Vector-Based Realization of Area-Weight Proportional Multiplicatively Weighted Voronoi Diagrams With the ArcGIS Engine

Song Tian, South China Sea Institute of Planning and Environmental Research, SOA, China

Zheng Wei, South China Sea Institute of Planning and Environmental Research, SOA, China*

ABSTRACT

In the geographical field, the studies have been on service areas using the Voronoi diagram and its derived models is extensive, but there is a lack of effective methods to achieve a good area-weight proportionality between generators and their exclusive regions. As a famous visualizing method, adaptive multiplicatively weighted Voronoi diagrams are able to achieve it, but are limited to displaying non-spatial data. The approach of the area-weight proportional multiplicatively weighted Voronoi diagram is proposed to solve these problems by allowing for spatial division with a point-fixed iteration approach and a vector-based multiplicatively weighted Voronoi diagram construction method from point features with spatial coordinates and references in GIS environments. It enables one to create a set of regions that is proportional to the weights of the generators. The method is successfully tested on a series of cases. The approach aims to establish a kind of spatial data model to represent demand and supply situations in real life.

KEYWORDS

Area-Weight Proportional Multiplicatively Weighted Voronoi Diagram, Demand and Supply, Multiplicatively Weighted Voronoi Diagram, Spatial Data Model, Point-Fixed Iteration, Service Area

INTRODUCTION

Spatial partitioning is an important method in the geographic field. It uses a set of specific constraints or criteria to divide a finite geographic area into a lot of non-overlapping subareas (Wang et al., 2014). A delimiting service area is one of the most interesting topics in spatial partitioning, as nearly everyone needs a variety of services provided by different facilities in their daily life (Wang et al., 2018). In most cases, people choose their interested facilities by considering some factors, such as the geographic distribution, the service area, and the transportation convenience of facilities. The methodology of service area delimitation has been applied to various fields, such as the delineation of school catchment areas (Caro, 2004), market areas (Ríos-Mercado & Fernández, 2009), residential care facilities (Cheng et al., 2012), healthcare facilities (Steiner et al., 2015), and political districts (Ricca et al., 2013).

*Corresponding Author

A variety of models have been proposed and developed to delineate service areas, in which the Voronoi diagram is a particularly famous one. The Voronoi diagram, named after Georgy Voronoi (Voronoi, 1908), is a method to partition space into several subareas (called Voronoi regions or Voronoi cells) from a predetermined set of points (called sites, seeds, or generators) by comparing their Euclidean distances, in which generators are abstracted as cities, hospitals, and schools, and the Voronoi regions represent their service areas. It has various names in different fields, such as Thiessen polygons in geography (Thiessen, 1911) and Dirichlet tessellation in mathematics (Dirichlet, 1850). In order to solve more real-world problems, many derived Voronoi diagrams have been developed, such as the weighted Voronoi diagram (Aurenhammer & Edelsbrunner, 1984; Mu, 2010; Gong et al., 2012), the centroidal Voronoi diagram (Du et al., 1999), the Voronoi treemap (Tian et al., 2015; Tian, 2021), and the city Voronoi diagram (Görke & Wolff, 2005).

A weighted Voronoi diagram can be divided into four major categories. They are the additively weighted Voronoi diagram, the multiplicatively weighted Voronoi diagram (MWVD), the additively weighted power Voronoi diagram, and the compoundly weighted Voronoi diagram (Okabe et al., 2000). The MWVD has a wider range of applications in service area delimitation than the other three weighted Voronoi diagrams for many reasons. First, each generator is located inside its own multiplicatively weighted Voronoi region. Second, all of the available space must be divided up by a set of generators, that is, there is no unallocated space in an WWVD. Third, it supports the notion of area-weight proportionality, that is, a generator with a larger weight dominates a larger multiplicatively weighted Voronoi region. Fourth, it puts distance and weight in a ratio relationship leading to that the changing of data units will not alter the result of the MWVD construction. Fifth, and the most important, several spatial interaction models can be expressed by the MWVDs, such as the Reilly model (Reilly, 1929), Converse model (Converse, 1949), and Huff model (Huff, 1964), which make it more attractive for service area delimitation. Mu (2004) classified the applications of the MWVD into four periods: (1) Early prototypes from the1800s to the 1940s. (2) Application in market and urban analysis from 1950s to 1970s. In this period, the MWVD was used as a mathematical and geometric solution in market and urban analysis (Huff & Jenks, 1967; Boots, 1975). (3) Parallel development in computer geometry and GIS from the 1980s to the 1990s. During this time, the MWVD began to be integrated in the GIS environment (Vincent & Daly, 1990). (4) From algorithm to implementation (1990s and beyond). In this period, the MWVD was widely used as a model to solve problems, such as Zhang et al (2018) introduced, to analyze ecosystem services coverage, and Mu and Wang (2006) utilized it to study spatial patterns of urban hierarchy in the United States.

However, the area-weight proportionality for an MWVD is only approximate, as the weights in an MWVD scale distance rather than area (Reitsma et al., 2007). As a result, regions with low weights tend to be squeezed by those with large weights. Then, some questions surfaced: how to achieve a good area-weight proportionality between generators and their exclusive regions based on the MWVD and whether this special type of the MWVD can be used to better delineate service area.

To solve the above problem, Related Works are introduced in the following section. The Definitions for the Area-Weight Proportional Multiplicatively Weighted Voronoi Diagram section introduces the definitions for the area-weight proportional multiplicatively weighted Voronoi diagram (or the APMWVD) and the relevant properties. The APMWVD Construction Algorithm section elaborates on the implementation of the APMWVD construction algorithm. The section Examples shows several examples to verify the feasibility of the algorithm. Results and Discussions and Conclusions are the final two sections.

RELATED WORKS

Two studies deserve to be mentioned. The one is the adaptive additively weighted Voronoi diagram proposed by Moreno-Regidora et al. (2012) on a discrete version. In this model, the generators are the centroids of zones, and their weights are computed iteratively until each zone reaches the expected

size based on a distance function of the shortest path. The other is the adaptive multiplicatively weighted Voronoi diagram (AMWVD) proposed by Reitsma et al. (2007) in a raster-based way. It is an information space partitioning model, which combines a fixed-point iteration method with an optional spatial resolution refinement method using quadtree decomposition. In this method, the weights of generators are updated in each iteration based on the weights and the error of the preceding iteration, and the multiplicatively weighted Voronoi regions are repeatedly constructed to achieve a proper area-weight proportionality. This approach was tested by the authors on many cases to prove that it could be used to visualize hierarchical data (Sethia et al., 2004; Reitsma & Trubin, 2006; Reitsma et al., 2007).

From the introduction we know that the MWVD has several advantages in service area delimitation; meanwhile, although the AMWVD proposed in the research of Reitsma et al. (2007) is a non-geographic approach, it still provided an inspiration for us because of its area-weight proportional partitioning property. Hence, we try to introduce this information visualization algorithm into the field of geography to generate an area-weight proportional layout with spatial information to better visualize and understand the service areas of spatial objects. It may assist us in understanding the balance of supply and demand of facilities in real life and provide us with some auxiliary decisions in several aspects, such as location and optimization of facilities and facility layout policy adjustment.

