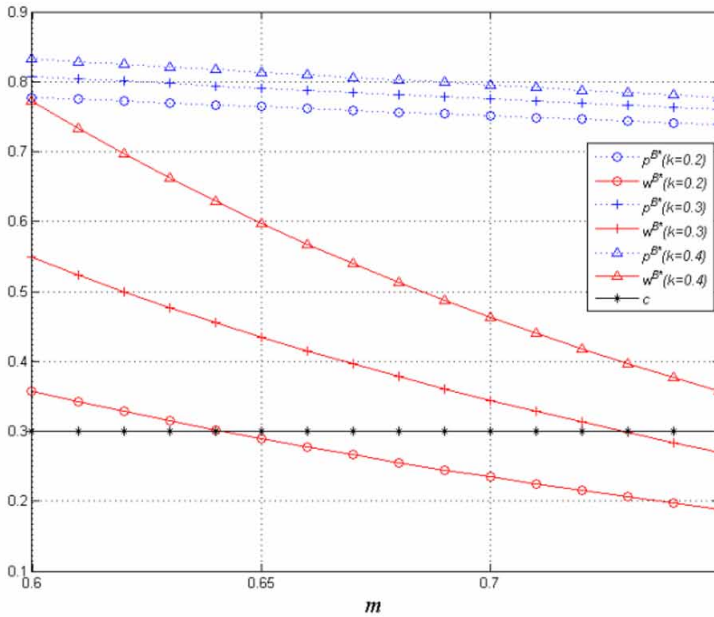


Figure 5 shows that as the matching rate increases, the optimal fraction α^* that the manufacturer shares with the retailers decreases. As the unit return loss increases, the optimal fraction α^* increases. This means that when online return losses are high, the manufacturer is willing to share more revenues with the offline stores.

Because the profits with coordination are equal to those in centralized decision-making, Figure 6 shows that the profits of the supply chain are higher with coordination than they are without

coordination under decentralized decision-making. This finding implies that coordination increases the profits of the entire supply chain under decentralized decision-making.

Moreover, the optimal α^* may be less than 0 or greater than 1, as shown in Figure 5. The author can explain this result using Figure 7. When the unit return loss is high, the wholesale and retail prices increase. It can be seen from Figure 7 that the manufacturer may set the wholesale price either close to or higher than the retail price, and then give the retailer a share α^* , which is larger than 1. This is possible when the manufacturer and the retailer are cooperative. Here, the manufacturer sets a high wholesale price, and the retailer accepts it. This is similar to a dominant manufacturer asking a retailer for a deposit and returning it after the cooperation has been consumed. Interestingly, the author finds that the fraction α^* may be less than 0. This is because the smaller the distribution proportion is, the lower the manufacturer's wholesale price is, which may be lower than the product cost. This means that the manufacturer sells the products to the retailer at a very low price, and the retailer returns to the manufacturer a portion of the revenues after the sales are fulfilled. These two types of situations can be observed in the market, which lends support to our results.

Then, the author assumes $\phi = 0.5, c = 0.3, m = 0.65, k = \{0.2, 0.3, 0.4\}, \theta = 0.25 \sim 0.4$ and analyze the effect of the transfer proportion θ on the optimal fraction α^* . The results are shown in Figure 8.

Figure 8 shows that as customers' transfer proportion with a BOPS strategy increases, the optimal α^* increases as well. This is because when more customers choose to pick up products at stores, the retailer bears more traffic. For the supply chain, the larger the return loss is, the bigger the fraction must be. This is because online returns cause significant losses to the manufacturer, and the manufacturer is therefore more motivated than the retailer to encourage more customers to shop in stores. The manufacturer therefore needs to increase the revenue shares offered to retailers to attract them.

Last, the author tests the impact of α on the profits of the retailer and the manufacturer. The author assumes $\phi = 0.5, c = 0.3, m = 0.65, k = \{0.2, 0.3, 0.4\}, \theta = 0.3$ and that α varies from -1 to 2. The results are shown in Figures 9 and 10.

