


Optimal Solution of Combined Heat and Power Dispatch Problem Using Whale Optimization Algorithm

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ABSTRACT

In this article, whale optimization algorithm (WOA) has been applied to solve the combined heat and power economic dispatch (CHPED) problem. The CHPED is an energy system that provides both heat and power. Due to the presence of valve point loading and the prohibited working region, the CHPED problem becomes more complex. The main objective of CHPED problem is to minimize the total cost of power generation and minimize the global warming of environment with fulfill the load demand. This optimization technique shows several advantages like having few input variables, best quality of solution with rapid computational time. The recommended approach is carried out on three test systems and compared with presently developed optimization techniques to judge the superiority of the proposed algorithm. The simulation results of the present work certify the activeness of the proposed technique.

KEYWORDS

Cogeneration, Combined Heat, and Power Economic Dispatch (CHPED), Prohibited Zone, Whale Optimization Algorithm (WOA)

1 INTRODUCTION

Heat is released, into the natural atmosphere from all thermal power generating plants through cooling towers, flue gas, or by other means during generation of electric power. Therefore, energy efficiency about the power generation units become very low within 50% to 60% and environment is polluted due to emission of byproduct (NO_x , SO_x , SO_2 & CO_2) during heating.

In order to use waste heat for improving the overall efficiency of power generation unit and reduction of emitted pollutants during heating CHPED has become an important area of research. In CHPED system, the heat recovery steam generator recovers the waste heat for heating or steam generation and cooling through the use of absorption Chillers. CHPED is a cogeneration system which produces power and process heat simultaneously.

For simplicity the cost function of power unit, heat unit and co-generation unit are represented by quadratic function and is solved by mathematical programming techniques. In practice the higher order nonlinearities and discontinuities due to valve point loading effects are introduced in

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mathematical formulations. Moreover, due to physical limitations on components of power generating units of CHPED problem, these units may have prohibited operating zones. In view of that, a unit with prohibited operating zones, its whole operating region will be broken into some isolated feasible sub-regions, which makes the CHPED problem discontinuous. So, the operation constraints and non-linearity make the CHPED problem a non-smooth optimization problem having complex and non-convex features with equality and inequality constraints.

To find quality solution, different optimization methods have been applied to get optimal point for power production such that the total demand matches the generation with minimum fuel cost, while satisfying required power demand and other constraints.

Many researchers performed a lot of researches on CHPED during last two decades. To solve CHPED, various optimizations techniques are adopted by various researchers. These methods are categorized into classical mathematical optimization algorithms and intelligent optimization algorithms. The classical algorithms include Lagrangian relaxation (Majd, et al., 2018), classical technique (CT) (Damodaran & Sunil kumar, 2014) etc. which have been successfully applied by the various researchers. Thomson et al. (Thomson, et al., 2000) proposed a statistical process control method to solve CHPED problem. Generally, these methods produce best optimal solutions if the fuel cost characteristics of generating units are linear. However, these traditional approaches cannot be applied directly to a practical CHPED problem because CHPED problem has complex and non-convex characteristics due to the presence of valve point effects, multiple fuel option, prohibited operating zone.

On the other hand, to obtain accurate dispatch result, various intelligent optimization algorithms i.e. heuristic techniques are applied to solve CHPED problem. Gravitational search algorithm proposed by Beigvand et al. (Beigvand et al. 2016) to solve CHPED problem of power system, where the effectiveness of GSA has been tested with considering valve point loading and transmission losses. Ghorbani et al. (Ghorbani et al. 2016) introduced exchange market algorithm to demonstrate nonlinear and nonconvex CHPED system. Cuckoo search algorithm has been implemented by Nguyen et al. (Nguyen et al. 2016) to analyze the CHPED problem considering with a set of control parameters. To perform CHPED problem, Haghrah et al. (Haghrah et al. 2016) introduced real coded genetic algorithm considering with improved muhlenbein mutation and the proposed real coded genetic algorithm was applied on different benchmark functions. Grey wolf optimization has been used to solve CHPED problem considering valve point loading, transmission losses, spinning reserve, and ramp rate by Jayakumar et al. (Jayakumar et al. 2016). Decomposition based optimization method has been used by Abdollahi et al. (Abdollahi et al. 2016) to judge its performances on CHPED problem. Hybrid gravitational search algorithm-particle swarm optimization with time varying acceleration coefficient for large scale CHPED problem has been used by Beigband et al. (Beigband et al. 2017). Group search optimization has been implemented by Basu (M. Basu 2016) to perform non smooth non convex based CHPED problem, where valve point loading and prohibited operating zone has been considered to judge the effectiveness of proposed algorithm. Shaabani et al. (Shaabani et al. 2017) proposed a multi-objective optimization technique to analyze the CHPED problem. Davoodi et al. (Davoodi et al. 2017) described combined heat and power economic dispatch problem using group search optimizer based algorithm. This approach implements adaptive scrounger and ranger strategies for improving GSO algorithm. Dynamic optimal power flow of combined heat and power system with valve point loading effect using krill herd algorithm has been analyzed by Adhvaryyu et al. (Adhvaryyu et al. 2017). Algorithm was tested on IEEE 30-bus and 118-bus systems. Optimal economic dispatch of FC-CHP based heat and power micro-grids has been solved by Heris et al. (Heris et al. 2017) where the uncertainties for load demand and price signals are taken into account. Murugan et al. (Murugan et al. 2018) proposed hybridized bat algorithm with artificial bee colony for solving combined heat and power economic dispatch, where disadvantages of bat algorithm with artificial bee colony has been eliminated through three search mechanisms. Li et al. (Li et al. 2018) implemented an optimization technique to solve CHPED problem considering with transmission

