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Project Procurement Method Decision-Making With Spearman Rank Correlation Coefficient Under Uncertainty Circumstances

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ABSTRACT

A project procurement method (PPM) defines the roles and responsibilities of the participates involved in the construction project. Selecting a suitable PPM is one of the critical issues to achieve the success of a construction project. The selection of PPM is a typical multi-criteria decision-making problem under uncertainty. Moreover, interval-valued intuitionistic fuzzy set (IVIFS) is a useful tool for depicting uncertainty of the multi-criteria decision-making (MCDM) problems. In this paper, the authors consider the PPM selection under IVIFS circumstance. Firstly, they introduce the concept of Spearman rank correlation coefficient (SRCC) between two IVIFSs and then calculate the SRCC between the ideal alternative and each alternative. The ideal option of PPM is determined according to the computed value of SRCC. Overall, the proposed method can avoid the calculation of the criteria weights, and the selection process is simple and straightforward. Finally, a real-world infrastructure project PPM selection has been illustrated the applicability and effectiveness of this methodology.

KEYWORDS

Decision Making, Interval-Valued Intuitionistic Fuzzy Set, Project Procurement Method, Spearman Rank Correlation Coefficient

1. INTRODUCTION

Project procurement method (PPM) describes how the project participants are organized to interact, and how the owner's goals and objectives are transformed into the finished facilities (Moon *et al.*, 2011; ASCE, 1988; Chen *et al.*, 2011). The PPM affects the objectives of a construction project, which are the schedule, cost, and quality (Chan *et al.*, 2001; Khalil, 2002; Blayse and Manley, 2004; Shane *et al.*, 2013; Mollaoglukorkmaz *et al.*, 2013). There are several PPMs in the construction industry. The most common approaches are design-bid-build (DBB), construction management at risk (CM-at risk), design-build (DB), engineering-procurement-construction (EPC) and integrated project delivery (IPD) (Chen *et al.*, 2010; Shi *et al.*, 2014; Qiang *et al.*, 2015; Li *et al.*, 2015). The PPM features its characteristics and meets different situations and owner's requirements (Alhazmi and Mccaffer, 2000). It was approved that the appropriate PPM can effectively get excellent project performance (Hong *et al.*, 2008; Ojiako *et al.*, 2008; Oyetunji and Anderson, 2006). Therefore, selecting a suitable PPM for a construction project is one of the vital decision-making issues for the owner in the planning stage.

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The PPM selection problem is also called the project delivery system (PDS) in the engineering field. Many researchers have done a lot of work on the selection of PDS (Li *et al*., 2015; Liu *et al.*, 2015; Konchar and Sanvido, 1998; Yngling and ShuHuiKerh, 2004; Ling and Liu, 2004). The aim of selecting the PDS is to achieve construction project performance better (Konchar and Sanvido, 1998; Yngling and ShuHuiKerh, 2004; Ling and Liu, 2004). As a powerful decision tool, analytical hierarchical process (AHP) was employed for PDS selection (Khalil, 2002; Alhazmi and Mccaffer, 2000; Mahdi and Alreshaid, 2005; Mafakheri *et al.*, 2007). However, AHP has been criticized for its incapability to deal with uncertainty and its lack of sound statistical theory (Belton and Stewart, 2002) adequately. Moreover, multi-attribute utility was also applied to deal with the PDS selection decision making (Chen *et al.*, 2011; Chan *et al*., 2001; Oyetunji and Anderson, 2006; Love *et al.*, 1998). Case-based reasoning (CBR) is the process of solving new problems based on the solutions of similar past cases, which is suitable for selecting PDS for construction projects (Luu *et al.*, 2003; Ng *et al*., 2005; Luu *et al.*, 2006; Kumaraswamy and Dissanayaka, 2001). Li *et al*. (2015) proposed a decision-making model for the selection of PDS based on information entropy and unascertained set. From the perspective of value-added, Wang *et al.* (2013) have made a comparison to select of PDSs between DB and DBB. Tran and Molenaar (2015) have considered the risk factors and presented a risk-based modeling methodology to the selection of a project delivery method for the highway project. Dai *et al.* (2016) used a hybrid cross-impact technique for PDS decision-making for the highway project. Nevertheless, some shortcomings should be overcome, such as imprecise of evaluation criteria in nature (Ng *et al.*, 2002). Therefore, the fuzzy set theory is also gradually applied to PDS selection (Ng *et al*., 2002; Khanzadi *et al.*, 2016; Wang *et al.*, 2014).

The selection of an appropriate PDS for a construction project is a typical multi-attribute decision-making problem under uncertainty(Ibbs *et al.*, 2011), and the evaluation criteria have intense fuzziness (Ng *et al.*, 2002). Many researchers have done much work on decision-making under uncertainty with the fuzzy set (Boran, 2011; Boran *et al.*, 2011; Ashraf *et al*., 2014; Gupta *et al.*, 2016; Büyüközkan and Güleryüz, 2016; Butt and Akram, 2016a,b; Nguyen, 2016; Habib *et al.*, 2016; Zafar and Akram, 2017; Sarwar and Akram, 2017). Interval-valued intuitionistic fuzzy set (IVIFS) can effectively elucidate the fuzziness and uncertainty of material things (Nguyen, 2016; Atanassov, 1989; Xu, 2007a; Wei et al., 2011; Chen and Huang, 2017). The Spearman rank correlation coefficient (SRCC) is considered as one of the best nonparametric measures of relationship (Dikbas, 2018).SRCC assesses the linear relationships between the ranks of monotonically related variables. Even if the relationship between the variables is not linear. In fact, SRCC had tried to prove that ranks of measurements instead of raw measurements have significant advantages in correlation calculations (Dikbas, 2018).

This paper aims to develop a more accurate and reliable PPM selection method. A decision-making model is established to support PPM selection based on the SRCC between two IVIFSs under IVIFS information. In order to give a comparison between two interval numbers, the concept of connection number in the Set Pair Analysis theory is introduced (Cao *et al.*, 2016; Kumar and Garg, 2018). The remaining parts of this paper are organized as follows. Section 2 gives the research methodology, including some preliminaries on IVIFS and multi-criteria decision-making methods. Section 3 presents the SRCC between IVIFSs. A decision-making algorithm is developed in Section 4. Section 5 provides a case study on PPM selection to verify the feasibility and practicability of the developed approach. Some conclusions and further suggestions are given in Section 6.

2. RESEARCH METHODOLOGY

PPM selection is a typical decision-making problem. Since the fuzziness of the evaluation data for criteria affecting PPM selection, the IVIFS theory is used to select suitable PPM. The following subsections show the preliminaries about IVIFS and multi-criteria decision-making methods.

2.1. Interval Number and Connection Number

Firstly, the definition of the connection number in the set pair analysis theory is given below.

Definition 2.1 (Cao *et al.*, 2016; Kumar and Garg, 2018) Let $X=\left\{x_1,x_2,...,x_n\right\}$ be the universe of discourse. A binary connection number set A is defined as

$$\mu=a+bi \tag{1}$$

where $i\in\left[-1,1\right]$ is the discrepancy degree, $j=-1$ is the contrary degree, $a,b>0$.

For any two connection numbers $\mu_1=a_1+b_1i$ and $\mu_2=a_2+b_2i$, then

if $a_1=a_2$ and $b_1=b_2$, then $\mu_1=\mu_2$;
if $a_1>a_2$ and $a_1-b_1\ge a_2+b_2$, then $\mu_1>\mu_2$;
if $a_1>a_2$, then $\mu_1>\mu_2$;
if $a_1=a_2$ and $b_1>b_2$, then $\mu_1>\mu_2$.

Definition 2.2 (Moore, 1979) Let $\bar{a}$ be an interval number, which represents a closed bounded set of real numbers:

$$\bar{a}=\left[a^L,a^R\right],$$

where a^L and a^R represent lower and upper bounds of the interval number $\bar{a}$, respectively. Especially if $a^L=a^R$, then, $\bar{a}$ is a real number.

For an interval number $\left[a^L,a^R\right]$, its corresponding connection number represents as (Cao *et al.*, 2016; Kumar and Garg, 2018):

$$\left[a^L,a^R\right]=a+bi=\left(a^L+a^R\right)/2+\left(\left(a^R-a^L\right)/2\right)i.$$

Therefore, for two interval numbers $\bar{a}_1=\left[a_1^L,a_1^R\right]$ and $\bar{a}_2=\left[a_2^L,a_2^R\right]$,

(C1) if $\left(a_1^L+a_1^R\right)/2>\left(a_2^L+a_2^R\right)/2$, then $\bar{a}_1>\bar{a}_2$;
(C2) if $\left(a_1^L+a_1^R\right)/2=\left(a_2^L+a_2^R\right)/2$, then
if $\left(a_1^R-a_1^L\right)/2=\left(a_2^R-a_2^L\right)/2$, then $\bar{a}_1=\bar{a}_2$;
if $\left(a_1^R-a_1^L\right)/2>\left(a_2^R-a_2^L\right)/2$, then $\bar{a}_1>\bar{a}_2$.

2.2 A Brief Introduction to IVIFS

As a generalization of fuzzy sets, the intuitionistic fuzzy set (IFS) has better agility in expressing uncertainly and ambiguous information. Because it can be used to describe the characteristics of affirmation, negation, and hesitation simultaneously. Since using crisp values to express the membership and non-membership degrees of IFS is difficult in practice, the concept of IVIFS was proposed by assigning membership and non-membership degrees in terms of intervals. In an IVIFS, for each $x \in X$, the membership degree $u_A(x)$ and non-membership degree $v_A(x)$ can be expressed by a closed interval. Their lower boundaries are denoted by $u_A^L(x)$ and $v_A^L(x)$. At the same time, the upper boundaries are presented as $u_A^R(x)$ and $v_A^R(x)$. Therefore, an IVIFS A in X is defined as:

$$A= \left\{\left(x,\left[u_A^L(x),u_A^R(x)\right],\left[v_A^L(x),v_A^R(x)\right]\right)\middle| x \in X\right\},$$

where $0 \le u_A^L(x) \le u_A^R(x) \le 1$, $0 \le v_A^L(x) \le v_A^R(x) \le 1$, and $0 \le u_A^L(x)+v_A^L(x) \le u_A^R(x)+v_A^R(x) \le 1$. Furthermore, for each element, the hesitation interval relative to A is given as:

$$\pi_A(x)=[\pi_A^L(x),\pi_A^R(x)]=[1-u_A^R(x)-v_A^R(x),1-u_A^L(x)-v_A^L(x)],$$

where $\pi_A^L(x) \in [0,1]$ and $\pi_A^R(x) \in [0,1]$ contain the lower and upper boundaries of hesitation degree, and $\pi_A^L(x) \le \pi_A^R(x)$.