Figure 8. Sharing fraction vs. Transfer proportion

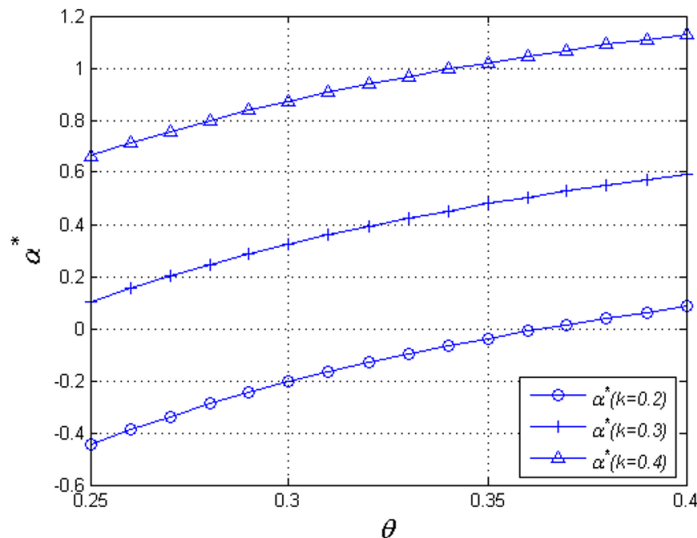
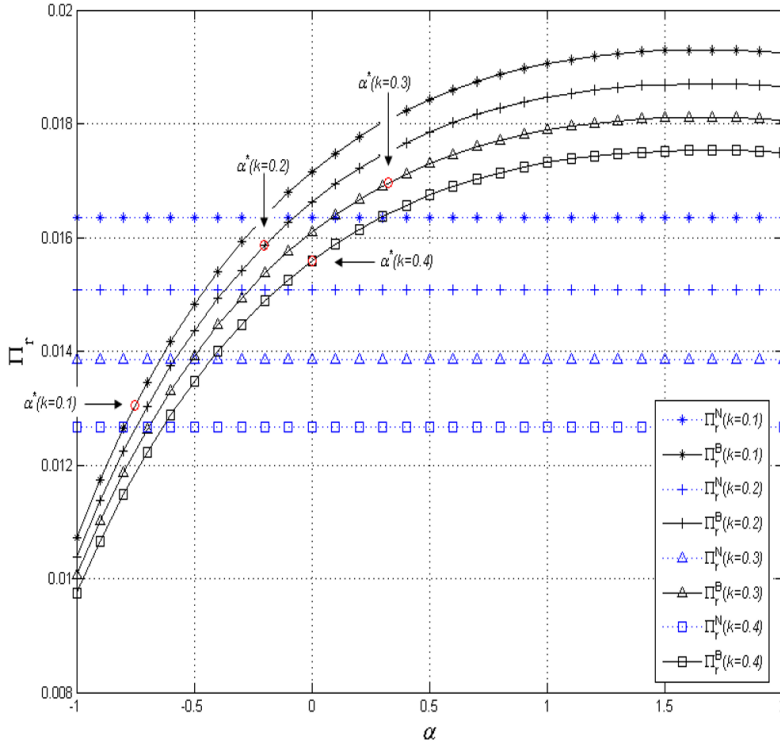