and valve point loading on three test cases. Rahman et al. (Rahman et al. 2018) introduced hybrid bio inspired computational intelligence in power system to obtain optimal solution. Levenberg marquardt algorithm was implemented on economic load dispatch problem by Daniel et al. (Daniel et al. 2018) to judge the performances of proposed algorithm. In the proposed process, ramp rate limit was considered to solve ELD problem. Pradhan et al. (Pradhan et al. 2018) applied oppositional based grey wolf optimization technique to solve economic dispatch problem of power system, where oppositional based learning has been incorporated with GWO to accelerate the convergence speed. To solve economic load dispatch problem Bulbul et al. (Bulbul et al. 2018) introduced oppositional based krill herd algorithm, where oppositional based learning is integrated to improve the performances of test area. Sekhar et al. (Sekhar et al. 2016) applied an optimization technique to demonstrate the security enhancement in economical load dispatch problem. Das et al. (Das et al. 2018) proposed point estimation method to analyze the performances of hydro thermal scheduling (HTS) problem. The wind and solar energy have been integrated with HTS to obtain optimal solution of generation cost and reduce the greenhouse effect. Biswas et al. (Biswas et al. 2018) suggested multi-objective algorithm to solve economic emission dispatch problem incorporating wind, solar and small hydro power. Zhang et al. (Zhang et al. 2018) worked on hydro thermal scheduling with environment emission using multi-objective optimization technique which is based on pareto dominance. Cuckoo bird inspired metaheuristic technique has been implemented on HTS problem combined with economic emission by Nguyen et al. (Nguyen et al. 2018) where optimal solution was reached with minimizing both cost and emission. Heris et al. (Heris et al. 2019) proposed harmony search method to analyze the large scale CHPED problem for optimal solution. Basu (M. Basu. 2019) introduced squirrel search algorithm to solve combined heat and power economic dispatch problem where renewable energy sources has been added with the system for cost and emission minimization. Zou et al. (Zou et al. 2019) proposed improve genetic algorithm with novel crossover to analyze CHPED problem. Nourianfar et al. (Nourianfar et al. 2019) applied the fast dominated TVAC-PSO combined with EMA to perform the combined heat and power economic emission dispatch and dynamic economic emission dispatch problem. Alomoush (Alomoush 2019) utilized an optimization technique to deal with multi-objective economic dispatch problem of combined heat and power in large microgrid. Gholamghasemi et al. (Gholamghasemi et al. 2019) applied an optimization technique to solve the ELD problem considering valve point loading, transmission losses, ramp rate and prohibited zone to judge the superiority of the proposed technique. Dey et al. (Dey et al. 2019) applied bio-inspired algorithm to solve economic emission problem on microgrid integrated with renewable sources. Dasgupta et al. (Dasgupta et al. 2019) proposed sine cosine algorithm to perform the hydro thermal scheduling problem, where wind energy has been integrated to minimize the generation cost and greenhouse effect. Montoya et al. (Montoya et al. 2019) introduced sequential quadratic programming to address the optimal power flow problem in dc grids. Fang et al. (Fang et al. 2019) introduced an optimization technique on overhead transmission line where both variation due to wind and load efficiently was handled. Abarghoee et al. (Abarghoee et al. 2015) proposed chance constrained and jointly distributed random variables methods on wind and solar photovoltaic based CHPED problem for energy saving and environmental conservation.

From the literature review, it is observed that the common drawback for most of evolutionary algorithms for solving non-linear problems is long computational time. So, there is still need to develop simple and effective methods, for obtaining better optimal solution and to accelerate the convergence time of the CHPED problem.

Advantages of WOA algorithm.

- a. In proposed WOA algorithm hunting behaviour of whales to provide optimal solution in order to reduce the generation cost.
- b. Overcome the local optimization problem and establish the global optimal solution.
- c. Improve the convergence speed, so computational time become less.

The aim of this paper is to introduce the efficient algorithm based on WOA for solving CHPED problems. The WOA is a new meta-heuristic algorithm, recently proposed by Mirjalili et al. (Mirjalili, & Lewis, 2016). In (Aziz et al., 2017), it has been proved by Aziz et al. that WOA is better than other existing algorithms. The WOA algorithm emulates the natural co-operative behavior of whales. It is flexible and gradient free mechanism, because it includes exploration and two approaches of exploitation. Moreover it has an ability to avoid local optima and get the global optimal solution that makes it suitable for real solution. In addition, it does not need structural adjustments in the algorithm for solving different optimization problems. This versatile property of WOA algorithm encourages the present authors to apply this newly developed algorithm for solving CHPED problems. The developed algorithm is illustrated on three test systems in order to show the strength of the proposed method. Results obtained from the proposed method are compared with classic PSO (CPSO) (Mohammadi-Ivatloo et al., 2013), time varying acceleration co-efficient particle swarm optimization (TVAC-PSO) (Mohammadi-Ivatloo et al., 2013), teaching learning based optimization (TLBO) (Roy, 2013), and group search optimization (GSO) (Basu, 2016).

The main contribution of the authors in this paper are mentioned below:

- a. The proposed technique is tested on three test systems for two cases.
- b. The test systems have been analysed considering valve point loading, prohibited operating zone and transmission losses.

The remainder of the paper is structured as follows. In section 2, CHPED problem formulation is introduced. Brief description of WOA algorithm have been made in section 3 and different steps of WOA algorithm applied to CHPED problem is presented in section 4. The simulation results along with cost convergence of the three test cases for prohibited and without prohibited zone are presented in section 5. The conclusion of the paper has been depicted in section 6 and future scopes are discussed in section 7.

2. PROBLEM FORMULATION OF CHPED

The main objective of the CHPED problem is to minimize the cost of the heat generation and the power generation by determining the heat generation and power generation of each unit while satisfying the heat demand, power demand, and capacity of each unit and heat-power feasible operation region of a cogeneration unit. To provide completeness for the CHPED problem formulation a variety of practical operation such as valve point effects, prohibited zone are taken into consideration. The mathematical model of the CHPED problem can be stated as follow.

Objective function

The objective function of CHPED problem is given by:

$$\text{Min } C = \sum_{i=1}^{N_p} C_{pi}(P_{pi}) + \sum_{j=1}^{N_c} C_{cj}(P_{cj}, H_{cj}) + \sum_{k=1}^{N_h} C_{hk}(H_{hk}) \quad (1)$$

Where C is the total generation cost; $C_{pi}(P_{pi})$ represents the fuel cost function of the i^{th} power unit. $C_{cj}(P_{cj}, H_{cj})$ and $C_{hk}(H_{hk})$ represents the production cost of co-generation and heat units. P_{pi} is the power of the i^{th} unit and H_{hk} is the heat of the k^{th} unit. N_p , N_c , and N_h are the number of thermal power units, co-generation units and heat only units respectively.

The thermal unit represented by quadratic cost function.

$$C_{pi}(P_{pi}) = \alpha_{pi}(P_{pi})^2 + \beta_{pi}P_{pi} + \gamma_{pi} \quad (2)$$

Where $\alpha_{pi}, \beta_{pi}, \gamma_{pi}$ are the cost coefficient of i^{th} thermal unit.

$$C_{pi}(P_{pi}) = \alpha_{pi}(P_{pi})^2 + \beta_{pi}P_{pi} + \gamma_{pi} + \left| \delta_{pi} \text{Sin}(\varepsilon_{pi} \times (P_{pi}^{\min} - P_{pi})) \right| \quad (3)$$

Equation (3) is the cost function with valve point effects. For valve point loading, a sinusoidal term is added to the quadratic costfunction, which makes the problem non-convex and non-differentiable. Where $\delta_{pi}, \varepsilon_{pi}$ are the cost coefficients of the i^{th} unit for modeling valve point effects.

$$C_{cj}(P_{cj}, H_{cj}) = \alpha_{cj}(P_{cj})^2 + \beta_{cj}P_{cj} + \gamma_{cj} + \delta_{cj}(H_{cj})^2 + \varepsilon_{cj}H_{cj} + \kappa_{cj}H_{cj}P_{cj} \quad (4)$$

$$C_{hk}(H_{hk}) = \alpha_{hk}(H_{hk})^2 + \beta_{hk}H_{hk} + \gamma_{hk} \quad (5)$$

Where $C_{cj}(P_{cj}, H_{cj})$ represents the cost function of the j^{th} cogeneration unit; $C_{hk}(H_{hk})$ is the cost function of the k^{th} heat only unit.