2.3 Weighted Averaging Operator

Let X be a finite universe of discourse, $A=(A_1,A_2,...,A_n)$ be a vector of IVIFSs, $A_i = \left\{\left\langle x_i,\left[u_A^L(x_i),u_A^R(x_i)\right],\left[v_A^L(x_i),v_A^R(x_i)\right]\right\rangle \middle| x_i \in X\right\}$, $i=1,2,\cdots,n$. The weighted averaging operator is defined as (Das *et al.*, 2016):

$$A_0=\left(\left[1-\prod_{i=1}^{n}(1-u_A^L(x_i))^{w_i},1-\prod_{i=1}^{n}(1-u_A^R(x))^{w_i}\right],\left[\prod_{i=1}^{n}v_A^L(x_i)^{w_i},\prod_{i=1}^{n}v_A^R(x_i)^{w_i}\right]\right), \tag{2}$$

where $W=(w_1,w_2,\cdots,w_n)$ is the weight of each vector and $w_i \in \left[0,1\right], \sum_{i=1}^{n} w_i = 1$.

2.4 Score Function and Accuracy Function

For an IVIFS with $A=\left\{\left(x,\left[u_A^L(x),u_A^R(x)\right],\left[v_A^L(x),v_A^R(x)\right]\right)\middle| x \in X\right\}$, the score function is defined as follows (Xu, 2007b):

$$S\left(x\right)=\frac{u_A^R\left(x\right)+u_A^L\left(x\right)-v_A^R\left(x\right)-v_A^L\left(x\right)}{2}. \tag{3}$$

The value of the accuracy function of an IVIFS $A=\left\{\left(x,\left[u_A^L\left(x\right),u_A^R\left(x\right)\right],\left[v_A^L\left(x\right),v_A^R\left(x\right)\right]\right)\middle| x \in X\right\}$ can be computed by the following Equation (Xu, 2007b):

$$H\left(x\right)=\frac{u_A^R\left(x\right)+u_A^L\left(x\right)+v_A^R\left(x\right)+v_A^L\left(x\right)}{2}. \tag{4}$$

For two IVIFSs $A_1 = \left\{\left(x_1, \left[u_A^L\left(x_1\right), u_A^R\left(x_1\right)\right], \left[v_A^L\left(x_1\right), v_A^R\left(x_1\right)\right]\right) \middle| x_1 \in X\right\}$ and $A_2 = \left\{\left(x_2, \left[u_A^L\left(x_2\right), u_A^R\left(x_2\right)\right], \left[v_A^L\left(x_2\right), v_A^R\left(x_2\right)\right]\right) \middle| x_2 \in X\right\}$, Xu (2007b) proposed the following comparison method based on the score function and accuracy function:

If $S\left(x_1\right) < S\left(x_2\right)$, then $A_1 < A_2$;
If $S\left(x_1\right) = S\left(x_2\right)$, but $H\left(x_1\right) < H\left(x_2\right)$, then $A_1 < A_2$;
If $S\left(x_1\right) = S\left(x_2\right)$ and $H\left(x_1\right) = H\left(x_2\right)$, then $A_1 = A_2$.

3. SPEARMAN RANK CORRELATION COEFFICIENT BETWEEN IVIFSS

Definition 3.1 (McGraw Hill, 1989) Let $\left(X_1, \ldots, X_n\right)$ be a sample from a population, the corresponding sample observations $\left(x_1, \ldots, x_n\right)$ are sorted in ascending order, that is, $x_{(1)} < \ldots < x_{(n)}$. If $x_i = x_{(k)}$, then, k is called the rank of the sample X_i, i.e., $R_i = k$, $i = 1, 2, \ldots, n$.

In each repeated sampling, R_i is a random variable. If there a case occurs that some x are the same; for instance, there exists $x_i = x_j$ for $i \neq j$, then their ranks are the average of those ranks. For example, if there is a sequence of a sample as: $1 \quad 1 \quad 2 \quad 2 \quad 2 \quad 3$, then the ranks of the two 1 are all $\frac{1+2}{2} = 1.5$, and the three ranks of 2 are all $\frac{3+4+5}{3} = 4$.

In statistics, the SRCC is the Pearson correlation coefficient applied to the ranks R. When there are not two values of X or two values of Y with the same rank (so-called ties), the Spearman correlation coefficient can be computed as (McGraw−Hill, 1989; Myers and Well, 2013):

$$r_s = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n\left(n^2 - 1\right)}, \tag{5}$$

where $d_i = R\left(x_i\right) - R\left(y_i\right)$ $i = 1, 2, \ldots, n$ are the differences between the ranks of x_i and y_i. If there are ties (two values of X or two values of Y with the same rank), but the number of ties is smaller compared with n, and Equation (5) still holds.

The Spearman correlation coefficient fulfills the requirements of the correlation measures. As Equation (5) was obtained from the Pearson coefficient for ranks, it fulfills the same properties as the Pearson coefficient:

(P1) $r_s\left(A, B\right) = r_s\left(B, A\right)$; (P2) If $A = B$, then $r_s\left(A, B\right) = 1$; (P3) $\left|r_s\left(A, B\right)\right| \leq 1$.

When the variables X and Y are perfectly positively related, i.e., when X increasing with Y increasing, then r_s it is equal to 1. When X and Y are perfectly negatively related, i.e., when X increases whenever Y decreases, r_s is equal to -1. r_s is equal to zero when there is no relation between

X and Y. Values between -1 and 1 give a relative indication of the degree of relationship between X and Y, in other words, $-1 \le r_s \le 1$.

Based on the correlation coefficient between two Atanassov's IFSs proposed by Szmidt and Kacprzyk (Szmidt and Kacprzyk, 2011), the SRCC between two IVIFSs is presented as below.

Definition 3.2 (Szmidt and Kacprzyk, 2011) The SRCC between two IFSs A and B is defined as:

$$r_{s-\mathrm{IFS}} = \frac{1}{3}\left(r_{s1} + r_{s2} + r_{s3}\right), \tag{6}$$

where r_{s1}, r_{s2} and r_{s3} are the SRCCs between A and B with respect to their membership function, non-membership function, and hesitation function, respectively. r_{s1} is given as

$$r_{s1} = 1 - \frac{6\sum_{i=1}^{n} d_{1i}^2}{n\left(n^2 - 1\right)}, \tag{7}$$

where d_{1i}, $i = 1,2,\ldots,n$ are the differences in the ranks with respect to the non-membership functions: $d_{1i} = R\left(u_A\left(x_i\right)\right) - R\left(u_B\left(x_i\right)\right)$. r_{s2} is given as

$$r_{s2} = 1 - \frac{6\sum_{i=1}^{n} d_{2i}^2}{n\left(n^2 - 1\right)}, \tag{8}$$

where d_{2i}, $i = 1,2,\ldots,n$ are the differences in the ranks with respect to the hesitation functions: $d_{2i} = R\left(v_A\left(x_i\right)\right) - R\left(v_B\left(x_i\right)\right)$. r_{s3} is given as

$$r_{s3} = 1 - \frac{6\sum_{i=1}^{n} d_{3i}^2}{n\left(n^2 - 1\right)}, \tag{9}$$

where d_{3i}, $i = 1,2,\ldots,n$ are the differences in the ranks with respect to the membership functions: $d_{3i} = R\left(\pi_A\left(x_i\right)\right) - R\left(\pi_B\left(x_i\right)\right)$.

Definition 3.3 The SRCC between two IVIFSs A and B is defined as:

$$r_{s-\mathrm{IVIFS}} = \frac{1}{3}\left(r_{su} + r_{sv} + r_{s\pi}\right), \tag{10}$$

where r_{su}, r_{sv} and $r_{s\pi}$ are the SRCCs between A and B with respect to their membership function, non-membership function, and hesitation function, respectively. r_{su} is given as

$$r_{su} = 1 - \frac{6\sum_{i=1}^{n} d_{ui}^2}{n\left(n^2 - 1\right)}, \tag{11}$$

where d_{ui}, $i = 1,2,\ldots,n$ are the differences in the ranks with respect to the membership functions: $d_{ui} = R\left(u_A\left(x_i\right)\right) - R\left(u_B\left(x_i\right)\right)$. r_{sv} is given as

$$r_{sv} = 1 - \frac{6\sum_{i=1}^{n} d_{vi}^2}{n\left(n^2 - 1\right)}, \tag{12}$$

where d_{vi}, $i = 1,2,\ldots,n$ are the differences in the ranks with respect to the non-membership functions: $d_{vi} = R\left(v_A\left(x_i\right)\right) - R\left(v_B\left(x_i\right)\right)$. $r_{s\pi}$ is given as

$$r_{s\pi} = 1 - \frac{6\sum_{i=1}^{n} d_{\pi i}^2}{n\left(n^2 - 1\right)}, \tag{13}$$

where $d_{\pi i}$, $i = 1,2,\ldots,n$ are the differences in the ranks with respect to the hesitation functions: $d_{\pi i} = R\left(\pi_A\left(x_i\right)\right) - R\left(\pi_B\left(x_i\right)\right)$.

Obviously, for the Spearman rank correlation (10), the same properties as the Pearson correlation coefficient are valid, i.e.:

(P1) $r_{s-IVIFS}(A,B) = r_{s-IVIFS}(B,A)$; then $r_{s-\mathrm{IVIFS}}\left(A,B\right) = 1$;
(P3) $\left|r_{s-\mathrm{IVIFS}}\left(A,B\right)\right| \leq 1$.

The separate components of the Spearmen rank correlation (10) i.e., Equations (11)-(13) fulfill the above properties, too. Obviously, in the case of crisp sets, $r_{s-IVIFS}$ in (10) reduces to r_s in (5), and in the case of the upper bound and lower bound are equal, $r_{s-IVIFS}$ in (10) reduces to $r_{s-\mathrm{IFS}}$ in (6).

