Preface

Providing relevant and current research, this book and its author's individual publications would be useful for academics, researchers, scholars, and practitioners interested in improving decision making models and business functions.

Probabilistic modeling represents a subject arising in many branches of mathematics, economics and computer science. Such modeling connects pure mathematics with applied sciences. Operations research similarly is situated on the border between pure mathematics and applied sciences. So when probabilistic modeling meets operations research, it is very interesting occasion. Our life and work are impossible without planning, time-tabling, scheduling, decision making, optimization, simulation, data analysis, risk analysis and process modeling. Thus, it is a part of management science or decision science.

This book looks to discuss and address the difficulties and challenges that occur during the process of planning or decision making. The editors have found the chapters that address different aspects of probabilistic modeling, stochastic methods, probabilistic distributions, data analysis, optimization methods, probabilistic methods in risk analysis, and related topics. Additionally, the book explores the impact of such probabilistic modeling with other approaches.

This comprehensive and timely publication aims to be an essential reference source, building on the available literature in the field of probabilistic modeling, operational research, planning and scheduling, data extrapolation in decision making, probabilistic interpolation and extrapolation in simulation, stochastic processes, and decision analysis. It is hoped that this text will provide the resources necessary for economics and management sciences, also for mathematics and computer sciences.

Decision makers, academicians, researchers, advanced-level students, technology developers, and government officials will find this text useful in furthering their research exposure to pertinent topics in operations research and assisting in furthering their own research efforts in this field. Proposed method, called Probabilistic Features Combination (PFC), is the method of 2D curve interpolation and extrapolation using the set of key points (knots or nodes). Nodes can be treated as characteristic points of data for modeling and analyzing. The model of data can be built by choice of probability distribution function and nodes combination. PFC modeling via nodes combination and parameter γ as probability distribution function enables value anticipation in risk analysis and decision making. Two-dimensional curve is extrapolated and interpolated via nodes combination and different functions as discrete or continuous probability distribution functions: polynomial, sine, cosine, tangent, cotangent, logarithm, exponent, arc sin, arc cos, arc tan, arc cot or power function.

Book topics include the following:

- Probabilistic Modeling,
- Operations Research,
- Stochastic Methods,
- Probabilistic Methods in Planning,
- Decision Making,
- Data Analysis,
- Optimization Methods,
- Probabilistic Methods in Risk Analysis,
- Probabilistic Interpolation and Extrapolation,
- Process Modeling,
- Data Simulation,
- Decision Analysis,
- Stochastic Processes,
- Probabilistic Optimization,
- Data Mining,
- Mathematical Modeling,
- Probabilistic Models in Scheduling,
- Time-Tabling,
- Data Extrapolation in Planning and Decision Making.

The chapters are as follows:

1. Introduction: MHR Method,
2. Probabilistic Nodes Combination (PNC): Formulas and Examples,
3. PNC in 2D Curve Modeling: Interpolation and Extrapolation,
4. Contour Reconstruction: 2D Object Modeling,
5. PNC in 3D Surface Modeling,
6. PNC in 4D Object and Multi-Dimensional Data Modeling,
7. Applications of PNC in Numerical Methods,
8. Applications of PNC in Artificial Intelligence.

Risk analysis needs suitable methods of data extrapolation and decision making. Proposed method of Hurwitz-Radon Matrices (MHR) can be used in extrapolation and interpolation of curves in the plane. For example quotations from the Stock Exchange, the market prices or rate of a currency form a curve. This chapter contains the way of data anticipation and extrapolation via MHR method and decision making: to buy or not, to sell or not. Proposed method is based on a family of Hurwitz-Radon (HR) matrices. The matrices are skew-symmetric and possess columns composed of orthogonal vectors. The operator of Hurwitz-Radon (OHR), built from these matrices, is described. Two-dimensional data are represented by the set of curve points. It is shown how to create the orthogonal and discrete OHR and how to use it in a process of data foreseeing and extrapolation. MHR method is interpolating and extrapolating the curve point by point without using any formula or function.