In order to expand applications of the weighted Voronoi diagrams in the geographic field, more research made effort to bring various types of weighted Voronoi diagram nowadays into GIS. Mu (2004) provided a precise computation of the MWVD. The result can be saved in shapefile formats and used in GIS environment. Dong (2008) tried to create MWVDs for points, lines, and areas in a raster-based approach based on ArcObjects. But the algorithm efficiency is low; creating a MWVD for 200 points requires about 15 minutes. Tian (2015, August 2-5; 2021) proposed a universal vector-based algorithm based on ArcGIS engine (AE) to generate the MWVDs and the geographical Voronoi treemaps for points through the methods of regions division and regions merging with high precision and practicability.

Mixed with the theoretical research on the AMWVD of Reitsma et al. (2007) and method study on the weighted Voronoi diagrams of Tian (2015, August 2-5; 2021), the approach of the area-weight proportional multiplicatively weighted Voronoi diagram is presented.

Definitions for the Area-Weight Proportional Multiplicatively Weighted Voronoi Diagram

Let $q(x,y)$ be a point and $P=\{p_1,p_2,...,p_n\}$ be a set of n disjoint generators (points) in the plane E^2 and let (x_i,y_i) be the coordinate of the generator p_i. Each generator p_i is assigned a positive weight $w(p_i)$. The following is a multiplicatively weighted distance function between q and p_i:

$$d_{mw}(q,p_i) = \frac{\sqrt{(x-x_i)^2 + (y-y_i)^2}}{w(p_i)}, w(p_i) > 0 \qquad (1)$$

Then the multiplicatively weighted dominance of the generator p_i over a generator p_j, called $Dom(p_i, p_j)$, can be represented by:

$$Dom(p_i, p_j) = \{q \in E^2 \mid d_{mw}(q,p_i) \leq d_{mw}(q,p_j), p_i \in P, p_j \in P - \{p_i\}\} \qquad (2)$$

To construct the multiplicatively weighted Voronoi region of a generator p_i, we should create the multiplicatively weighted dominances of p_i over every generator other than p_i in P. Then the

intersection of these dominances should be computed. Aurenhammer and Edelsbrunner (1984) presented a formulation to define the multiplicatively weighted Voronoi region for a generator p_i as:

$$V(p_i) = \bigcap_{p_i \in P, p_j \in P - \{p_i\}} Dom(p_i, p_j) \tag{3}$$

Consequently, the multiplicatively weighted Voronoi diagram of P in E^2, denoted as the MWVD, is defined as:

$$V(P) = \bigcup_{p_i \in P} V(p_i) \tag{4}$$

For an MWVD $V(P)=\{V(p_1),V(p_2),\ldots,V(p_n)\}$, let $s(V(p_i))$ be the area of the multiplicatively weighted Voronoi region $V(p_i)$ in $V(P)$ and $a(p_i)$ be an desired dominance area of the generator p_i, if each region $V(p_i)$ in $V(P)$ satisfies as:

$$\frac{s(V(p_i))}{a(p_i)} = \frac{s(V(p_j))}{a(p_j)}, V(p_i) \in V(P), V(p_j) \in V(P) - \{V(p_i)\} \tag{5}$$

then the MWVD $V(P)$ is called the area-weight proportional multiplicatively weighted Voronoi diagram.

It can be known from the above definition that an APMWVD is an MWVD in fact in which the areas of the multiplicatively weighted Voronoi regions satisfy a certain proportional relationship with the weights of their corresponding generators. For an APMWVD $V(P)=\{V(p_1),V(p_2),\ldots,V(p_n)\}$ and its corresponding generators $P=\{p_1,p_2,...,p_n\}$ with weights $\{a(p_1),a(p_2),\ldots,a(p_n)\}$ in finite region R, it has the following characteristics:

a) The location of each generator p_i cannot change:

$$\Delta p_i = 0 \tag{6}$$

b) Each generator p_i is located inside its multiplicatively weighted Voronoi region $V(p_i)$:

$$p_i \cap V(p_i) \neq 0 \tag{7}$$

c) Total area of the multiplicatively weighted Voronoi regions equals the area of R, called $s(R)$:

$$\sum_{i=1}^{n} s(V(p_i)) = s(V(P)) = s(R) \tag{8}$$

d) For each generator p_i, the ratio of its weight $a(p_i)$ to its multiplicatively weighted Voronoi region area $s(V(p_i))$ is a fixed value:

$$\frac{s(V(p_i))}{a(p_i)} = k \qquad (9)$$

e) The area of a multiplicatively weighted Voronoi region $V(p_i)$ satisfies a certain proportion with the area of region R as follows:

$$s(V(p_i)) = \frac{a(p_i)}{\sum_{j=1}^{n} a(p_j)} s(R) \qquad (10)$$

We define a finite range R because a borderless APMWVD is not solvable as a region with an infinite area, it makes area calculations and proportions impossible (Trubin, 2006).

It can be known that an APMWVD is derived from an MWVD. The weights scale distance in an MWVD, and the scale area in an APMWVD is shown in Figure 1. As regions with large weights in an MWVD tend to be oversized whereas those with low weights are undersized (Reitsma et al., 2007), it is difficult for an MWVD to achieve a good area-weight proportionality, that is, it is hard to construct a real APMWVD from an MWVD that satisfies Equations (5), (9), and (10). Nevertheless, an approximate APMWVD can be generated by using an adaptive version of an MWVD. In the following section, an adaptive construction method of an APMWVD from an MWVD is introduced.

**Figure 1. An MWVD and an APMWVD of three same generators with weights 1, 3, and 7**

(a)   (b)
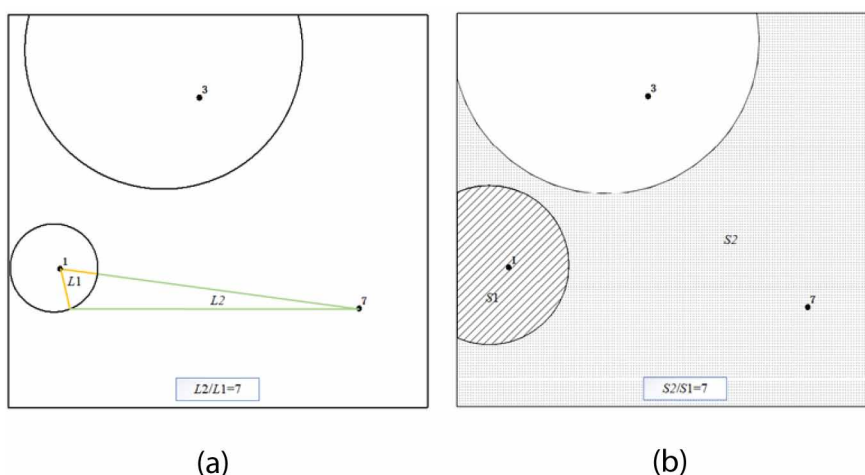
Note. (a) The weights of generators mean "distance" in an MWVD; (b) the weights of generators mean "area" in an APMWVD.