Example 1 There are two IVIFSs *A* and *B* described below:

$$\begin{aligned} A = \Big\{ & \left(x_1, \left[0.6, 0.7\right], \left[0.1, 0.2\right]\right), \left(x_2, \left[0.4, 0.6\right], \left[0.2, 0.3\right]\right), \\ & \left(x_3, \left[0.4, 0.5\right], \left[0.1, 0.3\right]\right), \left(x_4, \left[0.3, 0.4\right], \left[0.1, 0.4\right]\right) \Big\} \end{aligned}$$

and

$$B = \left\{\left(x_1, \left[0.3, 0.4\right], \left[0.1, 0.2\right]\right), \left(x_2, \left[0.5, 0.6\right], \left[0.1, 0.3\right]\right), \left(x_3, \left[0.4, 0.5\right], \left[0.2, 0.4\right]\right), \left(x_4, \left[0.2, 0.5\right], \left[0.2, 0.3\right]\right)\right\}.$$

From equations (10)-(13) and the results in Tables 1-3, $r_{s-\mathrm{IVIFS}}\left(A, B\right) = 0.0833$ is get.

4. DECISION-MAKING MODEL FOR PPM SELECTION

For a PPM selection problem, it is assumed that the sets of experts, alternatives, and criteria are $H = \left\{h_1, h_2, \ldots, h_l\right\}$, $O = \left\{o_1, o_2, \ldots, o_m\right\}$ and $C = \left\{c_1, c_2, \ldots, c_n\right\}$, respectively. In the process of PPM selection, the individual evaluation matrix is determined by each expert in the first step. And then, through aggregating the individual evaluation matrix, the evaluation matrix involved all experts' evaluation information is obtained. The following step is the determination of the intuitionistic fuzzy ideal alternative. And then, SRCCs can be calculated between the ideal alternative and each alternative. Finally, the ranking order is received according to all SRCCs, and the suitable PPM is selected. The detailed description is shown in Figure 1. The aggregation of experts' evaluation information needs to calculate the experts' weights. There are a lot of methods to calculate the experts' weights in decision making, for instance, subjective weigh method, objective weighs method, *etc*. In this study, an averaging weight method is used to obtain the experts' weights.

4.1 Determination of The Individual Decision Matrices

The alternatives are ordered based on the criteria characterized by IVIFSs. For the first step of PPM selection, every expert should give the evaluation values of all the alternatives under the criteria in terms of the decision matrices. Assume that the decision matrices given by the experts are $H = \left\{h_1, h_2, \ldots, h_l\right\}$, where

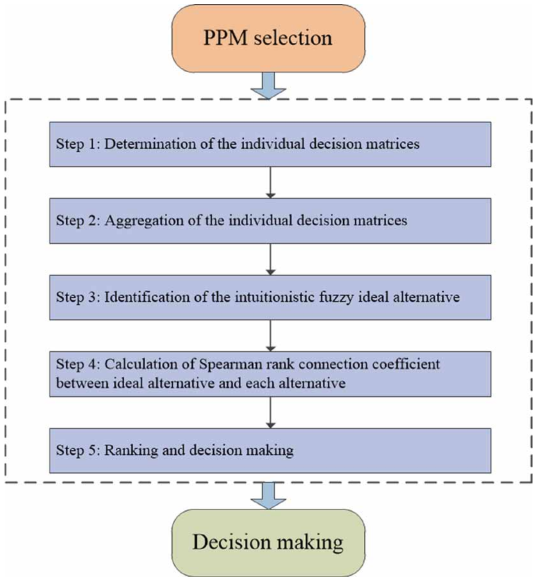
$$h_k = \begin{pmatrix} h_{11}^{(k)} & h_{12}^{(k)} & \cdots & h_{1n}^{(k)} \\ h_{21}^{(k)} & h_{22}^{(k)} & \cdots & h_{2n}^{(k)} \\ \cdots & \cdots & \cdots & \cdots \\ h_{m1}^{(k)} & h_{m2}^{(k)} & \cdots & h_{mn}^{(k)} \end{pmatrix} \tag{14}$$

where $h_{ij}^{(k)} = \left(\left[u_{ij}^{L(k)}, u_{ij}^{R(k)}\right], \left[v_{ij}^{L(k)}, v_{ij}^{R(k)}\right]\right)$ is the evaluation value given by the expert h_k for the alternative O_i under the criteria C_j, $k = 1,2,\ldots,l$, $i = 1,2,\ldots,m$, $j = 1,2,\ldots,n$.

4.2 Aggregation of The Individual Decision Matrices

After determining the weights of the experts, the collective decision matrix H^o can be obtained according to the weighted average operator by equation (2) as follows:

Figure 1. The process of PPM selection



$$H^o = \begin{pmatrix} h_{11}^o & h_{12}^o & \cdots & h_{1n}^o \\ h_{21}^o & h_{22}^o & \cdots & h_{2n}^o \\ \cdots & \cdots & \cdots & \cdots \\ h_{m1}^o & h_{m2}^o & \cdots & h_{mn}^o \end{pmatrix} \tag{15}$$

where $h_{ij}^o = \left\{ \left[1 - \prod_{k=1}^{l} \left(1 - u_{ij}^{L(k)}\right)^{w_{ij}^{(k)}}, 1 - \prod_{k=1}^{l} \left(1 - u_{ij}^{R(k)}\right)^{w_{ij}^{(k)}} \right], \left[\prod_{k=1}^{l} \left(v_{ij}^{L(k)}\right)^{w_{ij}^{(k)}}, \prod_{k=1}^{l} \left(v_{ij}^{R(k)}\right)^{w_{ij}^{(k)}} \right] \right\}, i = 1,2,\ldots,m$, and $j = 1,2,\ldots,n$.

4.3 Identification of the Intuitionistic Fuzzy Ideal Alternative

Based on the SRCC between two IVIFSs, a new PPM selection model is developed, which starts with the determination of the intuitionistic fuzzy ideal solution. The decision information appears as the form of IVIFSs. The score function and the accuracy function are employed to identify the intuitionistic fuzzy ideal solution. However, they usually do not exist intuitionistic fuzzy ideal solution in the real selection process. In other words, the intuitionistic fuzzy ideal solution vector O^* is usually not the feasible alternative, namely, $O^* \notin O$. Otherwise, the intuitionistic fuzzy ideal solution vector O^* is the optimal alternative vector of the selection decision-making problem. In this paper, the following equation is employed to identify the intuitionistic fuzzy ideal solution O^*:

for benefit type criteria

$$O^* = \left\{O_1^*, O_2^*, \ldots, O_n^*\right\} \text{ where } O_j^* = \left\{C_j, \max_i \left\{h_{ij}^o\right\} \middle| j = 1,2,\ldots,n\right\} \tag{16}$$

and for type cost criteria

$$O^* = \left\{O_1^*, O_2^*, \ldots, O_n^*\right\} \text{ where } O_j^* = \left\{C_j, \min_i \left\{h_{ij}^o\right\} \middle| j = 1, 2, \ldots, n\right\} \quad (17)$$

4.4 Calculation of Spearman Rank Connection Coefficient Between The Ideal Alternative and Each Alternative

According to equation (10), the SRCC between ideal alternative and each alternative can be obtained as follows:

$$r_{i-O^*} = \frac{1}{3}\left(r_{su} + r_{sv} + r_{s\pi}\right) \quad (18)$$

where r_{su}, r_{sv} and $r_{s\pi}$ are described in Definition 3.3, where $i = 1, 2, \ldots, m$.

4.5 Ranking and decision making

From the result obtained in subsection 4.4, the SRCC values for each alternative present the rank of all the alternatives. That is, we can get the ranking of the alternatives and the optimal alternative employing the values of all SRCCs.

5. CASE STUDY

5.1 PPM Selection Problem Statement

In this section, the proposed decision-making support method is applied to a real-world infrastructure project PPM selection. For a construction project, four experts h_1, h_2, h_3, h_4 from different fields, including academic, engineering, client, and contractor, are invited to select the appropriate PPM. After the preliminary analysis, four PPMs are considered for selection: IPD (O_1), EPC (O_2), DBB (O_3) and DB (O_4)

5.2 Indicators of PPM Selection

According to the research results of An *et al.* (2018), the indicator system of PPM selection can be determined, as shown in Figure 2.

5.3 PPM Decision

The process of decision making is as follows.

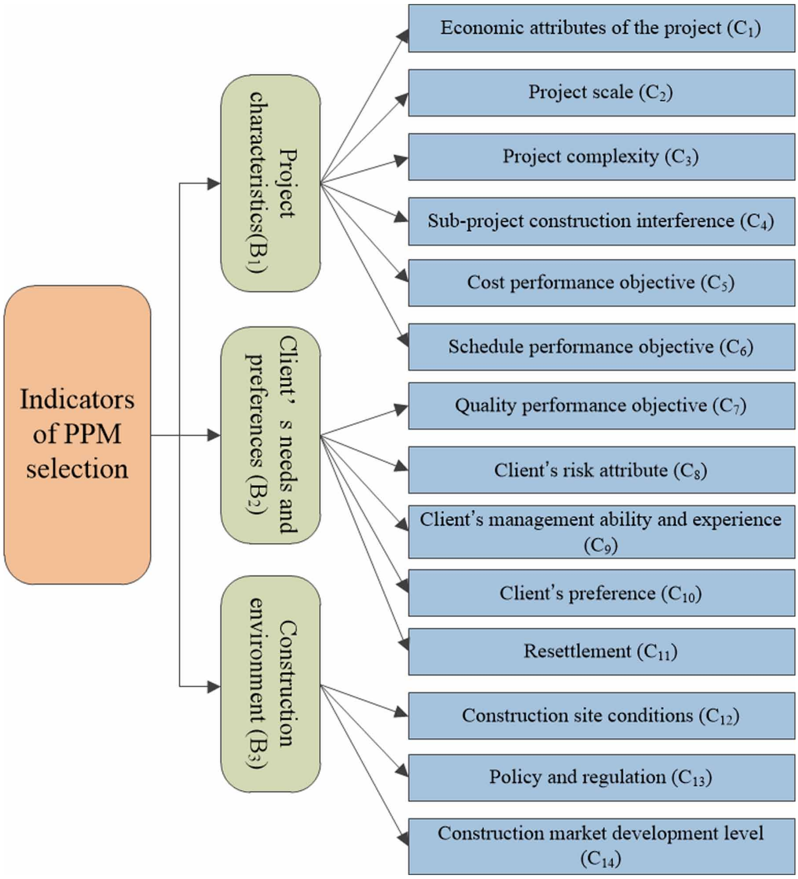
Step 1: Determination of the individual decision matrices.

Each expert needs to give the evaluation values of all alternatives under the criteria. There are four experts, and the evaluation results are shown in Table 4.