The following constraints of CHPED are given below:

i) Power balance Constraint

$$\sum_{i=1}^{N_p} P_{pi} + \sum_{j=1}^{N_c} P_{cj} = P_D + P_L \quad (6)$$

$$P_L = \sum_{i=1}^{N_p} \sum_{m=1}^{N_p} P_i B_{im} P_m + \sum_{i=1}^{N_p} \sum_{j=1}^{N_c} P_i B_{ij} P_j + \sum_{j=1}^{N_c} \sum_{r=1}^{N_c} P_j B_{jr} P_r \quad (7)$$

$$\sum_{j=1}^{N_c} H_j + \sum_{k=1}^{N_h} H_{hk} = H_D \quad (8)$$

Equation (6) represents power production and demand balance, equation (7) denotes power loss in transmission line, equation (8) represents heat production and demand balance. Where H_D is the thermal demand and B_{im}, B_{ij}, B_{jr} are the transmission loss coefficients of the system.

ii) Capacity Constraints

The total power and heat output of each generator unit, Cogeneration Unit, Heat unit must be in between its maximum and minimum limit for stable operation, i.e.,

$$P_{pi}^{\min} \leq P_{pi} \leq P_{pi}^{\max} \text{ where, } i = 1, 2, 3, \dots, N_p \quad (9)$$

$$P_{cj}^{\min}(H_{cj}) \leq P_{cj} \leq P_{cj}^{\max}(H_{cj}) \text{ where, } j = 1, 2, 3, \dots, N_c \quad (10)$$

$$H_{cj}^{\min}(P_{cj}) \leq H_{cj} \leq H_{cj}^{\max}(P_{cj}) \text{ where, } j = 1, 2, 3, \dots, N_c \quad (11)$$

$$H_{hk}^{\min} \leq H_{hk} \leq H_{hk}^{\max} \text{ where, } k = 1, 2, 3, \dots, N_h \quad (12)$$

Where P_{pi}^{\min} and P_{pi}^{\max} are the minimum and maximum power limit of the power only unit, $P_{cj}^{\min}(H_{cj})$ and $P_{cj}^{\max}(H_{cj})$ are minimum and maximum power limit of the j^{th} cogeneration unit, $H_{cj}^{\min}(P_{cj})$ and $H_{cj}^{\max}(P_{cj})$ are minimum and maximum heat limit of the j^{th} cogeneration unit and for k^{th} heat unit H_{hk}^{\min} , H_{hk}^{\max} are minimum and maximum limit of heat.

iii) Prohibited Operating Zones

Generating units can have prohibited operating zone, due to fault during physical operation of the machines or the associated auxiliaries such as boiler, feed pump, etc. Normally generators may experience amplification of vibrations in their shaft bearing those are operating in prohibited zone. It is very difficult to determine the I/P – O/P characteristics in the neighborhood of a prohibited zone because of discontinuity. The zones are to be shorted out and neglect for the best economy in actual operation. The feasible operating zones for unit, with POZ can be explained as given below.

$$P_{pi}^{\min} \leq P_{pi} \leq P_{pi,1}^l \text{ where, } i = 1, 2, 3, \dots, N \quad (13) \quad P_{pi,j-1}^u \leq P_{pi} \leq P_{pi,j}^l, \text{ where, } j = 1, 2, 3, \dots, N \quad (14)$$

$$P_{pi,N}^u \leq P_{pi} \leq P_{pi}^{\max} \text{ where, } i = 1, 2, 3, \dots, N \quad (15)$$

Where N is the number of prohibited zone of i^{th} unit, $P_{pi,j-1}^u$ is the upper generation limit and $P_{pi,j}^l$ lower generation limit of prohibited zone j and j-1, respectively, of the i^{th} unit.

iv) Feasible operating region of cogeneration units.

For three test systems the feasible regions of the cogeneration units are as follows:
 Test system 1:

$$\begin{aligned}
 1.781914894 \times H_5 - P_5 - 105.7446809 &\leq 0 \\
 0.1777777784 \times H_5 + P_5 - 247.0 &\leq 0 \\
 -0.169847328 \times H_5 - P_5 + 98.8 &\leq 0 \\
 1.158415842 \times H_6 - P_6 - 46.88118818 &\leq 0 \\
 0.151162791 \times H_6 + P_6 - 130.6976744 &\leq 0 \\
 -0.067681895 \times H_6 - P_6 + 45.07614213 &\leq 0
 \end{aligned} \tag{16}$$

Test system 2:

$$\begin{aligned}
 1.781914894 \times H_{14} - P_{14} - 105.7446809 &\leq 0 \\
 0.1777777784 \times H_{14} + P_{14} - 247.0 &\leq 0 \\
 -0.169847328 \times H_{14} - P_{14} + 98.8 &\leq 0 \\
 1.158415842 \times H_{15} - P_{15} - 46.88118818 &\leq 0 \\
 0.151162791 \times H_{15} + P_{15} - 130.6976744 &\leq 0 \\
 -0.067681895 \times H_{15} - P_{15} + 45.07614213 &\leq 0 \\
 1.781914894 \times H_{16} - P_{16} - 105.7446809 &\leq 0 \\
 0.1777777784 \times H_{16} + P_{16} - 247.0 &\leq 0 \\
 -0.169847328 \times H_{16} - P_{16} + 98.8 &\leq 0 \\
 1.158415842 \times H_{17} - P_{17} - 46.88118818 &\leq 0 \\
 0.151162791 \times H_{17} + P_{17} - 130.6976744 &\leq 0 \\
 -0.067681895 \times H_{17} - P_{17} + 45.07614213 &\leq 0 \\
 0.25 \times H_{18} - P_{18} + 20 &\leq 0 \\
 0.272727272 \times H_{18} + P_{18} - 60 &\leq 0 \\
 2.333333333 \times H_{18} - P_{18} - 83.33333333 &\leq 0 \\
 2.2 \times H_{19} - P_{19} - 9 &\leq 0 \\
 0.6 \times H_{19} + P_{19} - 105 &\leq 0
 \end{aligned} \tag{17}$$