THE APMWVD CONSTRUCTION ALGORITHM

Let $P=\{p_1,p_2,...,p_n\}$ be a set of n ($n>1$) disjoint generators in the finite range R and $a(p_i)$ be the final desired dominance area of p_i. Let $w_k(p_i)$ be the weight of p_i, $V_k(p_i)$ be the multiplicatively weighted Voronoi region of p_i, $s(V_k(p_i))$ be the area of $V_k(p_i)$ and $V_k(P)$ be the MWVD at k-th iteration. The basic thought of the APMWVD construction method is as follows: calculate the difference between $a(p_i)$ and $s(V_k(p_i))$ at k-th iteration and adapt the weight $w_k(p_i)$ in accordance with the area difference. Generate a new MWVD $V_{k+1}(P)$ based on new weights $w_k(P)=\{w_k(p_1), w_k(p_2),...,w_k(p_n)\}$ and judge if $V_{k+1}(p_i)$ and $a(p_i)$ are satisfying Equation 5. If it satisfies, the process ends, otherwise the above process is repeated.

Figure 2. The process of an APMWVD construction
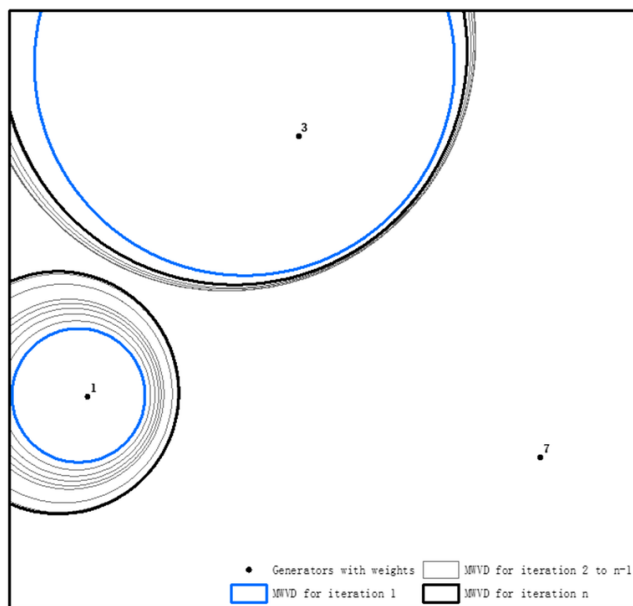
To clarify the process of the APMWVD construction clearly, the steps and pseudocode are given as follows:

Step a. Normalize $a(p_i)$ and get $a'(p_i)$.
Step b. Let $w_k(p_i)= a'(p_i)$, in which $k=1$.
Step c. Create the MWVD $V_k(P)$.
Step d. Calculate the normalized area of $V_k(p_i)$ and get $s'(V_k(p_i))$.
Step e. Calculate the difference between $a'(p_i)$ and $s'(V_k(p_i))$. If the area difference or the iteration is less than a default threshold, then the algorithm ends, otherwise let $k=k+1$ and adapt $w_k(p_i)$ based on the area difference to create $w_{k+1}(p_i)$.
Step d. Repeat step c to step e.

Algorithm 1. APMWVD construction

```
Input: P={p₁,p₂,…,pₙ}, a set of n generators with the property that
each pᵢ∈P corresponds to a weight value a(pᵢ);R: the range of P; a
positive, nonzero scaling constant β; thresh_a, thresh_b:
thresholds to deceive whether to end the algorithm
Output: an APMWVD V(P)
Begin
k=1
err=∞
a'(P)=Normalization(a(P)) %Data normalization for a(P)
initialize wₖ(P): a set of n weights at k-th iteration
for i:=1; step 1 until n do
    wₖ(pᵢ)=a'(pᵢ)
endfor
while(err<thresh_a or k <thresh_b) do
    Vₖ(P)=CreateMWVD(P,wₖ(P), R) %Create a MWVD
    %Data normalization for s(Vₖ(P)), s(Vₖ(P))={s(Vₖ(p₁)),
    s(Vₖ(p₂)),…, s(Vₖ(pₙ))}
    s'(Vₖ(P))=Normalization(s(Vₖ(P)))
    %Adapt the weight set wₖ(P) and generate the weight set wₖ₊₁(P)
    wₖ₊₁(P)=CalculateNewWeights(s'(Vₖ(P)), a'(P), wₖ(P), β);
    %Calculate difference between s'(Vₖ(P)) and a'(P)
    err=GetError(s'(Vₖ(P)), a'(P))
     k++
endwhile
End
```

Data Normalization for an APMWVD

It can be known from Algorithm1 that $s(V_k(p_i))$ is the area belonging to generator p_i after iteration k and $a(p_i)$ is the weight of the generator p_i in P. In order to get a better iteration, both $s(V_k(p_i))$ and $a(p_i)$ need to be normalized such that:

$$a'(p_i) = \frac{a(p_i)}{\sum a(p_i)} \tag{11}$$

and:

$$s'(V_k(p_i)) = \frac{s(V_k(p_i))}{\sum s(V_k(p_i))} \tag{12}$$

where $a'(p_i)$ is the target area of the generator p_i and $s'(V_k(p_i))$ is the area of the multiplicatively weighted Voronoi regon after normalization. Both $a'(p_i)$ and $s'(V_k(p_i))$ are between 0 and 1 satisfying such that:

$$\sum_{i=1}^{n} a'(p_i) = 1 \tag{13}$$

and:

$$\sum_{i=1}^{n} s'(V_k(p_i)) = 1 \tag{14}$$

The function *Normalization* () is used for data normalization as follows.

Algorithm 2. Normalization ()

```
Input: a set of n values V={v₁, v₂, …, vₙ}
Output: a set of n normalized values nV={nv₁, nv₂, …, nvₙ}
Begin
sumV=sum(V) %get a summation of V
Initialize an array nV for a set of n normalized values.
for i:=1; step 1 until n do
    nvᵢ=vᵢ/sumV
endfor
End
```

The Construction of an MWVD

In Algorithm 1, we utilize the function *CreateWVD*() proposed by Tian et al. (2015, August 2-5) to generate an MWVD. This function uses the idea of an incremental method as shown in Figure 3: Every time a new generator p_j is inserted, the original multiplicatively weighted Voronoi regions $V(p_i)$ (i=1,2,…,j-1) will be redivided to generate a new multiplicatively weighted Voronoi region$V(p_j)$ belonging to p_j by using methods of region division(function *DivideRegionWithPB*() and *DivideRegionWithArc*()) and region union(function *MergeRegions*()). In Figure 3c, dominances of p_j, that is, $Dom(p_i,p_j)$ mentioned in Equation 2, are divided from the existing multiplicatively weighted Voronoi regions $V(P)$={$V(p_1),V(p_2),…,V(p_{j-1})$} by using function *DivideRegionWithPB*() and *DivideRegionWithArc*(). In Figure 3d, $Dom(p_i,p_j)$ are merged to generate the multiplicatively weighted Voronoi region of p_j by using function *MergeRegions*().