Step 2: Identification of the intuitionistic fuzzy ideal alternative.

According to Table A4, equations (16) and (17), the ideal solution is obtained as follows:

Figure 2. Indicator system of PPM selection



$$
\begin{aligned}
O^{*}=\{&([0.3137,0.4654],[0.2163,0.3240]),([0.4892,0.5897],[0.1257,0.2617])\\
&([0.4892,0.5897],[0.1257,0.2617]),([0.5661,0.6904],[0.0931,0.1861])\\
&([0.6147,0.7919],[0.0783,0.1456]),([0.6550,0.7940],[0.0595,0.1612])\\
&([0.6147,0.7154],[0.0783,0.1831]),([0.2692,0.4091],[0.2523,0.3807])\\
&([0.5686,0.7154],[0.1316,0.2300]),([0.2402,0.3803],[0.2603,0.4122])\\
&([0.4313,0.5661],[0.1355,0.2364]),([0.5897,0.7590],[0.0595,0.1355])\\
&([0.2284,0.3827],[0.3234,0.4467]),([0.6904,0.8345],[0.0500,0.1000])\}.
\end{aligned}
$$

Step 3: Aggregation of the individual decision matrices.

According to equation (2), with equal weight for each expert, the aggregated values can be calculated, as shown in Table 5.

Step 4: According to equations (10)-(13), the differences in the ranks concerning membership, non-membership, and hesitation degree for O_1 can be calculated. The detailed results are shown in Tables 6-8. Then SRCCs between alternative O_1 and ideal alternative: $r_{O_1} = 0.0579$.

Similarly, SRCCs between alternative O_2 and ideal alternative is $r_{O_2} = 0.2425$ from the results in Tables 9-11. Moreover, SRCCs between O_3 and the ideal alternative is $r_{O_3} = 0.222$, the calculate results are as shown in Tables 12-14. The SRCCs between O_4 and ideal alternative is $r_{O_4} = 0.4593$, the detailed results are shown in Tables 14-17.

Step 5: From the results in Step 4, the ranking for O_1 , O_2 , O_3 and O_4 is:

$$r_{O_1} < r_{O_3} < r_{O_2} < r_{O_4}$$

that is, the order of the four PPMs is: $IPD < DBB < EPC < DB$. It can be seen that DB is the most suitable PPM for this project and followed by *EPC*. *IPD* is the least suitable PPM. From the results, it is can be seen that the ranking order is acceptable for the practical application.

5.4 Discussion and Comparison Analysis

In this section, comparison analysis and discussion are given to state the advantage of the proposed method by comparing it with another two decision-making methods.

The comparative methods are the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS)(Tan, 2011) and Multi-Objective Optimization by Ratio Analysis(MULTIMOORA)(Brauers and Zavadskas, 2010). The main principle of the TOPSIS method is that the optimal alternative should have the shortest distance measured from the positive ideal solution and the farthest distance measure from the negative one. MULTIMOORA is a decision-making method based on dimensionless measurement: ratio system, reference point method, and full multiplicative optimization. It employs the dominance theory to obtain a final integrative ranking.

Using the line of classical TOPSIS method and MULTIMOORA method, the case study can be calculated. The ranking results are $DBB < IPD < EPC < DB$ and $IPD < DBB < EPC < DB$, respectively. Obviously, the ranking result using the MULTIMOORA method is the same as that using the proposed method, and the result of the TOPSIS method is different to a small extent. However, the *DB* PPM is always at the first rank based on the three methods. Besides, the worst PPM provided by the proposed method is *IPD*, and the TOPSIS method is *DBB*. The reliability and feasibility of the proposed method are shown from the stated above.

6. CONCLUSION

PPM determines not only project performance, but also critical for project success. For a given project, selecting the proper PPM is one of the essential tasks for the owners. PPM selection is a typical multi-criteria decision-making problem. Moreover, IVIFS is a useful tool for depicting the uncertainty of multi-criteria decision-making problems. This paper firstly introduces the concept of Spearman rank correlation coefficient (SRCC) between two IVIFSs, and then the SRCC between ideal alternative and each alternative is calculated. The ideal option of PPM is determined according to the computed value of SRCC. Finally, to illustrate the applicability and effectiveness of this methodology, a real-world infrastructure project PPM selection was demonstrated.

The main contributions of this paper are as follows: (1) this study introduces the concept of SRCC between two IVIFSs to measure the "closeness" degree of alternative and ideal alternative; (2) the ranking orders of all alternatives are obtained through calculating the value of SRCC between each alternative and the ideal alternative, which enriches the theoretical knowledge of PPM selection; (3) this study utilizes connection number in Set Pair Analysis theory to deal with interval number, which is an effective way to assess the degree of closeness between two evaluated objects. In the process of PPM selection, the applications and effectiveness of the proposed decision-making method under interval intuitionistic fuzzy environments can be shown. The proposed method differs from interval intuitionistic fuzzy multi-criteria decision making; it not only can quickly and clearly be calculated but also d avoiding the calculation of criteria weights. It makes the method more flexible and practical than existing decision-making methods. Throughout the whole process of research and practice, it is realized that the improved interval number theory is essential for the precise result. So, in the future, the work on the comparison of intervals should be done. Besides, this study does not consider how to design and innovate a PPM according to a project's characteristics. Therefore, how to design and innovate a PPM according to a project's characteristics may be a good objective for future research.

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REFERENCES

Alhazmi, T., & Mccaffer, R. (2000). Project Procurement System Selection Model. *Journal of Construction Engineering and Management*, *126*(3), 176–184. doi:10.1061/(ASCE)0733-9364(2000)126:3(176)

An, X., Wang, Z., Li, H., & Ding, J. (2018). Project Delivery System Selection with Interval-Valued Intuitionistic Fuzzy Set Group Decision-Making Metho. *Group Decision and Negotiation*, *27*(4), 689–704. doi:10.1007/s10726-018-9581-y

ASCE. (1988). *Quality in the Constructed Project: A Guideline for Owners, Designers, and Constructors*. New York Ny American Society of Civil Engineers.

Ashraf, A., Akram, M., & Sarwar, M. (2014). Fuzzy decision support system for fertilizer. *Neural Computing & Applications*, *25*(6), 1495–1505. doi:10.1007/s00521-014-1639-4 PMID:24892071

Atanassov, K., & Gargov, G. (1989). Interval valued intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, *31*(3), 343–349. doi:10.1016/0165-0114(89)90205-4

Belton, V., & Stewart, T. J. (2002). Multiple criteria decision analysis: An integrated approach. *International*, *142*, 192–202.

Blayse, A. M., & Manley, K. (2004). Key influences on construction innovation. *Construction Innovation*, *4*(3), 143–154. doi:10.1108/14714170410815060

Boran, F. E. (2011). An integrated intuitionistic fuzzy multicriteria decision making method for facility location selection. *Mathematical & Computational Applications*, *16*(2), 487–496. doi:10.3390/mca16020487

Boran, F. E., Gen, S., & Akey, D. (2011). Personnel selection based on intuitionistic fuzzy sets. *Human Factors and Ergonomics in Manufacturing & Service Industries*, *21*(5), 493–503. doi:10.1002/hfm.20252

Brauers, W. K. M., & Zavadskas, E. K. (2010). Project management by MULTIMOORA as an instrument for transition economies. *Technological and Economic Development of Economy*, *16*(1), 5–24. doi:10.3846/tede.2010.01

Butt, M. A., & Akram, M. (2016a). A new intuitionistic fuzzy rule-based decision-making system for an operating system process scheduler. *SpringerPlus*, *5*(1), 1547. doi:10.1186/s40064-016-3216-z PMID:27652120

Butt, M. A., & Akram, M. (2016b). A novel fuzzy decision-making system for CPU scheduling algorithm. *Neural Computing & Applications*, *27*(7), 1927–1939. doi:10.1007/s00521-015-1987-8

Bykzkan, G., & Gleryz, S. (2016). A new integrated intuitionistic fuzzy groUp decision making approach for product development partner selection. *Computers & Industrial Engineering*, *102*, 383–395. doi:10.1016/j.cie.2016.05.038

Cao, Y. X., Zhou, H., & Wang, J. Q. (2016). An approach to interval-valued intuitionistic stochastic multi-criteria decision-making using set pair analysis. *International Journal of Machine Learning and Cybernetics*, *9*(4), 629–640. doi:10.1007/s13042-016-0589-9

Chan, A. P. C., Yung, E. H. K., Lam, P. T. I., Tam, C. M., & Cheung, S. O. (2001). Application of Delphi method in selection of procurement systems for construction projects. *Construction Management and Economics*, *19*(7), 699–718. doi:10.1080/01446190110066128

Chen, S. M., & Huang, Z. C. (2017). Multiattribute decision making based on interval-valued intuitionistic fuzzy values and linear programming methodology. *Information Sciences*, *381*, 341–351. doi:10.1016/j.ins.2016.11.010

Chen, Y. Q., Liu, J. Y., Li, B. G., & Lin, B. S. (2011). Project delivery system selection of construction projects in China. *Expert Systems with Applications*, *38*(5), 5456–5462. doi:10.1016/j.eswa.2010.10.008

Chen, Y. Q., Lu, H., Lu, W., & Zhang, N. (2010). Analysis of project delivery systems in Chinese construction industry with data envelopment analysis (DEA). *Engineering (London)*, *17*(6), 598–614.

Complete business statistics. (1989). Mcgraw−Hill Companies.

Dai, Q. T., Molenaar, K. R., & Alarcn, L. F. (2016). A Hybrid Cross-Impact Approach to Predicting Cost Variance of Project Delivery Decisions for Highways. *Journal of Infrastructure Systems*, *22*(1), 04015017. Advance online publication. doi:10.1061/(ASCE)IS.1943-555X.0000270

Das, S., Dutta, B., & Guha, D. (2015). Weight computation of criteria in a decision-making problem by knowledge measure with intuitionistic fuzzy set and interval-valued intuitionistic fuzzy set. *Soft Computing*, *20*(5), 3421–3442.