Test system 3:

$$\begin{aligned}
 &1.781914894 \times H_{27} - P_{27} - 105.7446809 \leq 0 \\
 &0.1777777784 \times H_{27} + P_{27} - 247.0 \leq 0 \\
 &-0.169847328 \times H_{27} - P_{27} + 98.8 \leq 0 \\
 &1.158415842 \times H_{28} - P_{28} - 46.88118818 \leq 0 \\
 &0.151162791 \times H_{28} + P_{28} - 130.6976744 \leq 0 \\
 &-0.067681895 \times H_{28} - P_{28} + 45.07614213 \leq 0 \\
 &1.781914894 \times H_{29} - P_{29} - 105.7446809 \leq 0 \\
 &0.1777777784 \times H_{29} + P_{29} - 247.0 \leq 0 \\
 &-0.169847328 \times H_{29} - P_{29} + 98.8 \leq 0 \\
 &1.158415842 \times H_{30} - P_{30} - 46.88118818 \leq 0 \\
 &0.151162791 \times H_{30} + P_{30} - 130.6976744 \leq 0 \\
 &-0.067681895 \times H_{30} - P_{30} + 45.07614213 \leq 0 \\
 &0.25 \times H_{31} - P_{31} + 20 \leq 0 \\
 &0.272727272 \times H_{31} + P_{31} - 60 \leq 0 \\
 &2.333333333 \times H_{31} - P_{31} - 83.33333333 \leq 0 \\
 &2.2 \times H_{32} - P_{32} - 9 \leq 0 \\
 &0.6 \times H_{32} + P_{32} - 105 \leq 0 \\
 &1.781914894 \times H_{33} - P_{33} - 105.7446809 \leq 0 \\
 &0.1777777784 \times H_{33} + P_{33} - 247.0 \leq 0 \\
 &-0.169847328 \times H_{33} - P_{33} + 98.8 \leq 0 \\
 &1.158415842 \times H_{34} - P_{34} - 46.88118818 \leq 0 \\
 &0.151162791 \times H_{34} + P_{34} - 130.6976744 \leq 0 \\
 &-0.067681895 \times H_{34} - P_{34} + 45.07614213 \leq 0 \\
 &1.781914894 \times H_{35} - P_{35} - 105.7446809 \leq 0 \\
 &0.1777777784 \times H_{35} + P_{35} - 247.0 \leq 0 \\
 &-0.169847328 \times H_{35} - P_{35} + 98.8 \leq 0 \\
 &1.158415842 \times H_{36} - P_{36} - 46.88118818 \leq 0 \\
 &0.151162791 \times H_{36} + P_{36} - 130.6976744 \leq 0 \\
 &-0.067681895 \times H_{36} - P_{36} + 45.07614213 \leq 0 \\
 &0.25 \times H_{36} - P_{36} + 20 \leq 0 \\
 &0.272727272 \times H_{37} + P_{37} - 60 \leq 0 \\
 &2.333333333 \times H_{37} - P_{37} - 83.33333333 \leq 0 \\
 &2.2 \times H_{38} - P_{38} - 9 \leq 0 \\
 &0.6 \times H_{38} + P_{38} - 105 \leq 0
 \end{aligned} \tag{18}$$

3. WHALE OPTIMIZATION ALGORITHM (WOA)

The whale optimization algorithm has been proposed by Mirjalili and Lewis (Mirjalili, & Lewis, 2016). The WOA algorithm is based on special hunting behavior of humpback whales. Encircling

prey, bubble net hunting method, search for prey (exploration phase) steps of WOA algorithm which has been discussed below.

i) Encircling Prey

To hunt krills or small fishes, humpback whales can identify the position of prey and encircle them. WOA algorithm considered the target prey is the best candidate solution. In encircling prey, the tendency of other search agents try to update their position towards best search agent.

$$\begin{aligned} \vec{X}(T+1) &= \vec{X}^*(T) - \vec{K} \cdot \vec{L} \\ \vec{L} &= \left| \vec{M} \cdot \vec{X}^*(T) - \vec{X}(T) \right| \end{aligned} \quad (19)$$

Where \vec{K}, \vec{M} are co-efficient vectors, are represented by

$$\vec{K} = 2 \cdot \vec{a} \cdot \vec{r} - \vec{a} \quad (20)$$

$$\vec{M} = 2 \cdot \vec{r} \quad (21)$$

Where T indicates the current iteration; $\vec{X}(T)$ is the position vector; $\vec{X}^*(T)$ is the position of the best solution and it can be updated if better solution is obtained. \vec{L} represents the distance between the position of $\vec{X}^*(T)$ and $\vec{X}(T)$; \vec{K}, \vec{M} are co-efficient vectors \vec{K} is a random value in the interval $[-a, a]$; where \vec{K} is decreased from 2 to 0 and \vec{a} also linearly decreased from 2 to 0. Here, \vec{K} position is setting down at random values in between $[-1, 1]$; \vec{r} is a random vector $[0, 1]$.

ii) Bubble net hunting method.

In Bubble net hunting method two techniques are there, firstly in shrinking encircling, the humpback whales swim around the prey and make a particular bubble along a circle. This behaviour is achieved by decreasing the value of \vec{a} .

In shrinking, encircling prey behaviour (\vec{a}) is represent by

$$\vec{a} = 2 - t \frac{2}{Maxiter} \quad (22)$$

Where t is the iteration number and $Maxiter$ is the max number of allowed iteration. \vec{K} is also decreased by \vec{a} . The update position of \vec{K} is obtained between original position and position of the current best agent.

The other one spiral position updating is then created between the position of the whale and prey to mimic the helix-shaped movement of humpback whales as given below

$$\vec{X}(T + 1) = \vec{L} \cdot e^{bn} \cos(2\pi n) + \vec{X}^*(T) \quad (23)$$

Where $\vec{L} = \left| \vec{X}^*(T) - X(T) \right|$ and indicates the distance of the i^{th} whale to best solution prey; b

is constant for defining the shape of the logarithmic spiral and n is random number in $[-1, 1]$.

During optimization the position of the whales which are swimming around the prey, it is to be assumed for simultaneous behaviour of whales that there is a probability of 50% to choose between either the shrinking encircling mechanism or the spiral model to update the position of the whales

$$\vec{X}(T + 1) = \text{Shrinking encircling, if } P < 0.5$$

$$\vec{X}(T + 1) = \text{Spiral encircling, if } P > 0.5$$

In above equation P is a random number $[0, 1]$ during the optimization process.

iii) Search for prey (Exploration phase)

Here a random search agent is selected to guide the search instead of updating the position of the search agents. So, the vector \vec{L} can be utilized with the random values greater than 1 or less than -1, is used to force search agent to move far away from the best search agent. This mechanism can be mathematically expressed by the following equation.

$$\vec{L} = \left| \vec{C} \cdot \vec{X}_{rand} - \vec{X} \right| \quad (24)$$

$$\vec{X}(T + 1) = \vec{X}_{rand} - \vec{K} \vec{L}$$

4. ALGORITHM OF WOA APPLIED TO COMBINED HEAT AND POWER ECONOMIC DISPATCH(CHPED):

The main steps of proposed whale optimization algorithm (WOA) approach applied to CHPED problem are discussed as follows.