Figure 9. Profits of the retailer vs. Sharing fraction

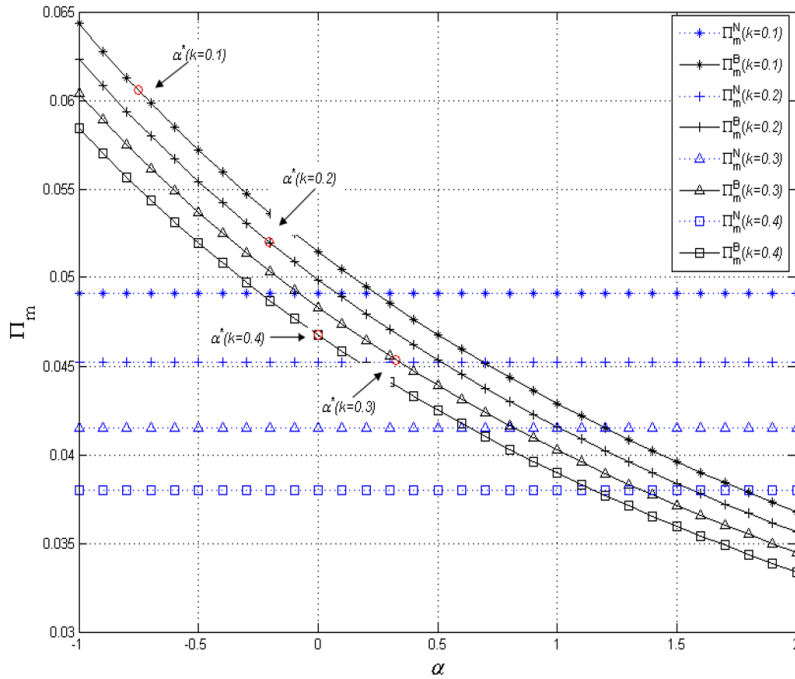


It can be seen from Figure 9 and Figure 10 that the profits of the manufacturer and retailer are not always higher in a coordinated setting than in a decentralized setting. When α is too low or too high, it will decrease the payoff of one of the players and thereby frustrate cooperation. The optimal revenue-sharing rate therefore needs to be determined. The author calculates the optimal rate in proposition 6 and mark it in the corresponding figures with red points. The author find that the optimal fraction can increase the profits of both the manufacturer and the retailer most of the time, thereby realizing a win-win outcome. The figures agree with our results in most cases, but there are some exceptions. In some extreme situations, for example, when the online unit return loss is low and the matching rate is high, the retailer's profits will decrease with coordination. That is because the manufacturer has no need to adopt a BOPS strategy, and if he does, the retailer may sacrifice her profits. Therefore, when the unit return loss is low, there is no need to encourage customers to switch from the online to the offline channel in our model. This finding therefore implies that the supply chain has to be appropriately coordinated. The author has thus proved our proposed propositions.

5. CONCLUSION

In this paper, the author studies the channel coordination problem with BOPS considering customer returns in the omni-channel environment. In many cases, customers return products because they do not have enough information on the product before purchasing it and only determine that the product does not meet their expectation after the purchase. Retailers should accordingly be willing to adopt a sale strategy that provides product information to customers before they purchase the product. Because

Figure 10. Profits of the manufacturer vs. Sharing fraction



more supply chains shift from multi-channel to omni-channel mode and sometimes different channels are managed by different firms, the author focuses on an omni-channel strategy BOPS to verify that it reduces customer returns and consider how to allocate profits between different channels optimally. In our study, the author presents analytical models to carry out the analysis.

With the analytical models and a detailed analysis, the author can answer the questions presented in the introduction. Customer returns indeed lower the profits of the supply chain and its members. Profits decrease faster as the return losses increase. Although it is efficient for the chain members to make a centralized decision, it is hard to implement this strategy for different channels that are operated by different firms in some cases. The author therefore considers decentralized decision-making to be an imperfect setting and centralized decision-making to be an optimal setting that serves as a benchmark for supply chain coordination. The author then puts forward a revenue-sharing contract to allocate profits between the manufacturer and the retailer. In addition, the author proposes an optimal revenue-sharing rate and prove that it can maximize the profits of the entire supply chain. The author also shows that a BOPS strategy can induce some customers to shop at stores, thereby indirectly reducing return losses and directly increasing traffic in offline stores. Moreover, the author tests the revenue-sharing contract with different sharing rates. As the author expected, in most cases, the optimal rate benefits the entire supply chain with decentralized decision-making. However, when return losses are small (the product's matching rate is high and unit return loss is low), there is no need to attract online customers to stores, for a BOPS strategy may reduce the retailer's profits. Therefore, external conditions determine whether it is necessary for decision makers to adopt a BOPS strategy to coordinate the online and offline channels. If a BOPS strategy is adopted, the proposed revenue-sharing contract can help the controller to effectively coordinate the offline and online channels.

However, there are still limitations that can be addressed in future research. For example, the author assume that only online customers use BOPS and will switch to the offline stores, but

customers who prefer offline channels may also switch to online ones when BOPS give them more online information. In response to the limitations, future research needs to consider both online and offline customer behaviors, so as to provide a more comprehensive classification of consumers and omni-channel impacts. Moreover, with the development of omni-channel, more and more new strategies emerge, such as showing rooms, order online and delivery offline, which also bring new research directions.

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APPENDIX

Proof of Proposition 1

According to (4)-(11), we can obtain:

$$p^C - p^N = -\frac{\phi[m(1-c) - (1-\phi)(1-m)k]}{2m(1+\phi)} < 0$$

$$\Delta\Pi = \Pi^C - (\Pi_r^N + \Pi_m^N) = \frac{\phi^2[m(1-c) - (1-\phi)(1-m)k]^2}{4m(1+\phi)^2} > 0$$

Proof of Proposition 2

According to (4) and (9), we have $\frac{dp^C}{dm} = -\frac{k(1-\phi)}{2m^2}$, $\frac{dp^N}{dm} = -\frac{k(1-\phi)}{2m^2(1+\phi)}$, which implies that $\frac{dp^C}{dm} < \frac{dp^N}{dm} < 0$. Similarly, we have $\frac{dp^C}{dk} = \frac{(1-\phi)(1-m)}{2m}$, $\frac{dp^N}{dk} = \frac{(1-\phi)(1-m)}{2m(1+\phi)}$, and therefore $\frac{dp^C}{dk} > \frac{dp^N}{dk} > 0$.