A Fixed-Point Iteration for an APMWVD

To construct an APMWVD from an MWVD, the key is to dynamically adjust the weight of each generator to minimize the difference between the final desired dominance area and the actual area of each multiplicatively weighted Voronoi region at each iteration. Thus, we use a fixed-point iteration method proposed by Reitsma et al. (2007) to adjust the weights of generators:

$$w_{k+1}(p_i) = w_k(p_i) + \Delta w(p_i) \tag{15}$$

where $w_{k+1}(p_i)$ and $w_k(p_i)$ are separately weights of the generator p_i after iteration k and iteration k+1, $\Delta w(p_i)$ is the weight adjustment factor.

We make the weight adjustment factor $\Delta w(p_i)$ proportional to three quantities: $w_k(p_i)$, the weight of the generator p_i at k-th iteration; $a'(p_i)$-$s'(V_k(p_i))$, the difference between the normalized desired

Figure 3. The basic thought to create an MWVD

(Tian et al., 2015, August 2-5)
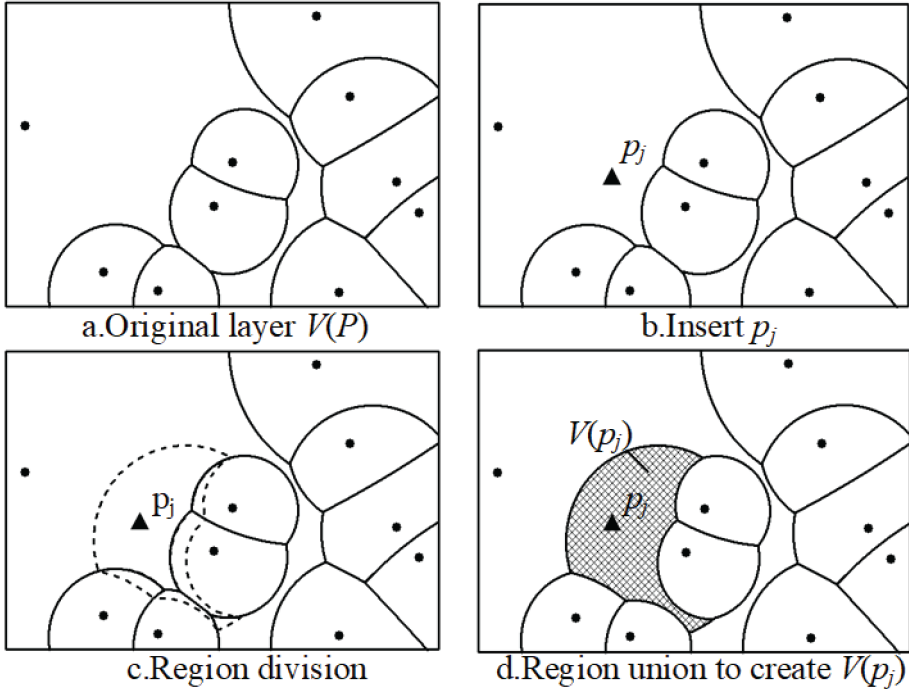
Note. (a) The existing multiplicatively weighted Voronoi regions $V(P)=\{V(p_1),$ $V(p_2),\ldots, V(p_{j-1})\}$; (b) Insert a generator; (c) Divide the multiplicatively weighted Voronoi regions $V(P)$ to generate some dominances of p_j; and (d) Merge dominances of p_j to get the multiplicatively weighted Voronoi region belonging to p_j.

dominance area of the generator p_i and the normalized area of the multiplicatively weighted Voronoi region of the generator p_i after iteration k; and β, a positive, nonzero scaling constant:

$$\Delta w(p_i) = w_k(p_i)\beta(a'(p_i)\text{-}s'(V_k(p_i))) \tag{16}$$

Replacing $\Delta w(p_i)$ in Equation 15 with Equation 16 gives:

$$w_{k+1}(p_i) = w_k(p_i)(1 + \beta(a'(p_i))\text{-}s'(V_k(p_i))) \tag{17}$$

In Equation 17, if $a'(p_i)$ is greater than $s'(V_k(p_i))$ at k-th iteration, we can get a positive increased weight of the generator p_i for $(k+1)$-th iteration with a proper set of β, thus the area of $V_{k+1}(p_i)$ will increase and the difference between $a'(p_i)$ and $s'(V_{k+1}(p_i))$ will be less. The same result will be acquired while $a'(p_i)$ is less than $s'(V_k(p_i))$ with a proper set of β. After enough iterations, we can minimize the difference between the desired dominance area and the actual area of each multiplicatively weighted Voronoi region.

It can be known from Equations 11 and 12 that $0< a'(p_i) <1$ and $0< s'(V_k(p_i)) <1$. Thus, the difference between $a'(p_i)$ and $s'(V_k(p_i))$ is in the range:

$$-1 < a'(p_i)\text{-}s'(V_k(p_i)) < 1 \qquad (18)$$

After multiplying by β and adding 1, we obtain:

$$1 - \beta < 1 + \beta(a'(p_i)\text{-}s'(V_k(p_i))) < 1 + \beta, \beta > 0 \qquad (19)$$

Accordingly, choosing $\beta > 1$ may get the minimized area difference in fewer iterations and the algorithm converges more quickly. However, it may result in negative weights and increases the likelihood of error function oscillation because larger values of β produce larger adjustments of the weights (Reitsma et al., 2007). To assure $w_{k+1}(p_i) > 0$ in Equation 17, β should be in the range (0,1].

The function *CalculateNewWeights*() is a realization of the fixed-point iteration method as mentioned above:

Algorithm 3. Calculate New Weights()

```
Input: a set of n actual areas S={s₁,s₂,..,sₙ}, a set of n desired
areas A={a₁,a₂,…,aₙ}, a set of n weights W={w₁,w₂,…,wₙ}, a positive,
nonzero scaling constant β
Output: a set of n new weights V={v₁, v₂,…, vₙ}
Begin
Initialize an array V for a set of n values.
for i:=1; step 1 until n do
    vᵢ=wᵢ(1+β(aᵢ−sᵢ))
endfor
End
```

Iterative Termination Metric

The APMWVD construction algorithm uses an iterative way to achieve an area-weight proportional balance for actual areas of multiplicatively weighted Voronoi regions and their desired areas. So, we must choose an operational metric to end the algorithm. Although it is possible for so many error metrics, the method of Wood (1974) is used in this paper as follows:

$$err = \frac{1}{n}\sum_{i=1}^{n}\frac{|\,s'(V_k(p_i)) - a'(p_i)\,|}{s'(V_k(p_i))} \qquad (20)$$

where, *err* is the mean proportional absolute error across all multiplicatively weighted Voronoi regions for an APMWVD.