Proposed method, called Probabilistic Nodes Combination (PNC), is the method of 2D curve interpolation and extrapolation using the set of key points (knots or nodes). Nodes can be treated as characteristic points of data for modeling and analyzing. The model of data can be built by choice of probability distribution function and nodes combination. PNC modeling via nodes combination and parameter γ as probability distribution function enables value anticipation in risk analysis and decision making. Two-dimensional curve is extrapolated and interpolated via nodes combination and different functions as discrete or continuous probability distribution functions: polynomial, sine, cosine, tangent, cotangent, logarithm, exponent, arc sin, arc cos, arc tan, arc cot or power function. Novelty of this book consists of two generalizations: generalization of previous MHR method with various nodes combinations and generalization of linear interpolation with different (no basic) probability distribution functions and nodes combinations.

Computer vision needs suitable methods of shape representation and contour reconstruction. One of them, invented by the author and called method of Hurwitz-Radon Matrices (MHR), can be used in representation and reconstruction of shapes of the objects in the plane. Proposed method is based on a family of Hurwitz-Radon (HR) matrices. The matrices are skew-symmetric and possess columns composed of orthogonal vectors. 2D shape is represented by the set of successive nodes. It is shown how to create the orthogonal and discrete OHR operator and how to use it in a process of shape representation and reconstruction. Then MHR method is generalized to Probabilistic Nodes Combination (PNC) method.

This work clarifies the significance and novelty of the proposed method compared to existing methods (for example polynomial interpolations and Bézier curves). Previous published papers of the author were dealing with the method of Hurwitz-Radon Matrices (MHR method). Novelty of this monograph and proposed method consists in the fact that calculations are free from the family of Hurwitz-Radon Matrices. Problem statement of this monograph is: how to reconstruct (interpolate) missing points of 2D curve having the set of interpolation nodes (key points) and using the information about probabilistic distribution of unknown points. For example the simplest basic distribution leads to the easiest interpolation – linear interpolation. Apart from probability distribution, additionally there is the second factor of proposed interpolation method: nodes combination. The simplest nodes combination is zero. Thus proposed curve modeling is based on two agents: probability distribution and nodes combination. Significance of this book consists in generalization for MHR method: the computations are done without matrices in curve fitting and shape modeling, with clear point interpolation formula based on probability distribution function (continuous or discrete) and nodes combination. This book also consists of generalization for linear interpolation with different (no basic) probability distribution functions and nodes combinations. So this book answers the question: "Why and when should we use PNC method?".

Curve interpolation represents one of the most important problems in mathematics and computer science: how to model the curve via discrete set of two-dimensional points? Also the matter of shape representation (as closed curve - contour) and curve parameterization is still opened. For example pattern recognition, signature verification or handwriting identification problems are based on curve modeling via the choice of key points. So interpolation is not only a pure mathematical problem but important task in computer vision and artificial intelligence. The monograph wants to approach a problem of curve modeling by characteristic points. Proposed method relies on nodes combination and functional modeling of curve points situated between the basic set of key points. The functions that are used in calculations represent whole family of elementary functions with inverse functions: polynomials, trigonometric, cyclometric, logarithmic, exponential and power function. These functions are treated as probability distribution functions in the range [0;1].

Significant problem in machine vision and computer vision is that of appropriate 2D shape representation and reconstruction. Classical discussion about shape representation is based on the problem: contour versus skeleton. This monograph is voting for contour which forms boundary of the object. Contour of the object, represented by successive contour points, consists of information which allows us to describe many important features of the object as shape coefficients. 2D curve modeling and generation is a basic subject in many branches of industry and computer science, for example in the cad/cam software. The representation of shape can have a great impact on the accuracy and effectiveness of object recognition. In the literature, shape has been represented by many options including curves, graph-based algorithms and medial axis to enable shape-based object recognition. Digital 2D curve (open or closed) can be represented by chain code (Freeman's code). Chain code depends on selection of the started point and transformations of the object. So Freeman's code is one of the method how to describe and to find contour of the object. Analog (continuous) version of Freeman's code is the curve α - s. Another contour representation and reconstruction is based on Fourier coefficients calculated in discrete Fourier transformation (DFT). These coefficients are used to fix similarity of the contours with different sizes or directions. If we assume that contour is built from segments of a line and fragments of circles or ellipses, hough transformation is applied to detect contour lines. Also geometrical moments of the object are used during the process of object shape representation. Contour is also applied in shape decomposition. Many branches of medicine, industry and manufacturing are looking for methods connected with geometry of the contour. Why and when should we use MHR and PNC methods? Interpolation methods and curve fitting represent so huge problem that each individual interpolation is exceptional and requires specific solutions. PNC method is such a novel tool with its all pros and cons. The user has to decide which interpolation method is the best in a single situation. The choice is yours if you have any choice. Presented method is such a new possibility for curve fitting and interpolation when specific data (for example handwritten symbol or character) starts up with no rules for polynomial interpolation. This book consists of two generalizations: generalization of previous MHR method with various nodes combinations and generalization of linear interpolation with different (no basic) probability distribution functions and nodes combinations.