Proof of Proposition 3

According to (7), (10) and (11), we have $\frac{d\Pi^N}{dm} = \frac{d\Pi^C}{dm} \cdot \frac{(1+2\phi)}{(1+\phi)^2}$ and $\frac{d\Pi^C}{dm} = \frac{(1-c+k-k\phi)^2}{4} - \frac{(k-k\phi)^2}{4m^2}$. With (5), we know that $1-c > \frac{(1-m)(k-k\phi)}{m} > 0$, then:

$$\begin{aligned} \frac{d\Pi^C}{dm} &= \frac{(1-c+k-k\phi)^2}{4} - \frac{(k-k\phi)^2}{4m^2} = \frac{(1-c)^2}{4} + \frac{(1-c)(k-k\phi)}{2} + \frac{(k-k\phi)^2}{4} - \frac{(k-k\phi)^2}{4m^2} \\ &> \frac{(1-m)^2(k-k\phi)^2}{4m^2} + \frac{(1-m)(k-k\phi)^2}{2m} + \frac{(k-k\phi)^2}{4} - \frac{(k-k\phi)^2}{4m^2} = 0. \end{aligned}$$

Therefore, we can find $\frac{d\Pi^C}{dm} > \frac{d\Pi^N}{dm} > 0$.

Similarly, with $\Pi^N = \Pi_r^N + \Pi_m^N$ we have $\frac{d\Pi^N}{dk} = \frac{d\Pi^C}{dk} \cdot \frac{(1+2\phi)}{(1+\phi)^2}$. Because $\frac{d\Pi^C}{dk} = \frac{(1-m)(1-\phi)[(1-m)(1-\phi)k - m(1-c)]}{2m} < 0$ it follows that $\frac{d\Pi^C}{dk} < \frac{d\Pi^N}{dk} < 0$.

Proof of Proposition 4

According to (8) and (9), we have $\frac{dp^N}{dm} = -\frac{k(1-\phi)}{2m^2(1+\phi)}$ and $\frac{dw^N}{dm} = -\frac{k(1-\phi)}{m^2(1+\phi)}$, so it's easy to get $\frac{dw^N}{dm} < \frac{dp^N}{dm} < 0$. We also have $\frac{dp^N}{dk} = \frac{(1-\phi)(1-m)}{2m(1+\phi)}$ and $\frac{dw^N}{dk} = \frac{(1-\phi)(1-m)}{m(1+\phi)}$, which implies that $\frac{dw^N}{dk} > \frac{dp^N}{dk} > 0$.

Proof of Proposition 5

According to (10) and (11), we have $\frac{d\Pi_m^N}{dm} = \frac{(1-c+k-k\phi)^2}{4(1+\phi)} - \frac{(k-k\phi)^2}{4m^2(1+\phi)}$ and $\frac{d\Pi_r^N}{dm} = \frac{d\Pi_m^N}{dm} \cdot \frac{\phi}{1+\phi}$. Following the same proof of Corollary 2 we get $\frac{d\Pi_m^N}{dm} > 0$. We then have $\frac{d\Pi_m^N}{dm} > \frac{d\Pi_r^N}{dm} > 0$. Similarly, we have $\frac{d\Pi_m^N}{dk} = \frac{(1-m)(1-\phi)[(1-m)(1-\phi)k - m(1-c)]}{2m(1+\phi)} < 0$ and $\frac{d\Pi_r^N}{dk} = \frac{d\Pi_m^N}{dk} \cdot \frac{\phi}{1+\phi}$. It therefore follows that $\frac{d\Pi_m^N}{dk} < \frac{d\Pi_r^N}{dk} < 0$.

Proof of Proposition 6

If the revenue-sharing contract can coordinate the supply chain with omni-channel BOPS, the sum of the profits of the retailer and the manufacturer should be equal to the total profits of the supply chain under a centralized decision-making; that is $\Pi^C = (\Pi_r^B + \Pi_m^B)$ when $\alpha = \alpha^*$.