The function *GetError* () is used to calculate the above-mentioned mean proportional absolute error:

Algorithm 4. Get Error ()

```
Input: a set of n actual areas S={s₁,s₂,..,sₙ}, a set of n desired
areas A={a₁,a₂,…,aₙ}
Output: err
Begin
sum=0
```

```
for i:=1; step 1 until n do
    sum = sum +|s_i-a_i|/s_i
endfor
err=sum/n
End
```

Complexity Analysis

It can be known from research of Tian et al. (2015, August 2-5) that the complexity of the MWVD algorithm for *n* generators is:

$$O(MWVD) = O(n^2) \tag{21}$$

The complexity of an APMWVD construction algorithm mainly depends on that of the MWVD construction. Considering the iterative weight adjustments, the complexity of the APMWVD construction algorithm with *k* iterations is:

$$O(APMWVD) = O(kO(MWVD)) = O(kn^2) \tag{22}$$

EXAMPLES

Visualization of an APMWVD for Random Spatial Points

Figure 4a shows the results for an MWVD of nine, uniformly distributed generators with integer weights $1 \leq W_j \leq 9$. Figure 4b exhibits the results for an APMWVD of the same generators and weights. We show the trajectory of *err* (Equation 20) for every iteration with β=0.1, 0.5, 0.9, 1, respectively in Figure 5, and the iteration numbers for different β while *err*<0.5,0.1,0.01,0.001,0.0001, respectively in Table 1. It can be apparently seen from Figure 5 and Table 1 that *err* could reduce to a certain value,

Figure 4. The MWVD and APMWVD for nine generators with the same locations and weights
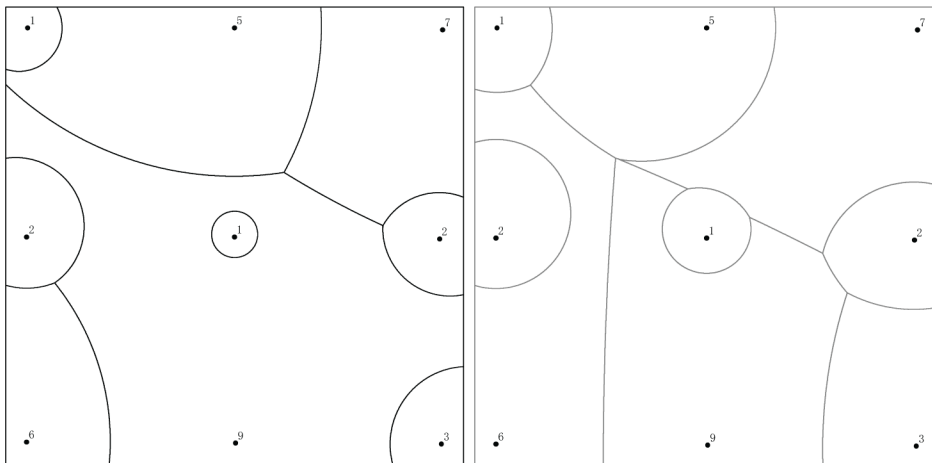
(a) the MWVD   (b) the APMWVD

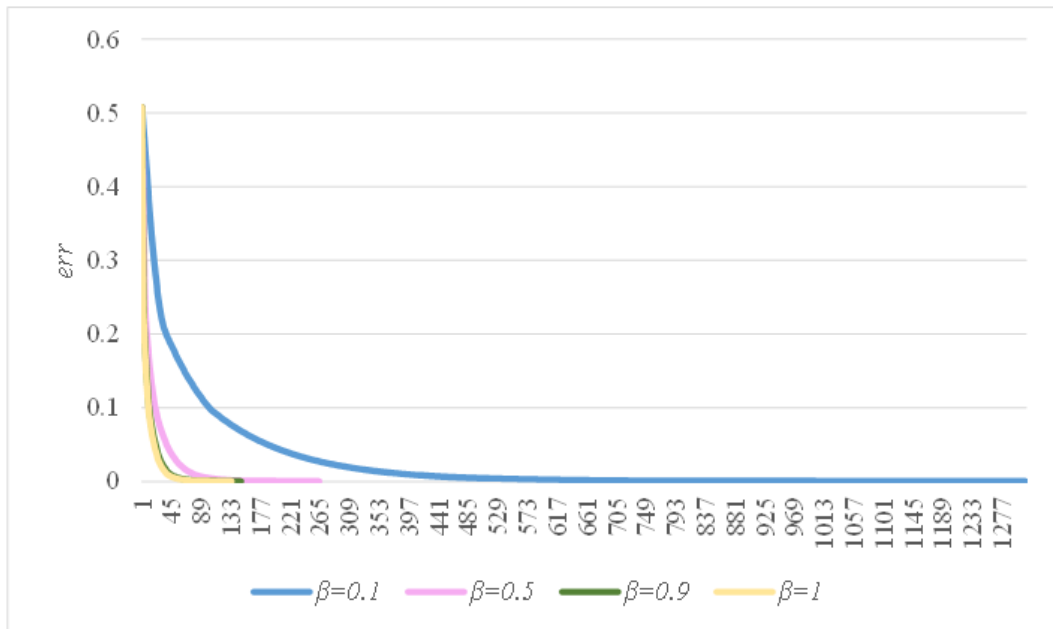**Figure 5. The trajectory of err for the APMWVD of nine generators at every iteration while β=0.1, 0.5, 0.9, 1**

Table 1. Iteration numbers for the APMWVD of nine generators in Figure 4 with different errs while β=0.1, 0.5, 0.9, 1

	err<0.5	*err<0.1*	*err<0.01*	*err<0.001*	*err<0.0001*
$\beta=1$	2	11	39	78	134
$\beta=0.9$	2	12	43	87	148
$\beta=0.5$	2	21	78	155	264
$\beta=0.1$	2	101	391	772	1312

such as 0.5,0.1,0.01,0.001,0.0001, with less iterations when β equals 1 than others. Thus, in order to get a quick rate of algorithm convergence, we suggest one lets $\beta=1$ in Equation 17.

We implemented the APMWVD construction algorithm in C# and ArcGIS Engine (a library of embeddable GIS components). Using four Core i5-3337U CPUs, each with 1.8 GHz, the constructions of an APMWVD with $\beta=0.1$, 0.5, 0.9 and 1 shown in Figure 4 required 5min, 57s, 32s and 30s, respectively. The running time is acceptable for many GIS applications.

## Visualization of the APMWVDs for Residential Care Facilities

The goal of the APMWVD is to delineate the service area of spatial objects from the aspect of area-weight proportionality in the geographic field. In this section, we express how to use an APMWVD to express the relationship between demand and supply of residential care facilities (RCFs).