- Step 1. Randomly Initialize active power of power only units and cogeneration units, heat of cogeneration units and heat only units within their maximum and minimum operating limits. Also, specify the input parameters of WOA technique.
- Step 2. Check the feasibility of heat and power of the cogeneration units so that the combined heat and power units are operated in a bounded heat versus power plane. If infeasible solution is reached, the solution is replaced by generating new feasible solution.
- Step 3. For actual operation, the best secure solution is achieved by avoiding the power generation in prohibited operating zone. Each prohibited operating zone is divided into two subzones, namely, left and right prohibited subzones. When a unit operates in one of its prohibited zones, the unit to move either towards the upper limit of that zone from the right sub-zone or towards the lower limit of that zone from the left sub-zone.
- Step 4. Evaluate the power generation and heat generation of slack power unit and slack heat unit, respectively and check whether these values are between maximum and minimum operating limit or not. Moreover, it is checked whether the power generation of slack unit satisfies the prohibited operating zone constraints or not. The infeasible solutions are replaced by newly generated feasible solution set.
- Step 5. Evaluate the fitness value of each feasible solution set. Thereafter, sort the fitness values in increasing order among the generated population.
- Step 6. In order to prevent the best solutions, few solutions are kept as elite solutions.
- Step 7. To modify the independent variables apply encircling prey, bubble net hunting method, search for prey(exploration phase) steps of WOA algorithm on non-elite solutions. In WOA algorithm the current best candidate solution is considered as the target prey. The other search agents try to updates their positions towards best search agent using the aforesaid approaches.
- Step 8. Check whether the independent variables of CHPED problem are within operating limits or not. The independent variable is made equal to minimum value if it less than minimum value and made equal to maximum value if it greater then maximum value.
- Step 9. Check the feasibility of slack units. If not satisfied then solution is replaced by currently generated best feasible solution. Duplicate solutions are replaced by a newly generated solution set.
- Step 10. If stopping iteration is satisfied, go to final optimal solution.
- Step 11. Otherwise, go to step7.

5. SIMULATION RESULTS AND DISCUSSION

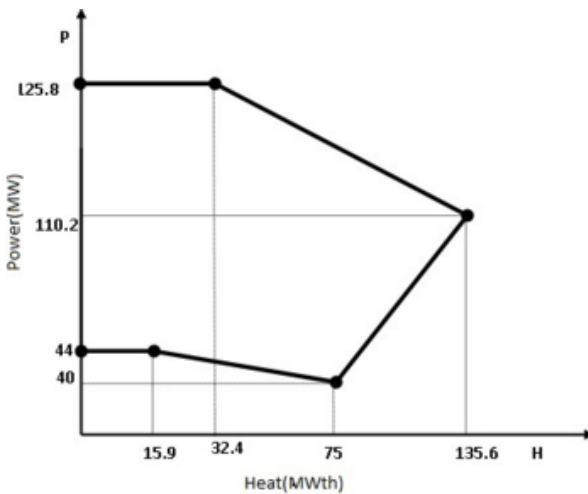
Three test systems are taken in this simulation study, to check the effectiveness and efficiency of the proposed WOA algorithms for CHPED problem. To show the superiority of the proposed algorithm the results from the WOA algorithm are compared with CPSO (Mohammadi-Ivatloo et al., 2013), TVAC-PSO (Mohammadi-Ivatloo et al., 2013), TLBO (Roy, 2013), GSO (Basu, 2016). In MATLAB 7.8, the program is written and executed on a computer having 2.5 GHZ core i5 processor with 4 GB RAM. For test systems 1,2 and 3, the feasible operating region of different CHP units and simulation results of the proposed algorithm for the three test systems are presented below. The best part of this method is that it does not have any algorithm-specific control parameter for its operation, only population size and maximum iteration is defined for its functionality. The proposed WOA algorithm is run for 50 population size and 100 iterations for each case with prohibited zone and without prohibited zone.

i) Test System 1

The study system is composed of four power only units, two CHP units and one heat –only unit. In Fig.1 and Fig.2, the feasible operating regions of two cogeneration units are shown. Power and heat demands are taken as 600 MW and 150MWth, respectively. The proposed WOA algorithm's convergence graph for prohibited zone is depicted in Fig.3. Simulation results are given in Table 1

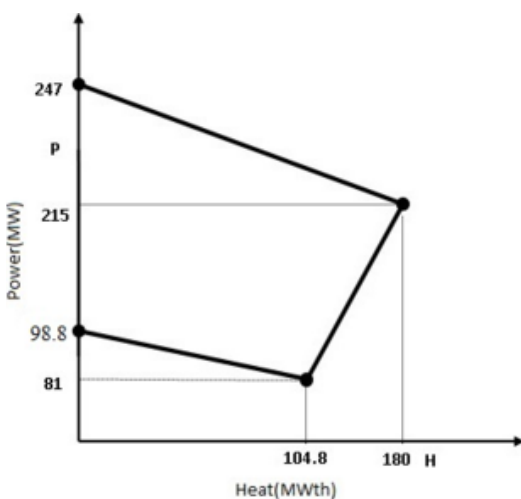
and Table 2. As seen from Table 1 (without prohibited zone) that the minimum cost is found to be 10094.2091\$/hr using WOA. This clearly suggests that the cost achieved by WOA is much less than that obtained by the other techniques. Moreover, the computation time required for WOA is 2.3216 sec, which means that the proposed WOA approach is computationally most efficient as time requirement of WOA algorithm is min among all the algorithms. Again from Table 2(with prohibited zone) it is seen that WOA algorithm is more economical and computationally much faster than other algorithms.

Figure 1.



Feasible operation region of cogeneration unit units (6th unit for case I, 15th and 17th unit for case II)

Figure 2.



Feasible operation region of cogeneration units (5th unit for case I, (14th and 16th units for case II)