Substitute (7), (17) and (18) into this equation, sorting and simplification, we obtain the following equation:

$$[m(1-c) - (1-\phi)(1-m)k]^2 = \frac{(1+2\phi+2\theta\alpha^*)[m(1-c) - (1-\phi-\theta)(1-m)k]^2}{(1+\phi+\theta\alpha^*)^2}$$

Incorporating the constraints (5) and (18) and simplifying the above equation yields the following:

$$[m(1-c) - (1-\phi)(1-m)k] = \frac{\sqrt{1+2\phi+2\theta\alpha^*}[m(1-c) - (1-\phi-\theta)(1-m)k]}{1+\phi+\theta\alpha^*}$$

Rewrite and get:

$$\frac{\sqrt{1+2\phi+2\theta\alpha^*}}{1+\phi+\theta\alpha^*} = \frac{m(1-c) - (1-\phi)(1-m)k}{m(1-c) - (1-\phi-\theta)(1-m)k}$$

We have therefore proved the proposition with the constraints (5) and (18).

Proof of Proposition 7

According to (10) and (17), we have:

$$\Pi_r^B = \frac{(\phi + \theta\alpha)[m(1-c) - (1-\phi-\theta)(1-m)k]^2}{4m(1+\phi+\theta\alpha)^2}, \Pi_r^N = \frac{\phi[m(1-c) - (1-\phi)(1-m)k]^2}{4m(1+\phi)^2}$$

We then solve $\frac{\Pi_r^B}{\Pi_r^N}$ to obtain:

$$\frac{\Pi_r^B}{\Pi_r^N} = \frac{(\phi + \theta\alpha)(1+\phi)^2[m(1-c) - (1-\phi-\theta)(1-m)k]^2}{\phi(1+\phi+\theta\alpha)^2[m(1-c) - (1-\phi)(1-m)k]^2}$$

Setting $\alpha = \alpha^*$ and simplifying we get:

$$\frac{\Pi_r^{B*}}{\Pi_r^{N*}} = \frac{(\phi + \theta\alpha^*)(1+\phi)^2}{\phi(1+2\phi+2\theta\alpha^*)}$$

Letting $\frac{\Pi_r^{B*}}{\Pi_r^{N*}} \geq 1$, we obtain $\phi^3 + \theta\alpha^*(1+\phi^2) \geq 0$.

We can therefore show that $\Pi_r^{B*} \geq \Pi_r^{N*}$ when $\phi^3 + \theta\alpha^*(1+\phi^2) \geq 0$ and that $\Pi_r^{B*} < \Pi_r^{N*}$ when $\phi^3 + \theta\alpha^*(1+\phi^2) < 0$.

Proof of Proposition 8

According to (11) and (18), we have:

$$\Pi_m^N = \frac{[m(1-c) - (1-\phi)(1-m)k]^2}{4m(1+\phi)}, \Pi_m^B = \frac{[m(1-c) - (1-\phi-\theta)(1-m)k]^2}{4m(1+\phi+\theta\alpha)}$$

The ratio $\frac{\Pi_m^B}{\Pi_m^N}$ is therefore:

$$\frac{\Pi_m^B}{\Pi_m^N} = \frac{(1+\phi)[m(1-c) - (1-\phi-\theta)(1-m)k]^2}{(1+\phi+\theta\alpha)[m(1-c) - (1-\phi)(1-m)k]^2}$$

Assuming there exists a $\hat{\alpha}$ such that $\frac{\Pi_m^B}{\Pi_m^N} = 1$ with $\phi + \theta\hat{\alpha} \geq 0$, we obtain:

$$\sqrt{\frac{1+\phi}{1+\phi+\theta\hat{\alpha}}} = \frac{m(1-c) - (1-\phi)(1-m)k}{m(1-c) - (1-\phi-\theta)(1-m)k}$$

Letting $\Phi = \frac{[m(1-c) - (1-\phi-\theta)(1-m)k]^2}{[m(1-c) - (1-\phi)(1-m)k]^2}$ and simplifying gives:

$$\Delta\Pi = \frac{\Pi_m^B}{\Pi_m^N} = \frac{1+\phi}{1+\phi+\theta\alpha} \cdot \Phi$$

Using the first-order condition, we get:

$$\frac{d\Delta\Pi}{d\alpha} = -\frac{\theta(1+\phi)}{(1+\phi+\theta\alpha)^2} \cdot \Phi < 0$$

We thus showed that $\Pi_m^B \geq \Pi_m^N$ when $\alpha \leq \hat{\alpha}$ and that $\Pi_m^B < \Pi_m^N$ when $\alpha > \hat{\alpha}$.

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