To generate an APMWVD for RCFs, we first need a point file in shapefile format representing the spatial distribution of RCFs and a polygon file used as the boundary of them. Then, the weights of RCFs should be confirmed. We choose the Dongcheng and Xicheng Districts of Beijing in China as the study area. The spatial data are acquired from the State Bureau of Surveying and Mapping with WGS 1984 UTM projection, which contains 32 subdistricts. The data of RCFs are acquired from the Beijing Municipal Civil Affair Bureau on its website. The latest data are from 2010 (the website

stopped updating data later). There were 35 RCFs in the Dongcheng and Xicheng Districts of Beijing from 2010. Motivated by the research on residential care facilities (Cheng et al., 2012; Zhou et al., 2013; Tao et al., 2014), the number of beds of the RCF is chosen as the weight, which ranges from 8 to 340. The total number of beds of 35 RCFs is 2936.

Let $\beta=1$, $thresh\_a$=5000, $thresh\_b$=0.01, $P=\{p_1,p_2,\ldots,p_n\}$ denote the RCFs in the Dongcheng District and Xicheng Districts of Beijing and R denote the administrative boundary of the Dongcheng District and Xicheng Districts of Beijing, an APMWVD for 35 RCFs can be constructed by using Algorithm 1 as is shown in Figure 6a. The Algorithm 1 ends at the 5000th iteration while err=0.046. Because an APMWVD is derived from a MWVD. To compare the effect of service area delineation, we also constructed an MWVD by using Tian's method (Tian et al., 2015, August 2-5) as is shown in Figure 6b.

Figure 6. The service area delineation of the APMWVD and MWVD for RCFs in Dongcheng and Xicheng districts of Beijing

a) the APMWVD   b) the MWVD
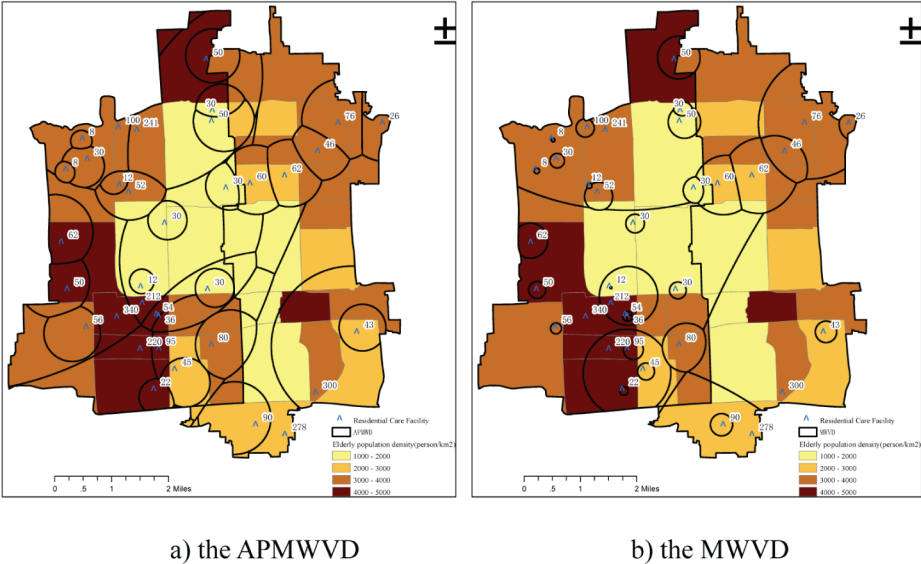
The population data of the elderly aging above 65 in the Dongcheng and Xicheng Districts at the subdistrict level are chosen to further explore the demand and supply situation between the elderly and RCFs by the APMWVD and MWVD, which is from the sixth population census. The total number of the elderly aging above 65 in the Dongcheng and Xicheng Districts is 270463. Then, the demand and supply ratio of the elderly population to bed number of RCFs (or DSR) in the Dongcheng and Xicheng Districts of Beijing delineated by the APMWVD and MWVD (Figure 7) can be acquired by using ArcGIS software as follows:

1. Link the elderly population data to the map of the Dongcheng and Xicheng Districts. Each subdistrict is assigned a 'population density' value, which is the ratio of the elderly population in the subdistrict compared to the area of subdistrict.
2. Obtain the elderly population in each service area of the APMWVD and MWVD through the Intersect Tool and Dissolve Tool. Because the Beijing municipal government has made a ''9064'' policy, that is, by 2020, 90% of the elderly would depend on family care, 6% of the elderly would depend on community care, and 4% of the elderly would depend on residential care, we can get

the number of the elderly who need to stay in RCFs in each service area of the APMWVD and MWVD, that is, 4% of the elderly in each service area of the APMWVD and MWVD.

3. Calculate the DSR in each service area of the APMWVD and MWVD. Then, use ArcGIS to visualize the DSR (Figure 7).

Figure 7. The DSR in the Dongcheng and Xicheng districts of Beijing delineated by the APMWVD and MWVD

a) the APMWVD   b) the MWVD
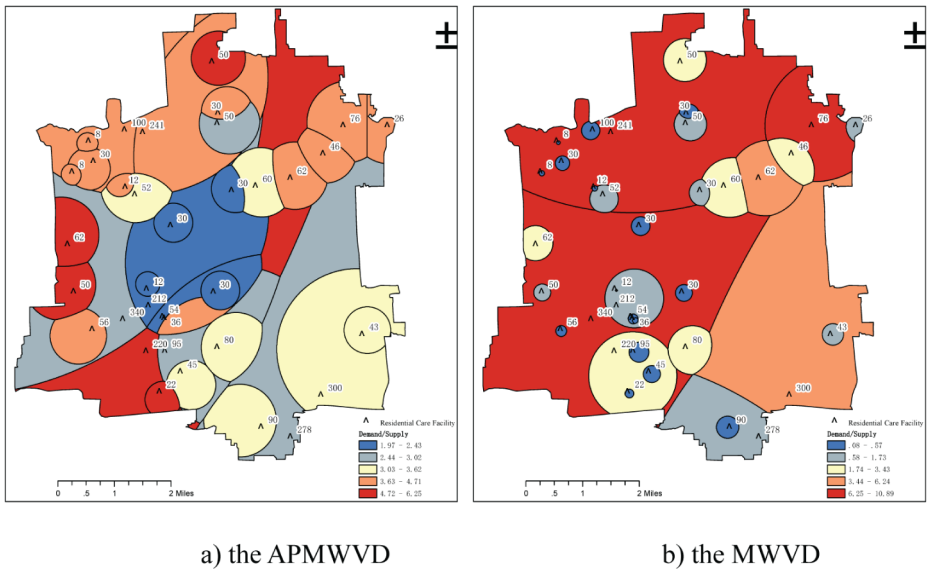
The total DSR in the Dongcheng and Xicheng Districts is 270463×0.04/2936=3.68. The maximum value, minimum value, mean value and standard deviation of the DSRs of 35 service areas for the APMWVD in Figure 7a are respectively 6.25, 1.97, 3.74 and 1, while that for the MWVD in Figure 7b are 10.89, 0.08, 1.93 and 2.64, respectively. From above statistic data, Figures 7 and 8, it can be known that (1) the mean value of the DSRs of 35 service areas for the APMWVD are closer to the actual demand and supply situation of the elderly and RCFs; and (2) because the MWVD has a property that regions with large weights tend to be oversized whereas those with low weights are undersized, it leads to that the DSRs of 35 service areas in the MWVD have a large fluctuation range. Supply exceeds demand in some small service areas, whereas demand far exceeds supply in some large service areas; whereas the fluctuation range of the DSRs in the APMWVD is smaller. In summary, the APMWVD have more advantages in depicting demand and supply in the geographic field.