Dikbas, F. (2018). A New Two-Dimensional Rank Correlation Coefficient. *Water Resources Management*, *32*(2), 1539–1553. doi:10.1007/s11269-017-1886-0

Gupta, P., Mehlawat, M. K., & Grover, N. (2016). Intuitionistic fuzzy multi-attribute group decision-making with an application to plant location selection based on a new extended VIKOR method. *Information Sciences*, *370-371*, 184–203. doi:10.1016/j.ins.2016.07.058

Habib, S., Akram, M., & Ashraf, A. (2016). Fuzzy Climate Decision Support Systems for Tomatoes in High Tunnels. *International Journal of Fuzzy Systems*, *19*(3), 751–775. doi:10.1007/s40815-016-0183-z

Hong, H. K., Kim, J. S., Kim, T., & Leem, B. H. (2008). The effect of knowledge on system integration project performance. *Industrial Management & Data Systems*, *108*(3), 385–404. doi:10.1108/02635570810858787

Ibbs, W., Chih, Y. Y., & Hartmann, A. (2011). Alternative methods for choosing an appropriate project delivery system (PDS). *Facilities*, *29*(13/14), 527–541. doi:10.1108/02632771111178418

Khalil, M. I. A. (2002). Selecting the appropriate project delivery method using AHP. *International Journal of Project Management*, *20*(6), 469–474. doi:10.1016/S0263-7863(01)00032-1

Khanzadi, M., Nasirzadeh, F., Hassani, S. M. H., & Mohtashemi, N. N. (2016). An integrated fuzzy multi-criteria group decision making approach for project delivery system selection. *Scientia Iranica*, *23*(3), 802–814. doi:10.24200/sci.2016.2160

Konchar, M., & Sanvido, V. (1998). Comparison of U.S. Project Delivery Systems. *Journal of Construction Engineering and Management*, *124*(6), 435–444. doi:10.1061/(ASCE)0733-9364(1998)124:6(435)

Kumar, K., & Garg, H. (2016). TOPSIS method based on the connection number of set pair analysis under interval-valued intuitionistic fuzzy set environment. *Computational & Applied Mathematics*, *37*(2), 1319–1329. doi:10.1007/s40314-016-0402-0

Kumaraswamy, M. M., & Dissanayaka, S. M. (2001). Developing a decision support system for building project procurement. *Building and Environment*, *36*(3), 337–349. doi:10.1016/S0360-1323(00)00011-1

Li, H., Qin, K., & Li, P. (2015). Selection of project delivery approach with unascertained model. *Kybernetes*, *44*(2), 238–252. doi:10.1108/K-01-2014-0012

Ling, F. Y. Y., & Kerh, S. H. (2004). Comparing the Performance of Design-Build and Design-Bid-Build Building Projects in Singapore. *Architectural Science Review*, *47*(2), 163–175. doi:10.1080/00038628.2004.9697040

Ling, F. Y. Y., & Liu, M. (2004). Using neural network to predict performance of design-build projects in Singapore. *Building and Environment*, *39*(10), 1263–1274. doi:10.1016/j.buildenv.2004.02.008

Liu, B., Huo, T., Shen, Q., Yang, Z., Meng, J., & Xue, B. (2015). Which Owner Characteristics Are Key Factors Affecting Project Delivery System Decision Making? Empirical Analysis Based on the Rough Set Theory. *Journal of Management Engineering*, *31*(4), 05014018. Advance online publication. doi:10.1061/(ASCE)ME.1943-5479.0000298

Love, P. E. D., Skitmore, M., & Earl, G. (1998). Selecting a suitable procurement method for a building project. *Construction Management and Economics*, *16*(2), 221–233. doi:10.1080/014461998372501

Luu, D. T., Ng, S. T., & Chen, S. E. (2005). Formulating Procurement Selection Criteria through Case-Based Reasoning Approach. *Journal of Computing in Civil Engineering*, *19*(3), 269–276. doi:10.1061/(ASCE)0887-3801(2005)19:3(269)

Luu, D. T., Ng, S. T., Chen, S. E., & Jefferies, M. (2006). A strategy for evaluating a fuzzy case-based construction procurement selection system. *Advances in Engineering Software*, *37*(3), 159–171. doi:10.1016/j.advengsoft.2005.05.004

Luu, D. T., Thomas, N. S., & Chen, S. E. (2003). A case-based procurement advisory system for construction. *Advances in Engineering Software*, *34*(7), 429–438. doi:10.1016/S0965-9978(03)00043-7

Mafakheri, F., Dai, L., Slezak, D., & Nasiri, F. (2007). Project Delivery System Selection under Uncertainty: Multicriteria Multilevel Decision Aid Model. *Journal of Management Engineering*, *23*(4), 200–206. doi:10.1061/(ASCE)0742-597X(2007)23:4(200)

Mahdi, I. M., & Alreshaid, K. (2005). Decision support system for selecting the proper project delivery method using analytical hierarchy process (AHP). *International Journal of Project Management*, *23*(7), 564–572. doi:10.1016/j.ijproman.2005.05.007

Mollaoglu-Korkmaz, S., Swarup, L., & Riley, D. (2013). Delivering Sustainable, High-Performance Buildings: Influence of Project Delivery Methods on Integration and Project Outcomes. *Journal of Management Engineering*, *29*(1), 71–78.

Moon, H., Cho, K., Hong, T., & Hyun, C. (2011). Selection Model for Delivery Methods for Multifamily-Housing Construction Projects. *Journal of Management Engineering*, *27*(2), 106–115.

Moore, A. R. (1979). Methods and Applications of Interval Analysis. *SIAM Studies in Applied Mathematics*.

Myers, J. L., & Well, A. D. (2003). *Research design and statistical analysis*. Lawrence Erlbaum Associates.

Ng, S. T., Luu, D. T., Chen, S. E., & Lam, K. C. (2002). Fuzzy membership functions of procurement selection criteria. *Construction Management and Economics*, *20*(3), 285–296.

Nguyen, H. (2016). A new interval-valued knowledge measure for interval-valued intuitionistic fuzzy sets and application in decision making. *Expert Systems with Applications*, *56*, 143–155.

Ojiako, U., Johansen, E., & Greenwood, D. (2008). A qualitative re-construction of project measurement criteria. *Industrial Management & Data Systems*, *108*(3), 405–417.

Oyetunji, A. A., & Anderson, S. D. (2006). Relative Effectiveness of Project Delivery and Contract Strategies. *Journal of Construction Engineering and Management*, *132*(1), 3–13.

Qiang, M., Wen, Q., Jiang, H., & Yuan, S. (2015). Factors governing construction project delivery selection: A content analysis. *International Journal of Project Management*, *33*(8), 1780–1794.

Sarwar, M., & Akram, M. (2017). Certain Algorithms for Computing Strength of Competition in Bipolar Fuzzy Graphs. *International Journal of Uncertainty, Fuzziness and Knowledge-based Systems*, *25*(6), 877–896.

Shane, J. S., Bogus, S. M., & Molenaar, K. R. (2013). Municipal Water/Wastewater Project Delivery Performance Comparison. *Journal of Management Engineering*, *29*(3), 251–258.

Shi, Q., Zhou, Y., Xiao, C., Chen, R., & Zuo, J. (2014). Delivery risk analysis within the context of program management using fuzzy logic and DEA: A China case study. *International Journal of Project Management*, *32*(2), 341–349.

Szmidt, E., & Kacprzyk, J. (2011). The Spearman and Kendall rank correlation coefficients between intuitionistic fuzzy sets. *Proceedings of the 7th conference of the European Society for Fuzzy Logic and Technology, Eusflat, European*, 521-528.

Tan, C. Q. (2011). A multi-criteria interval-valued intuitionistic fuzzy group decision making with choquet integral-based TOPSIS. *Expert Systems with Applications*, *38*, 3023–3033.

Tran, D. Q., & Molenaar, K. R. (2015). Risk-Based Project Delivery Selection Model for Highway Design and Construction. *Journal of Construction Engineering and Management*, *141*(12). Advance online publication. doi:10.1061/(asce)co.1943-7862.0001024

Wang, L., An, X., & Li, H. (2014). Applying fuzzy set model for selecting project delivery system. *WIT Transactions on Modelling and Simulation*, *60*, 1301–1308.

Wang, Z., Wang, D., Yang, G., & Ding, J. (2013). Selection of Construction Project Delivery Method Based on Value-Added Analysis: A Theoretical Framework. *Iccrem*, 403-414.

Wei, C. P., Wang, P., & Zhang, Y. Z. (2011). Entropy, similarity measure of interval-valued intuitionistic fuzzy sets and their applications. *Information Sciences*, *181*(19), 4273–4286.

Xu, Z. S. (2007a). Intuitionistic fuzzy aggregation operators. *IEEE Transactions on Fuzzy Systems*, *15*(6), 1179–1187.

Xu, Z. S. (2007b). Methods for aggregating interval-valued intuitionistic fuzzy information and their application to decision making. *Control and Decision*, *22*(2), 215–219.

Zafar, F., & Akram, M. (2017). A Novel Decision-Making Method Based on Rough Fuzzy Information. *International Journal of Fuzzy Systems*, *20*(3), 1000–1014.

APPENDIX 1

Table 1. Calculations of equation (11)

u_A	$R(u_A)$	u_B	$R(u_B)$	d_{u1}	d_{u1}^2
[0.6,0.7]	4	[0.3,0.4]	1.5	2.5	6.25
[0.4,0.6]	3	[0.5,0.6]	4	-1	1
[0.4,0.5]	2	[0.4,0.5]	3	-1	1
[0.3,0.4]	1	[0.2,0.5]	1.5	-0.5	0.25

Table 2. Calculations of equation (12)

v_A	$R(v_A)$	v_B	$R(v_B)$	d_{v1}	d_{v1}^2
[0.1,0.2]	1	[0.1,0.2]	1	0	0
[0.2,0.3]	3.5	[0.1,0.3]	2	1.5	2.25
[0.1,0.3]	2	[0.2,0.4]	4	-2	4
[0.1,0.4]	3.5	[0.2,0.3]	3	0.5	0.25

Table 3. Calculations of equation (13)

π_A	$R(\pi_A)$	π_B	$R(\pi_B)$	$d_{\pi 1}$	$d_{\pi 1}^2$
[0.1,0.3]	1	[0.4,0.6]	4	-3	9
[0.1,0.4]	2	[0.1,0.4]	1.5	0.5	0.25
[0.2,0.5]	3	[0.1,0.4]	1.5	1.5	2.25
[0.2,0.6]	4	[0.2,0.6]	3	1	1