The method of Probabilistic Nodes Combination (PNC) enables interpolation and modeling of two-dimensional curves using nodes combinations and different coefficients γ: polynomial, sinusoidal, cosinusoidal, tangent, cotangent, logarithmic, exponential, arc sin, arc cos, arc tan, arc cot or power function, also inverse functions. This probabilistic view is novel approach a problem of modeling and interpolation. Computer vision and pattern recognition are interested in appropriate methods of shape representation and curve modeling. PNC method represents the possibilities of shape reconstruction and curve interpolation via the choice of nodes combination and probability distribution function for interpolated points. It seems to be quite new look at the problem of contour representation and curve modeling in artificial intelligence and computer vision.

Function for γ calculations is chosen individually at each curve modeling and it is treated as probability distribution function: γ depends on initial requirements and curve specifications. PNC method leads to curve interpolation as handwriting modeling via discrete set of fixed knots. So PNC makes possible the combination of two important problems: interpolation and modeling. Main features of PNC method are:

1. The smaller distance between knots the better;
2. Calculations for coordinates close to zero and near by extremum require more attention because of importance of these points;
3. PNC interpolation develops a linear interpolation into other functions as probability distribution functions;
4. PNC is a generalization of MHR method via different nodes combinations;
5. Interpolation of L points is connected with the computational cost of rank $O(L)$ as in MHR method;
6. Nodes combination and coefficient γ are crucial in the process of curve probabilistic parameterization and interpolation: they are computed individually for a single curve.

Why and when should we use MHR and PNC methods? Interpolation methods and curve fitting represent so huge problem that each individual interpolation is exceptional and requires specific solutions. PNC method is such a novel tool with its all pros and cons. The user has to decide which interpolation method is the best in a single situation. The choice is yours if you have any choice. Presented method is such a new possibility for curve fitting and interpolation when specific data (for example handwritten symbol or character) starts up with no rules for polynomial interpolation.

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The method of Probabilistic Features Combination (PFC) enables interpolation and modeling of high-dimensional data using features' combinations and different coefficients γ as modeling function. Functions for γ calculations are chosen individually at each data modeling and it is treated as N-dimensional probability distribution function: γ depends on initial requirements and features' specifications. PFC method leads to data interpolation as handwriting or signature identification and image retrieval via discrete set of feature vectors in N-dimensional feature space. So PFC method makes possible the combination of two important problems: interpolation and modeling in a matter of image retrieval or writer identification. PFC interpolation develops a linear interpolation in multidimensional feature spaces into other functions as N-dimensional probability distribution functions. Future works are going to applications of PFC method in biometric recognition, computer vision and artificial intelligence.

Nodes are treated as characteristic points of data for modeling and analyzing. The model of data can be built by choice of probability distribution function and nodes combination. PFC modeling via nodes combination and parameter γ as probability distribution function enables value anticipation in risk analysis and decision making. Two-dimensional object is extrapolated and interpolated via nodes combination and different functions as discrete or continuous probability distribution functions: polynomial, sine, cosine, tangent, cotangent, logarithm, exponent, arc sin, arc cos, arc tan, arc cot or power function.

Functions for γ calculations are chosen individually at each data modeling and it is treated as 2-dimensional probability distribution function: γ depends on initial requirements and features' specifications. PFC method leads to data interpolation as handwriting or signature identification and image retrieval via discrete set of feature vectors in 2-dimensional feature space. So PFC method makes possible the combination of two important problems: interpolation and modeling in a matter of image retrieval or writer identification. PFC interpolation develops a linear interpolation in multidimensional feature spaces into other functions as two-dimensional probability distribution functions. Future works are going to applications of PFC method in biometric recognition, computer vision and artificial intelligence.

The editor, the publisher, and the authors hope that this book, *Probabilistic Nodes Combination (PNC) for Object Modeling and Contour Reconstruction*, will be a heavy brick in the construction of the House of Science. Please read it!

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