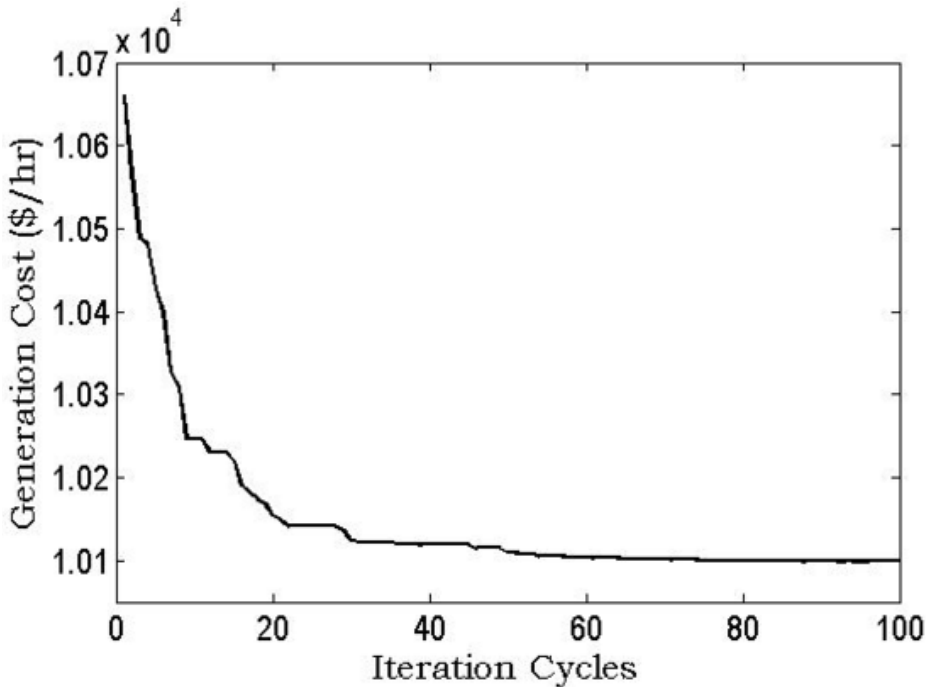
Table 1. Simulation results achieved by various approaches of test system 1 (without prohibited zone)

Control Variables	CPSO]	TVAC-PSO	TLBO	GSO	WOA
P ₁ (MW)	75.0000	47.3383	45.2660	45.6188	45.6072
P ₂ (MW)	112.3800	98.5398	98.5479	98.5401	98.5398
P ₃ (MW)	30.0000	112.6735	112.6786	112.6727	112.6735
P ₄ (MW)	250.0000	209.81582	209.8284	209.8154	209.8158
P ₅ (MW)	93.2701	92.3718	94.4121	94.1027	94.1021
P ₆ (MW)	40.1585	40.0000	40.0062	40.0001	40.0001
H ₅ (MWth)	32.5655	37.8467	25.8365	27.6600	27.6596
H ₆ (MWth)	72.6738	74.9999	74.9970	74.9987	75.0000
H ₇ (MWth)	44.7606	37.1532	49.1666	47.3413	47.3404
Statistical analysis					
Minimum cost (\$/hr)	10325.33	10100.32	10094.83	10094.2318	10094.2091
Mean cost (\$/hr)	-	-	10114.15	10095.6615	10094.8214
Maximum cost (\$/hr)	-	-	10133.61	10097.2406	10095.9102
Computational time (Sec)	-	-	2.86	2.4203	2.3216

Table 2. Simulation results achieved by various approaches of test system 1 (with prohibited zone)

Control Variables	GSO	WOA
P ₁ (MW)	44.1443	44.3065
P ₂ (MW)	100.0023	98.5373
P ₃ (MW)	112.6752	112.6795
P ₄ (MW)	209.8148	209.8155
P ₅ (MW)	94.1126	95.3859
P ₆ (MW)	40.0004	40.0137
H ₅ (MWth)	27.6002	20.1011
H ₆ (MWth)	74.9975	75.0117
H ₇ (MWth)	47.4023	54.8872
Cost (\$/hr.)		
Statistical analysis		
Minimum cost (\$/hr)	10101.3483	10098.4554
Mean cost (\$/hr)	10102.2168	10099.8627
Maximum cost (\$/hr)	10103.7203	10100.7893
Computational time (Sec)	2.5903	2.3715

Figure 3.



Cost convergence profile of WOA for Test system 1 with prohibited operating zone

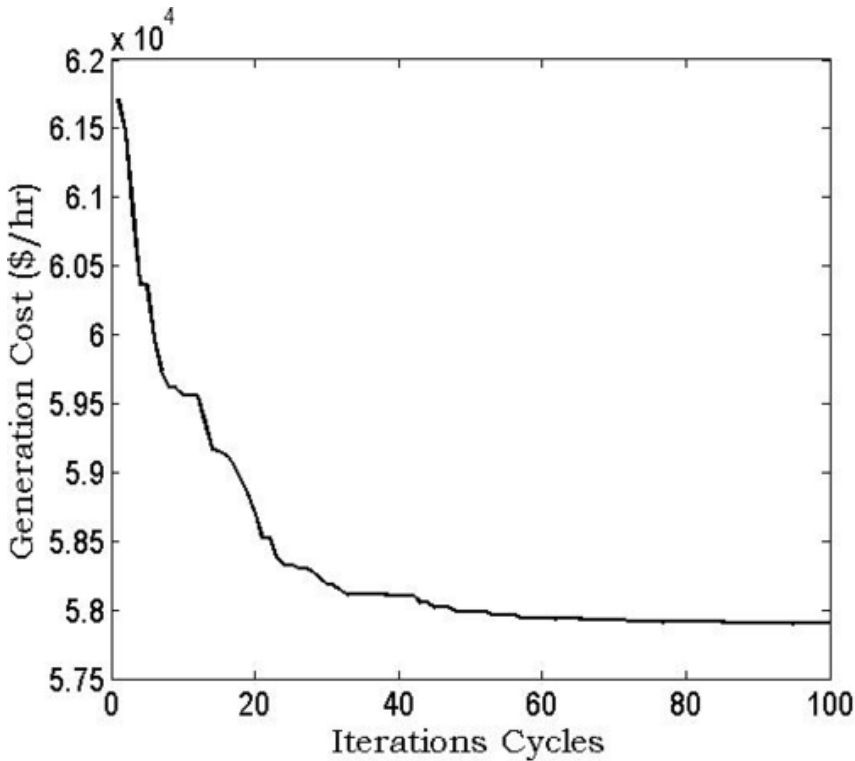
In order to judge the superiority of the proposed WOA method, its results are compared with the results obtained using GSO (Basu, 2016). After 100 iteration runs, computed generation costs obtained for all these iterations are displayed in convergence graph of Fig .3. Where after 60 iterations the solution reaches within 0.02% range best solution cost of prohibited zone. These results indicate clearly stability of the solutions given by proposed WOA method. So, it is clear that the proposed method WOA reported the global optimal solution.

ii) Test System 2

The presented algorithm is applied to a system with thirteen power only units, six CHP units and five heat only units. The total power and heat demand for the system is set to 2350 MW and 1250MWth. The convergence profile of the cost function from proposed WOA algorithms for the system without prohibited zone and with prohibited zone is shown in Fig.4 and Fig.5. The feasible operating regions of cogeneration units are shown in Fig.1, Fig.2, Fig.6 and Fig.7. For the system without prohibited zone, the mean, minimum and maximum cost and computation time from convergence graph for 100 iterations of presented algorithm and other algorithms CPSO (Mohammadi-Ivatloo et al., 2013), TVAC-PSO(Mohammadi-Ivatloo et al., 2013), TLBO(Roy, 2013), GSO (Basu, 2016) are shown in Table 3. For the system with prohibited zone, the mean, minimum and maximum cost and computation time from convergence graph for 100 iterations of presented algorithm and with GSO (Basu, 2016) are shown in Table 4. From the comparative results of Table 3 and Table 4, it is evident that WOA out performs all other methods in terms of achieving successfully the best minimum cost obtained by the proposed WOA method. It is given by 57898.6023\$/hr for without prohibited zone and 57997.0697 \$/hr for prohibited zone. After that the computation time for without prohibited zone is 5.2865 sec

and for prohibited zone is 5.3214 sec. The analyses of these comparative results demonstrate that the proposed approach shows superior performance compared to other methods reported in the literature.