Table 4. Evaluation results

Criteria		IVIFS	h_1				h_2				h_3				h_4			
			O_1	O_2	O_3	O_4	O_1	O_2	O_3	O_4	O_1	O_2	O_3	O_4	O_1	O_2	O_3	O_4
B_1	C_1	$u_A^L(x)$	0.40	0.45	0.40	0.35	0.40	0.50	0.40	0.25	0.35	0.55	0.40	0.30	0.35	0.45	0.50	0.35
		$u_A^R(x)$	0.60	0.60	0.55	0.45	0.55	0.65	0.50	0.40	0.55	0.65	0.60	0.45	0.65	0.60	0.60	0.55
		$v_A^L(x)$	0.15	0.15	0.20	0.10	0.20	0.25	0.30	0.35	0.15	0.10	0.10	0.25	0.10	0.15	0.15	0.25
		$v_A^R(x)$	0.25	0.20	0.35	0.20	0.30	0.30	0.40	0.45	0.30	0.25	0.20	0.35	0.25	0.20	0.20	0.35
	C_2	$u_A^L(x)$	0.40	0.60	0.30	0.55	0.35	0.50	0.45	0.35	0.60	0.70	0.40	0.35	0.45	0.55	0.35	0.50
		$u_A^R(x)$	0.65	0.70	0.40	0.65	0.55	0.65	0.60	0.45	0.75	0.85	0.55	0.50	0.55	0.65	0.50	0.60
		$v_A^L(x)$	0.15	0.10	0.35	0.15	0.15	0.20	0.15	0.30	0.10	0.05	0.25	0.15	0.10	0.15	0.25	0.20
		$v_A^R(x)$	0.20	0.25	0.50	0.20	0.25	0.35	0.25	0.45	0.20	0.10	0.35	0.30	0.25	0.30	0.35	0.35
	C_3	$u_A^L(x)$	0.55	0.50	0.65	0.55	0.45	0.55	0.60	0.55	0.45	0.50	0.65	0.45	0.50	0.55	0.65	0.45
		$u_A^R(x)$	0.65	0.65	0.75	0.70	0.55	0.75	0.70	0.65	0.55	0.70	0.80	0.50	0.60	0.65	0.75	0.60
		$v_A^L(x)$	0.10	0.10	0.05	0.05	0.10	0.05	0.15	0.15	0.25	0.15	0.05	0.10	0.10	0.15	0.05	0.15
		$v_A^R(x)$	0.25	0.20	0.15	0.20	0.25	0.10	0.20	0.25	0.30	0.30	0.15	0.20	0.25	0.20	0.15	0.20
	C_4	$u_A^L(x)$	0.35	0.50	0.45	0.25	0.30	0.55	0.50	0.20	0.45	0.65	0.55	0.35	0.50	0.55	0.55	0.45
		$u_A^R(x)$	0.55	0.65	0.60	0.45	0.45	0.70	0.65	0.35	0.60	0.75	0.70	0.55	0.65	0.65	0.60	0.50
		$v_A^L(x)$	0.20	0.10	0.15	0.25	0.25	0.15	0.15	0.35	0.10	0.05	0.10	0.25	0.15	0.10	0.05	0.20
		$v_A^R(x)$	0.35	0.20	0.30	0.35	0.40	0.20	0.25	0.45	0.20	0.20	0.15	0.30	0.25	0.15	0.25	0.30
B_2	C_5	$u_A^L(x)$	0.35	0.60	0.55	0.30	0.35	0.65	0.60	0.30	0.25	0.65	0.70	0.45	0.25	0.55	0.60	0.40
		$u_A^R(x)$	0.40	0.75	0.65	0.40	0.50	0.80	0.70	0.45	0.40	0.75	0.85	0.60	0.50	0.85	0.75	0.50
		$v_A^L(x)$	0.15	0.10	0.10	0.35	0.25	0.15	0.15	0.25	0.15	0.05	0.05	0.15	0.25	0.05	0.10	0.15
		$v_A^R(x)$	0.25	0.15	0.15	0.50	0.40	0.20	0.25	0.35	0.30	0.15	0.15	0.20	0.35	0.10	0.20	0.25
	C_6	$u_A^L(x)$	0.35	0.55	0.50	0.65	0.20	0.60	0.50	0.70	0.40	0.55	0.45	0.55	0.35	0.55	0.45	0.70
		$u_A^R(x)$	0.50	0.70	0.65	0.80	0.35	0.65	0.65	0.80	0.55	0.65	0.65	0.70	0.50	0.65	0.65	0.85
		$v_A^L(x)$	0.30	0.10	0.10	0.05	0.45	0.15	0.10	0.05	0.30	0.15	0.20	0.10	0.30	0.15	0.10	0.05
		$v_A^R(x)$	0.40	0.25	0.20	0.15	0.60	0.20	0.25	0.15	0.40	0.25	0.35	0.20	0.40	0.25	0.20	0.15

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Table 4. Continued

Criteria		IVIFS	h_1				h_2				h_3				h_4			
			O_1	O_2	O_3	O_4	O_1	O_2	O_3	O_4	O_1	O_2	O_3	O_4	O_1	O_2	O_3	O_4
	C_7	$u_A^L(x)$	0.65	0.55	0.45	0.35	0.60	0.55	0.45	0.30	0.55	0.50	0.45	0.30	0.65	0.55	0.45	0.30
		$u_A^R(x)$	0.75	0.75	0.55	0.45	0.75	0.75	0.60	0.50	0.70	0.65	0.60	0.40	0.85	0.70	0.60	0.40
		$v_A^L(x)$	0.10	0.15	0.25	0.25	0.05	0.15	0.20	0.35	0.15	0.20	0.15	0.25	0.05	0.10	0.15	0.30
		$v_A^R(x)$	0.20	0.25	0.30	0.40	0.15	0.20	0.30	0.45	0.25	0.35	0.25	0.45	0.15	0.15	0.25	0.40
	C_8	$u_A^L(x)$	0.50	0.45	0.40	0.35	0.40	0.45	0.40	0.25	0.40	0.45	0.50	0.10	0.40	0.55	0.40	0.35
		$u_A^R(x)$	0.60	0.55	0.65	0.50	0.50	0.60	0.55	0.35	0.55	0.65	0.65	0.25	0.50	0.70	0.55	0.50
		$v_A^L(x)$	0.10	0.25	0.20	0.15	0.15	0.15	0.20	0.30	0.15	0.05	0.10	0.45	0.10	0.15	0.20	0.20
		$v_A^R(x)$	0.20	0.35	0.30	0.25	0.25	0.20	0.35	0.40	0.30	0.20	0.20	0.60	0.30	0.20	0.35	0.35
	C_9	$u_A^L(x)$	0.65	0.55	0.45	0.25	0.45	0.65	0.55	0.35	0.55	0.45	0.35	0.25	0.40	0.60	0.45	0.35
		$u_A^R(x)$	0.70	0.65	0.60	0.45	0.75	0.75	0.60	0.45	0.65	0.70	0.55	0.40	0.65	0.75	0.65	0.40
		$v_A^L(x)$	0.15	0.20	0.15	0.15	0.15	0.10	0.15	0.20	0.10	0.15	0.20	0.25	0.15	0.10	0.15	0.35
		$v_A^R(x)$	0.20	0.35	0.25	0.35	0.20	0.20	0.25	0.35	0.25	0.20	0.35	0.35	0.25	0.20	0.20	0.45
	C_{10}	$u_A^L(x)$	0.45	0.55	0.40	0.30	0.50	0.65	0.45	0.20	0.55	0.60	0.45	0.15	0.60	0.65	0.35	0.30
		$u_A^R(x)$	0.70	0.65	0.55	0.45	0.65	0.75	0.60	0.35	0.75	0.70	0.65	0.25	0.70	0.75	0.55	0.45
		$v_A^L(x)$	0.10	0.15	0.10	0.25	0.15	0.10	0.15	0.35	0.15	0.10	0.20	0.35	0.15	0.05	0.20	0.15
		$v_A^R(x)$	0.20	0.25	0.30	0.35	0.25	0.15	0.20	0.50	0.20	0.15	0.30	0.55	0.20	0.25	0.25	0.30
B_3	C_{11}	$u_A^L(x)$	0.45	0.60	0.45	0.45	0.55	0.50	0.55	0.40	0.35	0.55	0.50	0.45	0.35	0.65	0.55	0.65
		$u_A^R(x)$	0.55	0.65	0.60	0.55	0.65	0.65	0.70	0.55	0.50	0.65	0.60	0.60	0.55	0.75	0.60	0.70
		$v_A^L(x)$	0.15	0.20	0.10	0.25	0.10	0.05	0.15	0.20	0.15	0.20	0.25	0.15	0.15	0.10	0.15	0.05
		$v_A^R(x)$	0.25	0.30	0.25	0.35	0.20	0.15	0.25	0.35	0.25	0.30	0.35	0.20	0.25	0.15	0.25	0.15
	C_{12}	$u_A^L(x)$	0.55	0.65	0.45	0.60	0.50	0.60	0.45	0.55	0.65	0.70	0.45	0.55	0.50	0.65	0.55	0.65
		$u_A^R(x)$	0.65	0.75	0.65	0.70	0.60	0.65	0.60	0.75	0.70	0.75	0.65	0.70	0.60	0.80	0.75	0.85
		$v_A^L(x)$	0.15	0.10	0.15	0.10	0.15	0.25	0.15	0.05	0.05	0.15	0.10	0.05	0.15	0.05	0.05	0.05
		$v_A^R(x)$	0.20	0.15	0.20	0.15	0.25	0.30	0.25	0.15	0.20	0.20	0.25	0.15	0.30	0.10	0.15	0.10
	C_{13}	$u_A^L(x)$	0.55	0.45	0.40	0.30	0.65	0.45	0.45	0.25	0.75	0.50	0.45	0.25	0.60	0.55	0.60	0.10
		$u_A^R(x)$	0.75	0.65	0.65	0.45	0.75	0.65	0.55	0.45	0.80	0.65	0.60	0.40	0.80	0.65	0.70	0.20
		$v_A^L(x)$	0.05	0.15	0.10	0.25	0.05	0.15	0.15	0.25	0.05	0.10	0.15	0.35	0.05	0.15	0.15	0.50
		$v_A^R(x)$	0.10	0.25	0.20	0.35	0.15	0.25	0.20	0.35	0.10	0.15	0.25	0.50	0.15	0.20	0.25	0.65