Since the main purpose of this paper is to introduce a method to generate a spatial layout by an APMWVD, the specific and deeper research on representing a demand and supply situation is not included in this paper; we leave this for future research.

RESULTS AND DISCUSSIONS

From the above sections, it can be known as follows:

1. The idea of the APMWVD is aroused by the research of Reitsma et al. (2007) on AMWVD. Both methods use adaptive multiplicatively weighted Voronoi regions to visualize area. However, there are some differences between them (Table 2). AMWVDs are composed of multiplicatively

Figure 8. The DSR of each service area in the Dongcheng and Xicheng districts of Beijing delineated by the APMWVD and MWVD

Table 2. Differences between the AMWVD and APMWVD

	The AMWVD	**The APMWVD**
Composition	multiplicatively weighted Voronoi regions	multiplicatively weighted Voronoi regions
Representation	Non-spatial objects	Spatial objects
Spatial information	None	Spatial coordinates and references
Generators	Pre-set	Pre-set
Construction method	A fixed-point iteration approach in a raster way	A fixed-point iteration approach in a vector way

weighted Voronoi regions without spatial information with the purpose of representing non-spatial objects, while our proposed APMWVD consists of the multiplicatively weighted Voronoi regions and the corresponding generators with spatial coordinates and references that are used to visualize spatial objects. Generators can be regarded as schools, hospitals, cities, etc., and the multiplicatively weighted Voronoi regions represent their dominant areas. The precision of the generated APMWVD layout will be higher by this method than by a raster-based method.

2. β determines the degree of algorithm convergence. In our experiment, $\beta=1$ may be the better choice (Figures 5 and Table 1). Overall convergence of the algorithm is good, at least for these small test cases (Figures 4 and Figure 6). The algorithm needs to loop through all point features one after another in each iteration using several complex topological operations, such as cell division and cell union, thus, the process may be relatively slow when facing massive inputting point features.

3. The proposed algorithm can seamlessly work with GIS applications. The weights for an APMWVD can be acquired from a numeric field in the spatial attribute table of a point feature in the shapefile format. The inputting point set and the outputting APMWVD layer can be stored together in a geodatabase. By combining them with other vector-based or raster-based data for

overlaying analyses, more in-depth information may be obtained. In Figure 6, we combined the APMWVD layer with the population data of the elderly aging above 65 in the Dongcheng and Xicheng Districts, then the demand and supply situation of the elderly and RCFs can be obtained in Figure 7.

4. In order to provide a clear interpretation and deep perception of the APMWVD visualization for users, enhancements like coloring, legends, scales, compass, etc., may also be used to decorate the generated layout using GIS symbols and styles (Figures 6 and 7).
5. We did some preliminary testing to compare how to depict demand and supply by using an MWVD and an APMWVD in the section Visualization of the APMWVDs for Residential Care Facilities. According to the experimental data, the APMWVD performed well, no matter the fluctuation range of the demand and supply ratio or the reasonable presentation of the demand and supply situation.

CONCLUSION

Ordinarily, the MWVD is used for the delineation of service areas in the geographic field. The organization of an MWVD layout can be abstracted into two parts: a set of points, which describes the locations of spatial objects, and a set of spheres, which reveals the service area of spatial objects. But weights in an MWVD scale distance, so it is difficult for an MWVD to achieve a good area-weight proportionality. Thus, we proposed an APMWVD construction algorithm to visualize a spatial demand and supply situation. It can be known from experiments that the APMWVD is a good candidate for the type of area-weight proportional partitioning applied to depict the demand and supply situations that are modeled by constraints Equations 6–10. Because the APMWVD construction algorithm needs to loop through all generators in each iteration based on several complex topological operations, the process will be relatively slow while facing massive inputting generators. Therefore, the methods for increasing the efficiency of the algorithm should be further researched in the future. Although this study implemented the APMWVDs for delineating the RCFs service areas that consider the distribution of elderly population, the method may also be used for delimiting the service area of other public facilities (e.g., schools and supermarkets) and may be extended through considering other socioeconomic factors. As such, we believe that the APMWVD will offer some attractive promises for the presentation of demand and supply situations in the geographic field, especially helping city managers to allocate public service resources more efficiently.

FUNDING STATEMENT

DATA AVAILABILITY

The data used to support the findings of this study are available from the corresponding author upon request.

CONFLICTS OF INTEREST

The authors declare no conflicts of interest.

REFERENCES

Aurenhammer, F., & Edelsbrunner, H. (1984). An optimal algorithm for constructing the weighted Voronoi diagram in the plane. *Pattern Recognition*, *17*(2), 251–257. doi:10.1016/0031-3203(84)90064-5

Boots, B. N. (1975). Patterns of urban settlements revisited. *The Professional Geographer*, *27*(4), 426–431. doi:10.1111/j.0033-0124.1975.00426.x

Caro, F., Shirabe, T., Guignard, M., & Weintraub, A. (2004). School redistricting: Embedding GIS tools with integer programming. *The Journal of the Operational Research Society*, *55*(8), 836–849. doi:10.1057/palgrave.jors.2601729

Cheng, Y., Wang, J. E., & Rosenberg, M. W. (2012). Spatial access to residential care resources in Beijing, China. *International Journal of Health Geographics*, *11*(1), 32–32. doi:10.1186/1476-072X-11-32 PMID:22877360

Converse, P. D. (1949). New laws of retail gravitation. *Journal of Marketing*, *14*(3), 379–384. doi:10.1177/002224295001400303

Dirichlet, G.L. (1850). Über die Reduktion der positiven quadratischen Formen mitdrei unbestimmten ganzen Zahlen. *Journal fiir die Heinc und Angewandte Mathenwtik. 40*, 209-227.

Du, Q., Max, G., & Vance, F. (1999). Centroidal Voronoi tessellations: Applications and algorithms. *SIAM Review*, *41*(4), 737–676. doi:10.1137/S0036144599352836

Gong, Y. X., Li, G. C., Tian, Y., Lin, Y. Y., & Liu, Y. A. (2012). Vector-based algorithm to generate and update multiplicatively weighted Voronoi diagrams for points, polylines, and polygons. *Computers & Geosciences*, *42*, 118–125. doi:10.1016/j.cageo.2011.09.003

Görke, R., & Wolff, A. (2005, March 9-11). *Constructing the city Voronoi diagram faster*. In *Proceedings of the 21st European workshop on computational geometry,* Eindhoven, The Netherlands.