Figure 4.



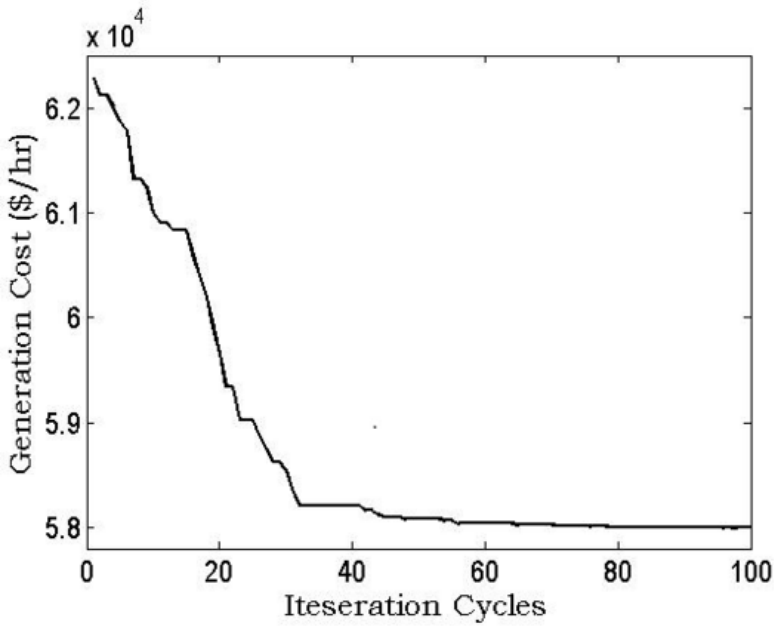
Cost convergence profile of WOA for Test system 2 without prohibited operating zone

iii) Test System 3

In this section a test system consists of twenty-six power only units, twelve CHP units and ten heat only units, where systems without prohibited and with prohibited zones are considered for power only units are used to demonstrate the performance of the proposed method. The system power and heat demands are 4700 MW and 2500 MWth respectively. Over 100 repeated trials, the mean cost achieved by WOA in without prohibited operating zone was 116242.3856 \$/hr with a minimum cost of 116239.7747 \$/hr and maximum cost 116247.0892 \$/hr and for with prohibited operating zone the mean cost was 116534.9214 \$/hr with a minimum cost of 116530.6922 \$/hr and maximum cost 116538.4239 \$/hr. The computational time of WOA for the case without prohibited zone is 8.54 sec and 8.93 sec with prohibited operating zone which is entirely reasonable for solving CHPED problem.

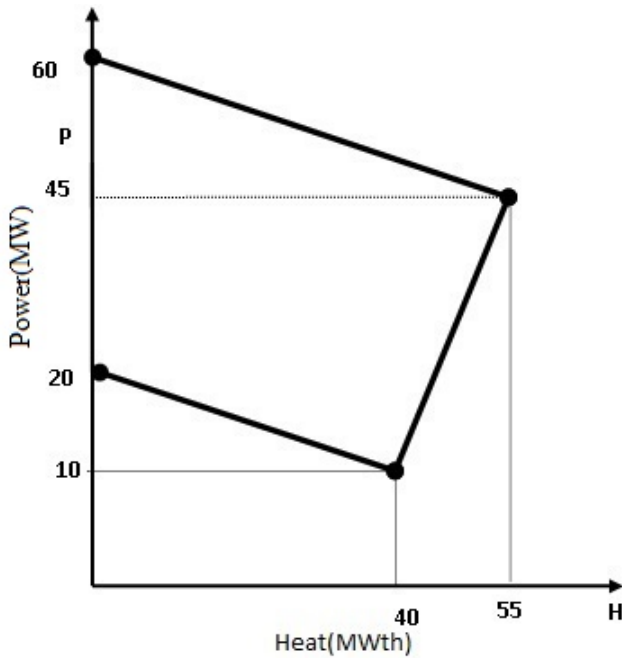
Moreover, for the 100 trials run, all generators output are within permissible limits. The best, worst and average optimization results found by WOA for systems without prohibited zone were compared in Table 5, with the results reported using GSO(Basu, 2016), TLBO(Roy, 2013), PSO-TVAC (Mohammadi-Ivatloo et al., 2013), CPSO (Mohammadi-Ivatloo et al., 2013) and for prohibited zone compared with GSO (Basu, 2016) in Table 6 . From the comparative results of Table 5 and Table 6, it

Figure 5.



Cost convergence profile of WOA for Test system 2 with prohibited operating zone.

Figure 6.

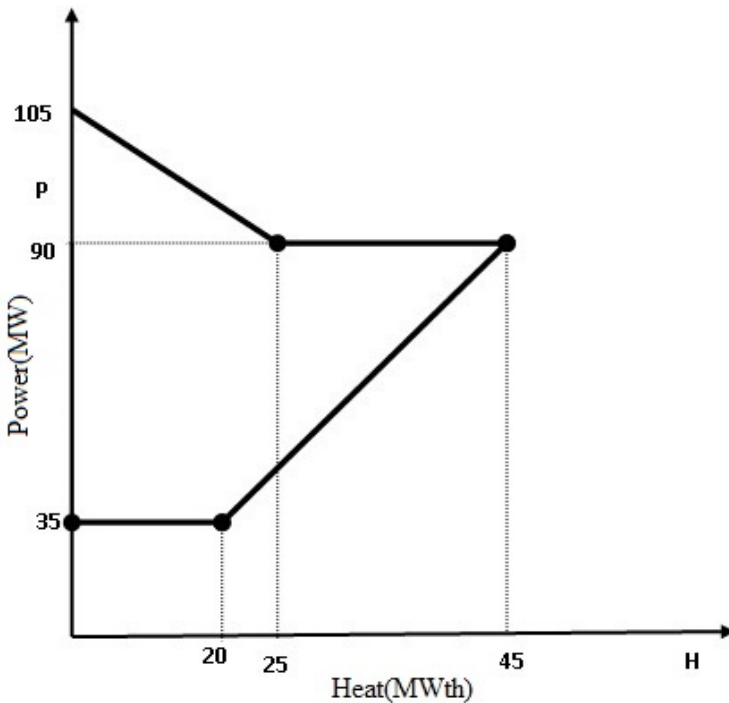


Feasible operation region of cogeneration units (18th units for case II).

is proved that WOA is best among the other optimization technique in terms of achieving successfully the best minimum cost. To clearly distinguish all three test system results with and without prohibited zone are highlighted in Table 7. The computed generation costs of the 100 trial runs are displayed in Fig. 8 and Fig.9. These figures show that the proposed method has generated satisfactory solutions, which lie close to the best solution cost for without prohibited zone and prohibited zone. These results indicate clearly stability of the solutions given by proposed WOA method.