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Table 4. Continued

Criteria		IVIFS	h_1				h_2				h_3				h_4			
			O_1	O_2	O_3	O_4	O_1	O_2	O_3	O_4	O_1	O_2	O_3	O_4	O_1	O_2	O_3	O_4
	C_{14}	$u_A^L(x)$	0.65	0.50	0.45	0.20	0.70	0.55	0.45	0.30	0.65	0.55	0.35	0.35	0.75	0.60	0.45	0.25
		$u_A^R(x)$	0.75	0.65	0.60	0.35	0.85	0.70	0.60	0.45	0.80	0.65	0.45	0.50	0.90	0.75	0.65	0.40
		$v_A^L(x)$	0.05	0.25	0.15	0.45	0.05	0.15	0.15	0.35	0.05	0.15	0.25	0.30	0.05	0.10	0.15	0.45
		$v_A^R(x)$	0.10	0.35	0.25	0.55	0.10	0.25	0.20	0.45	0.10	0.25	0.35	0.45	0.10	0.20	0.25	0.55

Table 5. Collective decision matrix

	IVIFS	C_1	C_2	C_3	C_4	C_5	C_6	C_7
O_1	$u_A(x)$	[0.3755,0.5897]	[0.4588,0.6352]	[0.4892,0.5897]	[0.4052,0.5686]	[0.3018,0.4523]	[0.3289,0.4800]	[0.6147,0.7697]
	$v_A(x)$	[0.1456,0.2739]	[0.1225,0.2236]	[0.1257,0.2617]	[0.1655,0.2893]	[0.1936,0.3201]	[0.3320,0.4427]	[0.0783,0.1831]
O_2	$u_A(x)$	[0.4892,0.6258]	[0.5946,0.7275]	[0.5257,0.6904]	[0.5661,0.6904]	[0.6147,0.7919]	[0.5631,0.6632]	[0.5380,0.7154]
	$v_A(x)$	[0.1540,0.2340]	[0.1107,0.2264]	[0.1030,0.1861]	[0.0931,0.1861]	[0.0783,0.1456]	[0.1355,0.2364]	[0.1456,0.2264]
O_3	$u_A(x)$	[0.4267,0.5644]	[0.3775,0.5179]	[0.6381,0.7525]	[0.5142,0.6400]	[0.6166,0.7495]	[0.4756,0.6500]	[0.4500,0.5880]
	$v_A(x)$	[0.1732,0.2736]	[0.2393,0.3518]	[0.0658,0.1612]	[0.1030,0.2303]	[0.0931,0.1831]	[0.1189,0.2432]	[0.1831,0.2739]
O_4	$u_A(x)$	[0.3137,0.4654]	[0.4447,0.5570]	[0.5025,0.6193]	[0.3195,0.4674]	[0.3659,0.4931]	[0.6550,0.7940]	[0.3128,0.4391]
	$v_A(x)$	[0.2163,0.3240]	[0.1917,0.3118]	[0.1030,0.2115]	[0.2572,0.3450]	[0.2106,0.3058]	[0.0595,0.1612]	[0.2846,0.4243]
	IVIFS	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}
O_1	$u_A(x)$	[0.4267,0.5394]	[0.5225,0.6904]	[0.5283,0.7021]	[0.4313,0.5661]	[0.5545,0.6400]	[0.6457,0.7764]	[0.6904,0.8345]
	$v_A(x)$	[0.1225,0.2590]	[0.1355,0.2236]	[0.1355,0.2115]	[0.1355,0.2364]	[0.1140,0.2340]	[0.0500,0.1225]	[0.0500,0.1000]
O_2	$u_A(x)$	[0.4769,0.6292]	[0.5686,0.7154]	[0.6147,0.7154]	[0.5787,0.6782]	[0.6518,0.7428]	[0.4892,0.6500]	[0.5514,0.6904]
	$v_A(x)$	[0.1295,0.2300]	[0.1316,0.2300]	[0.0931,0.1936]	[0.1189,0.2121]	[0.1170,0.1732]	[0.1355,0.2081]	[0.1540,0.2572]
O_3	$u_A(x)$	[0.4267,0.6031]	[0.4546,0.6016]	[0.4139,0.5897]	[0.5142,0.6278]	[0.4769,0.6673]	[0.4809,0.6292]	[0.4265,0.5811]
	$v_A(x)$	[0.1682,0.2928]	[0.1612,0.2572]	[0.1565,0.2590]	[0.1540,0.2719]	[0.1030,0.2081]	[0.1355,0.2236]	[0.1704,0.2572]
O_4	$u_A(x)$	[0.2692,0.4091]	[0.3018,0.4255]	[0.2402,0.3803]	[0.4980,0.6052]	[0.5897,0.7590]	[0.2284,0.3827]	[0.2772,0.4277]
	$v_A(x)$	[0.2523,0.3807]	[0.2264,0.3727]	[0.2603,0.4122]	[0.1392,0.2462]	[0.0595,0.1355]	[0.3234,0.4467]	[0.3819,0.4975]

Table 6. Calculations of differences in the ranks with respect to the membership about O_1

u_{O_1}	$R\left(u_{O_1}\right)$	u_{O^*}	$R\left(u_{O^*}\right)$	d_{u1}	d_{u1}^2
[0.3755,0.5897]	3	[0.3137,0.4654]	4	-1	1
[0.4588,0.6352]	8	[0.3775,0.5179]	5	3	9
[0.4892,0.5897]	7	[0.4892,0.5897]	7	0	0
[0.4052,0.5686]	5	[0.5661,0.6904]	8	-3	9
[0.3018,0.4523]	1	[0.6147,0.7919]	12	-11	121
[0.3289,0.4800]	2	[0.6550,0.7940]	13	-11	121
[0.6147,0.7697]	12	[0.6147,0.7697]	11	1	1
[0.4267,0.5394]	4	[0.2692,0.4091]	3	1	1
[0.5225,0.6904]	10	[0.5686,0.7104]	9	1	1
[0.5283,0.7021]	11	[0.2404,0.3803]	2	9	81
[0.4313,0.5661]	6	[0.4313,0.5661]	6	0	0
[0.5545,0.6400]	9	[0.5897,0.7590]	10	-1	1
[0.6457,0.7764]	13	[0.2284,0.3827]	1	12	144
[0.6904,0.8345]	14	[0.6904,0.8345]	14	0	0

Table 7. Calculations of differences in the ranks concerning non-membership about O_1

v_{O_1}	$R\left(v_{O_1}\right)$	v_{O^*}	$R\left(v_{O^*}\right)$	d_{v1}	d_{v1}^2
[0.1456,0.2739]	11	[0.2163,0.3240]	10	1	1
[0.1225,0.2236]	4	[0.2393,0.3518]	11	-7	49
[0.1257,0.2617]	10	[0.1257,0.2617]	9	1	1
[0.1655,0.2893]	12	[0.0931,0.1861]	6	6	36
[0.1936,0.3201]	13	[0.0783,0.1456]	4	9	81
[0.3320,0.4427]	14	[0.0595,0.1621]	3	11	121
[0.0783,0.1831]	3	[0.0783,0.1831]	5	-2	4
[0.1225,0.2590]	9	[0.2523,0.3807]	12	-3	9
[0.1355,0.2236]	7	[0.1316,0.2300]	7	0	0
[0.1355,0.2115]	5	[0.2603,0.4122]	13	-8	64
[0.1355,0.2364]	8	[0.1355,0.2364]	8	0	0
[0.1140,0.2340]	6	[0.0595,0.1355]	2	4	16
[0.0500,0.1225]	2	[0.3234,0.4467]	14	-12	144
[0.0500,0.1000]	1	[0.0500,0.1000]	1	0	0

Table 8. Calculations of differences in the ranks with respect to hesitation about O_1

π_{O_1}	$R\left(\pi_{O_1}\right)$	π_{O^*}	$R\left(\pi_{O^*}\right)$	$d_{\pi 1}$	$d_{\pi 1}^2$
[0.1364,0.4789]	12	[0.2106,0.4700]	12	0	0
[0.1412,0.4187]	10	[0.1303,0.3832]	8	2	4
[0.1486,0.3851]	9	[0.1486,0.3851]	9	0	0
[0.1421,0.4293]	11	[0.1235,0.3408]	7	4	16
[0.2276,0.5046]	14	[0.0625,0.3070]	5	9	81
[0.0773,0.3391]	4	[0.0448,0.2855]	2	2	4
[0.0472,0.3070]	2	[0.0472,0.3070]	3	-1	1
[0.0339,0.4508]	8	[0.2102,0.4785]	13	-5	25
[0.0860,0.3420]	6	[0.0546,0.2998]	4	2	4
[0.0864,0.3362]	5	[0.2075,0.4995]	14	-9	81
[0.1975,0.4332]	13	[0.1975,0.4332]	11	2	4
[0.1260,0.3315]	7	[0.1055,0.3508]	6	1	1
[0.1011,0.3043]	3	[0.1706,0.4482]	10	-7	49
[0.0655,0.2596]	1	[0.0655,0.2596]	1	0	0

Table 9. Calculations of differences in the ranks concerning the membership about O_2

u_{O_2}	$R\left(u_{O_2}\right)$	u_{O^*}	$R\left(u_{O^*}\right)$	d_{u2}	d_{u2}^2
[0.4892,0.6258]	2	[0.3137,0.4654]	4	-2	4
[0.5946,0.7275]	11	[0.3775,0.5179]	5	6	36
[0.5257,0.6904]	4	[0.4892,0.5897]	7	-3	9
[0.5661,0.6904]	8	[0.5661,0.6904]	8	0	0
[0.6147,0.7919]	14	[0.6147,0.7919]	12	2	4
[0.5631,0.6632]	5	[0.6550,0.7940]	13	-8	64
[0.5380,0.7154]	7	[0.6147,0.7697]	11	-4	16
[0.4769,0.6292]	1	[0.2692,0.4091]	3	-2	4
[0.5686,0.7154]	10	[0.5686,0.7104]	9	1	1
[0.6147,0.7154]	12	[0.2404,0.3803]	2	10	100
[0.5787,0.6782]	9	[0.4313,0.5661]	6	3	9
[0.6518,0.7428]	13	[0.5897,0.7590]	10	3	9
[0.4892,0.6500]	3	[0.2284,0.3827]	1	2	4
[0.5514,0.6904]	6	[0.6904,0.8345]	14	-8	64