Huff, D. L. (1964). Defining and estimating a trade area. *Journal of Marketing*, *28*(3), 34–38. doi:10.1177/002224296402800307

Huff, D. L., & Jenks, G. F. (1968). A graphic interpretation of the friction of distance in gravity models. *Annals of the Association of American Geographers*, *58*(4), 814–824. doi:10.1111/j.1467-8306.1968.tb01670.x

Moreno-Regidora, P., & Lacalleb, J.G.L., & Miguel-ÁngelManso-Callejoa. (2012). *International Journal of Geographical Information Science*, *31*(4), 763-782.

Mu, L. (2004). Polygon characterization with the multiplicatively weighted Voronoi diagram. *The Professional Geographer*, *56*(2), 223–239. doi:10.1111/j.0033-0124.2004.05602007.x

Mu, L., & Wang, X. (2006). Population landscape: A geometric approach to studying spatial patterns of the U.S. urban hierarchy. *International Journal of Geographical Information Science*, *20*(6), 649–667. doi:10.1080/13658810600661342

Okabe, A., Boots, B., Sugihara, K., & Chiu, S. N. (2000). *Spatial tessellations: Concepts and applications of Voronoi diagrams* (2nd ed.). John Wiley & Sons. doi:10.1002/9780470317013

Reilly, W. J. (1929). Methods for the study of retail relationship. *University of Texas Bulletin*, 29-44.

Reitsma, R., & Trubin, S. (2006). *Weight-proportional information space partitioning using adaptive multiplicatively-weighted Voronoi diagrams*. In *Proceedings of the twelfth Americas conference on information systems*, Acapulco, Mexico,

Reitsma, R. F., Trubin, S., & Mortensen, E. N. (2007). Weight-proportional space partitioning using adaptive Voronoi diagrams. *GeoInformatica*, *11*(3), 383–405. doi:10.1007/s10707-006-0006-8

Ricca, F., Scozzari, A., & Simeone, B. (2013). Political districting: From classical models to recent approaches. *Annals of Operations Research*, *204*(1), 271–299. doi:10.1007/s10479-012-1267-2

Ríos-Mercado, R. Z., & Fernández, E. (2009). A reactive GRASP for a commercial territory design problem with multiple balancing requirements. *Computers & Operations Research*, *36*(3), 755–776. doi:10.1016/j.cor.2007.10.024

Sethia, S., Trubin, S., & Reitsma, R. (2004, July 16). Information space regionalization using adaptive multiplicatively weighted Voronoi diagrams. In *Proceedings of 14th international conference on information visualisation*, London, UK.

Steiner, M. T. A., Datta, D., Steiner Neto, P. J., Scarpin, C. T., & Rui Figueira, J. (2015). Multi-objective optimization in partitioning the healthcare system of parana state in Brazil. *Omega*, *52*, 53–64. doi:10.1016/j.omega.2014.10.005

Tao, Z., Cheng, Y., Dai, T., & Rosenberg, M. W. (2014). Spatial optimization of residential care facility locations in Beijing, China: Maximum equity in accessibility. *International Journal of Health Geographics*, *13*(1), 33. doi:10.1186/1476-072X-13-33 PMID:25178475

Thiessen, A. H. (1911). Precipitation averages for large areas. *Monthly Weather Review*, *39*, 1082–1084.

Tian, S. (2021). Vector-based realisation of geographical Voronoi treemaps with the arcGIS engine. *Journal of Information Technology Research*, *14*(1), 14. doi:10.4018/JITR.2021010103

Tian, S., Cui, X. M., & Gong, Y. (2015). An approach to generate spatial Voronoi treemaps for points, lines, and polygons. *Journal of Electrical and Computer Engineering*, *2015*, 1–10. doi:10.1155/2015/787163

Tian, S., Cui, X. M., & Gong, Y. (2015, August 2-5). A general vector-based algorithm to generate weighted Voronoi diagrams based on arcGIS engine. In *Proceedings of 2015 IEEE international conference on mechatronics and automation* (pp. 941-946). IEEE.

Trubin, S. I. (2006). *Information space mapping with adaptive multiplicatively weighted Voronoi diagrams*. Oregon State University.

Vincent, P. J., & Daly, R. (1990). Thiessen polygon types and their use in GIS. *Mapping Awareness*, *4*(5), 40–42.

Voronoï, G. (1908). Nouvelles applications des paramètres continus à la théoriedes formes quadratiques. Deuxième mémoire. Recherches sur les parallélloèdres primitifs. *Journal für die Reine und Angewandte Mathematik*, *134*(134), 198–287. doi:10.1515/crll.1908.134.198

Wang, J., & Kwan, M. P. (2018). Hexagon-based adaptive crystal growth Voronoi diagrams based on weighted planes for service area delimitation. *ISPRS International Journal of Geo-Information*, *7*(7), 257. doi:10.3390/ijgi7070257

Wang, J., Kwan, M. P., & Ma, L. (2014). Delimiting service area using adaptive crystal-growth Voronoi diagrams based on weighted planes: A case study in Haizhu District of Guangzhou in China. *Applied Geography (Sevenoaks, England)*, *50*, 108–119. doi:10.1016/j.apgeog.2014.03.001

Wood, I. J. (1974). Spatial interactions and partitions of rural market space. *Tijdschrift voor Economische en Sociale Geografie*, *65*(1), 23–34. doi:10.1111/j.1467-9663.1974.tb01208.x

Zhang, P., Jing, W., & Chen, Y. (2018). Weighted Voronoi diagram-based simulation and comparative analysis of ecosystem service coverage: Case study of the Zhongyuan urban agglomeration. *Journal of Sensors*, *2018*(10), 1–11. doi:10.1155/2018/7147524

Zhou, S. Y., Cheng, U., Xiao, M. Z., & Bao, X. (2013). Assessing the location of public-and-community facilities for the elderly in Beijing, China. *GeoJournal*, *78*(3), 539–551. doi:10.1007/s10708-012-9450-1

Song Tian, PhD in GIS and remote sensing, works as senior engineer at the South China Sea Planning and Environment Research Institute of the State Oceanic Administration. In recent years, he has been committed to the application research of marine remote sensing and mapping technology and has undertaken a series of scientific research projects in marine and island monitoring of GIS algorithms.

Zheng Wei, PhD in photogrammetry and remote sensing, works as senior engineer at the South China Sea Planning and Environment Research Institute of the State Oceanic Administration. In recent years, he has been committed to the application research of marine remote sensing and mapping technology and has undertaken a series of scientific research projects in marine aerial photogrammetry and island monitoring.