Table 10. Calculations of differences in the ranks concerning non-membership about O_2

v_{O_2}	$R\left(v_{O_2}\right)$	v_{O^*}	$R\left(v_{O^*}\right)$	d_{v2}	d_{v2}^2
[0.1540,0.2340]	13	[0.2163,0.3240]	10	3	9
[0.1107,0.2264]	7	[0.2393,0.3518]	11	-4	16
[0.1030,0.1861]	4	[0.1257,0.2617]	9	-5	25
[0.0931,0.1861]	2	[0.0931,0.1861]	6	-4	16
[0.0783,0.1456]	1	[0.0783,0.1456]	4	-3	9
[0.1355,0.2364]	11	[0.0595,0.1621]	3	8	64
[0.1456,0.2264]	12	[0.0783,0.1831]	5	7	49
[0.1295,0.2300]	9	[0.2523,0.3807]	12	-3	9
[0.1316,0.2300]	10	[0.1316,0.2300]	7	3	9
[0.0931,0.1936]	3	[0.2603,0.4122]	13	-10	100
[0.1189,0.2121]	6	[0.1355,0.2364]	8	-2	4
[0.1170,0.1732]	5	[0.0595,0.1355]	2	3	9
[0.1355,0.2081]	8	[0.3234,0.4467]	14	-6	36
[0.1540,0.2572]	14	[0.0500,0.1000]	1	13	169

Table 11. Calculations of differences in the ranks concerning hesitation about O_2

π_{O_2}	$R\left(\pi_{O_2}\right)$	π_{O^*}	$R\left(\pi_{O^*}\right)$	$d_{\pi 2}$	$d_{\pi 2}^2$
[0.1402,0.3568]	12	[0.2106,0.4700]	12	0	0
[0.0461,0.2947]	2	[0.1303,0.3832]	8	-6	36
[0.1235,0.3713]	11	[0.1486,0.3851]	9	2	4
[0.1235,0.3408]	10	[0.1235,0.3408]	7	3	9
[0.0625,0.3070]	5	[0.0625,0.3070]	5	0	0
[0.1004,0.3014]	8	[0.0448,0.2855]	2	6	36
[0.0582,0.3164]	6	[0.0472,0.3070]	3	3	9
[0.1408,0.3936]	14	[0.2102,0.4785]	13	1	1
[0.0546,0.2998]	4	[0.0546,0.2998]	4	0	0
[0.0910,0.2922]	7	[0.2075,0.4995]	14	-7	49
[0.1097,0.3024]	9	[0.1975,0.4332]	11	-2	4
[0.0840,0.2312]	1	[0.1055,0.3508]	6	-5	25
[0.1419,0.3753]	13	[0.1706,0.4482]	10	3	9
[0.0524,0.2946]	3	[0.0655,0.2596]	1	2	4

Table 12. Calculations of differences in the ranks concerning the membership about O_3

u_{O_3}	$R\left(u_{O_3}\right)$	u_{O^*}	$R\left(u_{O^*}\right)$	d_{u3}	d_{u3}^2
[0.4267,0.5644]	2	[0.3137,0.4654]	4	-2	4
[0.3775,0.5179]	1	[0.3775,0.5179]	5	-4	16
[0.6381,0.7525]	14	[0.4892,0.5897]	7	7	49
[0.5142,0.6400]	12	[0.5661,0.6904]	8	4	16
[0.6166,0.7495]	13	[0.6147,0.7919]	12	1	1
[0.4756,0.6500]	9	[0.6550,0.7940]	13	-4	16
[0.4500,0.5880]	6	[0.6147,0.7697]	11	-5	25
[0.4267,0.6031]	5	[0.2692,0.4091]	3	2	4
[0.4546,0.6016]	7	[0.5686,0.7104]	9	-2	4
[0.4139,0.5897]	3	[0.2404,0.3803]	2	1	1
[0.5142,0.6278]	10	[0.4313,0.5661]	6	4	16
[0.4769,0.6673]	11	[0.5897,0.7590]	10	1	1
[0.4809,0.6292]	8	[0.2284,0.3827]	1	7	49
[0.4265,0.5811]	4	[0.6904,0.8345]	14	-10	100

Table 13. Calculations of differences in the ranks concerning non-membership about O_3

v_{O_3}	$R\left(v_{O_3}\right)$	v_{O^*}	$R\left(v_{O^*}\right)$	d_{v3}	d_{v3}^2
[0.1732,0.2736]	11	[0.2163,0.3240]	10	1	1
[0.2393,0.3518]	14	[0.2393,0.3518]	11	3	9
[0.0658,0.1612]	1	[0.1257,0.2617]	9	-8	64
[0.1030,0.2303]	4	[0.0931,0.1861]	6	-2	4
[0.0931,0.1831]	2	[0.0783,0.1456]	4	-2	4
[0.1189,0.2432]	6	[0.0595,0.1621]	3	3	9
[0.1831,0.2739]	12	[0.0783,0.1831]	5	7	49
[0.1682,0.2928]	13	[0.2523,0.3807]	12	1	1
[0.1612,0.2572]	8	[0.1316,0.2300]	7	1	1
[0.1565,0.2590]	7	[0.2603,0.4122]	13	-6	36
[0.1540,0.2719]	9	[0.1355,0.2364]	8	1	1
[0.1030,0.2081]	3	[0.0595,0.1355]	2	1	1
[0.1355,0.2236]	5	[0.3234,0.4467]	14	-9	81
[0.1704,0.2572]	10	[0.0500,0.1000]	1	9	81

Table 14. Calculations of differences in the ranks concerning hesitation about O_3

π_{O_3}	$R\left(\pi_{O_3}\right)$	π_{O^*}	$R\left(\pi_{O^*}\right)$	$d_{\pi 3}$	$d_{\pi 3}^2$
[0.1620,0.4010]	12	[0.2106,0.4700]	12	0	0
[0.1030,0.3832]	4	[0.1303,0.3832]	8	-4	16
[0.0863,0.2961]	2	[0.1486,0.3851]	9	-7	49
[0.1297,0.3828]	8	[0.1235,0.3408]	7	1	1
[0.0674,0.2903]	1	[0.0625,0.3070]	5	-4	16
[0.1068,0.4055]	7	[0.0448,0.2855]	2	5	25
[0.1381,0.3669]	5	[0.0472,0.3070]	3	2	4
[0.1041,0.4051]	6	[0.2102,0.4785]	13	-7	49
[0.1412,0.3842]	9	[0.0546,0.2998]	4	5	25
[0.1513,0.4296]	14	[0.2075,0.4995]	14	0	0
[0.1003,0.3318]	3	[0.1975,0.4332]	11	-8	64
[0.1246,0.4201]	11	[0.1055,0.3508]	6	5	25
[0.1472,0.3836]	10	[0.1706,0.4482]	10	0	0
[0.1617,0.4031]	13	[0.0655,0.2596]	1	12	144

Table 15. Calculations of differences in the ranks concerning membership about O_4

u_{O_4}	$R\left(u_{O_4}\right)$	u_{O^*}	$R\left(u_{O^*}\right)$	d_{u4}	d_{u4}^2
[0.3137,0.4654]	7	[0.3137,0.4654]	4	3	9
[0.4447,0.5570]	10	[0.3775,0.5179]	5	5	25
[0.5025,0.6193]	12	[0.4892,0.5897]	7	5	25
[0.3195,0.4674]	8	[0.5661,0.6904]	8	0	0
[0.3659,0.4931]	9	[0.6147,0.7919]	12	-3	9
[0.6550,0.7940]	14	[0.6550,0.7940]	13	1	1
[0.3128,0.4391]	6	[0.6147,0.7697]	11	-5	25
[0.2692,0.4091]	3	[0.2692,0.4091]	3	0	0
[0.3018,0.4255]	5	[0.5686,0.7104]	9	-4	16
[0.2402,0.3803]	2	[0.2404,0.3803]	2	0	0
[0.4980,0.6052]	11	[0.4313,0.5661]	6	5	25
[0.5897,0.7590]	13	[0.5897,0.7590]	10	3	9
[0.2284,0.3827]	1	[0.2284,0.3827]	1	0	0
[0.2772,0.4277]	4	[0.6904,0.8345]	14	-10	100

Table 16. Calculations of differences in the ranks concerning non-membership about O_4

v_{O_4}	$R\left(v_{O_4}\right)$	v_{O^*}	$R\left(v_{O^*}\right)$	d_{v4}	d_{v4}^2
[0.2163,0.3240]	7	[0.2163,0.3240]	10	-3	9
[0.1917,0.3118]	5	[0.2393,0.3518]	11	-6	36
[0.1030,0.2115]	3	[0.1257,0.2617]	9	-6	36
[0.2572,0.3450]	9	[0.0931,0.1861]	6	3	9
[0.2106,0.3058]	6	[0.0783,0.1456]	4	2	4
[0.0595,0.1612]	2	[0.0595,0.1621]	3	-1	1
[0.2846,0.4243]	12	[0.0783,0.1831]	5	7	49
[0.2523,0.3807]	10	[0.2523,0.3807]	12	-2	4
[0.2264,0.3727]	8	[0.1316,0.2300]	7	1	1
[0.2603,0.4122]	11	[0.2603,0.4122]	13	-2	4
[0.1392,0.2462]	4	[0.1355,0.2364]	8	-4	16
[0.0595,0.1355]	1	[0.0595,0.1355]	2	-1	1
[0.3234,0.4467]	13	[0.3234,0.4467]	14	-1	1
[0.3819,0.4975]	14	[0.0500,0.1000]	1	13	169

Table 17. Calculations of differences in the ranks concerning hesitation about O_4

π_{O_4}	$R\left(\pi_{O_4}\right)$	π_{O^*}	$R\left(\pi_{O^*}\right)$	$d_{\pi 4}$	$d_{\pi 4}^2$
[0.2106,0.4700]	13	[0.2106,0.4700]	12	1	1
[0.1312,0.3636]	4	[0.1303,0.3832]	8	-4	16
[0.1692,0.3945]	7	[0.1486,0.3851]	9	-2	4
[0.1876,0.4233]	8	[0.1235,0.3408]	7	1	1
[0.2011,0.4235]	10	[0.0625,0.3070]	5	5	25
[0.0448,0.2855]	1	[0.0448,0.2855]	2	-1	1
[0.1366,0.4026]	6	[0.0472,0.3070]	3	3	9
[0.2012,0.4785]	12	[0.2102,0.4785]	13	-1	1
[0.2018,0.4718]	11	[0.0546,0.2998]	4	7	49
[0.2075,0.4995]	14	[0.2075,0.4995]	14	0	0
[0.1486,0.3628]	5	[0.1975,0.4332]	11	-6	36
[0.1055,0.3508]	3	[0.1055,0.3508]	6	-3	9
[0.1706,0.4482]	9	[0.1706,0.4482]	10	-1	1
[0.0748,0.3409]	2	[0.0655,0.2596]	1	1	1

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