To clearly distinguish all three test system results with and without prohibited zone are highlighted in Table 7. The obtained generation cost of test system 1, for prohibited zone is 10098.4554 \$/hr and for without prohibited zone 10094.2091 \$/hr. Further the study has been extended with 24 units, where generation cost of test system 2, for prohibited zone is 57997.0697 \$/hr and for without prohibited zone 57898.6023 \$/hr. Finally, the proposed system implemented on several number of unit combination with 48 units to judge the effectiveness of WOA algorithm, where the obtained generation cost of test system 3, for prohibited zone is 116530.6922 \$/hr and for without prohibited zone 116239.7747 \$/hr.

Figure 7.



Feasible operation region of cogeneration units (19th units for case II)

Table 3. Simulation results achieved by various approaches of test system 2 (without prohibited zone)

Control Variables	Algorithms					Control Variables	Algorithms				
	CPSO	TVAC-PSO]	TLBO]	GSO	WOA		CPSO	TVAC-PSO	TLBO	GSO	WOA
P ₁ (MW)	680.0000	538.5587	628.3240	538.2192	538.5587	P ₁₆ (MW)	117.4854	88.3514	84.7710	81.2620	81.4314
P ₂ (MW)	0.0000	224.4608	227.3588	298.7686	299.2377	P ₁₇ (MW)	45.9281	40.5611	40.5874	40.0119	40.6510
P ₃ (MW)	0.0000	224.4608	225.9347	298.9086	224.9947	P ₁₈ (MW)	10.0013	10.0245	10.0010	10.0011	10.0000
P ₄ (MW)	180.0000	109.8666	110.3721	110.1919	109.8687	P ₁₉ (MW)	42.1109	40.4288	31.0978	35.0012	30.0000
P ₅ (MW)	180.0000	109.8666	110.2461	110.0846	109.8684	H ₁₄ (MWth)	125.2754	108.9256	105.6717	105.2110	105.4388
P ₆ (MW)	180.0000	109.8666	160.1761	110.1390	109.8684	H ₁₅ (MWth)	80.1175	75.4844	76.2843	76.5306	78.5139
P ₇ (MW)	180.0000	109.8666	108.3552	110.1045	109.8741	H ₁₆ (MWth)	125.2754	108.9256	106.9125	105.5119	105.0421
P ₈ (MW)	180.0000	109.8666	110.5379	110.2444	109.8724	H ₁₇ (MWth)	80.1174	75.484	75.5061	75.4706	75.5620
P ₉ (MW)	180.0000	109.8666	110.5672	110.1992	109.8767	H ₁₈ (MWth)	40.0005	40.0104	39.9986	39.9999	40.0000
P ₁₀ (MW)	50.5304	77.5210	75.7562	77.4989	77.4495	H ₁₉ (MWth)	23.2322	22.4676	18.2205	18.4014	17.7273
P ₁₁ (MW)	50.5304	77.5210	41.8698	77.7367	77.4027	H ₂₀ (MWth)	415.9815	458.7020	468.2278	468.9029	467.7172
P ₁₂ (MW)	55.0000	120.0000	92.4789	55.1036	92.4311	H ₂₁ (MWth)	60.0000	60.0000	59.9867	59.9995	60.0000
P ₁₃ (MW)	55.0000	120.0000	57.5140	55.1107	92.4056	H ₂₂ (MWth)	60.0000	60.0000	59.9814	59.9999	60.0000
P ₁₄ (MW)	117.4854	88.3514	82.5628	81.0624	82.1383	H ₂₃ (MWth)	120.0000	120.0000	119.6074	119.9856	119.9994
P ₁₅ (MW)	45.9281	40.5611	41.4891	40.3515	44.0706	H ₂₄ (MWth)	120.0000	120.0000	119.6030	119.9867	119.9993
Statistical analysis											
	CPSO			TVAC-PSO		TLBO		GSO#		WOA	
Minimum cost (\$/hr)	59736.2635			58122.7460		58006.9992		57843.5191		57898.6023	
Mean cost (\$/hr)	59853.4780			58198.3106		58014.3685		57849.3017		57900.2137	
Maximum cost (\$/hr)	60076.6903			58359.5520		58038.5273		57857.7938		57903.4420	
Computational time (Sec)	8.00			7.84		5.67		5.4106		5.2865	

Found infeasible Solution

Table 4. Simulation results achieved by various approaches of test system 2 (with prohibited zone)

Control Variables	Algorithms		Control Variables	Algorithms	
	GSO	WOA		GSO	WOA
P ₁ (MW)	628.3274	628.3185	P ₁₆ (MW)	86.1718	81.1391
P ₂ (MW)	299.2273	224.0183	P ₁₇ (MW)	44.9263	44.1953
P ₃ (MW)	0.0015	149.4003	P ₁₈ (MW)	10.0039	10.0016
P ₄ (MW)	60.0000	159.6179	P ₁₉ (MW)	36.4003	32.2199
P ₅ (MW)	159.7333	159.7363	H ₁₄ (MWth)	113.2357	104.8069
P ₆ (MW)	159.7334	109.4761	H ₁₅ (MWth)	83.9529	78.4535
P ₇ (MW)	159.7373	60.0157	H ₁₆ (MWth)	107.7004	104.8775
P ₈ (MW)	60.0025	109.5923	H ₁₇ (MWth)	79.2794	78.6214
P ₉ (MW)	159.7361	159.3956	H ₁₈ (MWth)	40.0006	39.9996
P ₁₀ (MW)	77.4139	77.2000	H ₁₉ (MWth)	20.6318	18.7361
P ₁₁ (MW)	114.8064	40.0084	H ₂₀ (MWth)	445.2006	465.0446
P ₁₂ (MW)	55.0003	91.1838	H ₂₁ (MWth)	60.0000	59.9996
P ₁₃ (MW)	92.4065	89.4657	H ₂₂ (MWth)	59.9988	59.9975
P ₁₄ (MW)	96.0351	81.0138	H ₂₃ (MWth)	120.0000	119.7252
P ₁₅ (MW)	50.3365	44.0014	H ₂₄ (MWth)	119.9999	119.7381
Statistical analysis					
		GSO			WOA
Minimum cost (\$/hr)		58110.0900		57997.0697	
Mean cost (\$/hr)		58114.6060		57999.3115	
Maximum cost (\$/hr)		58119.1635		58003.5016	
Computational time (Sec)		5.8017		5.3214	

Figure 8.

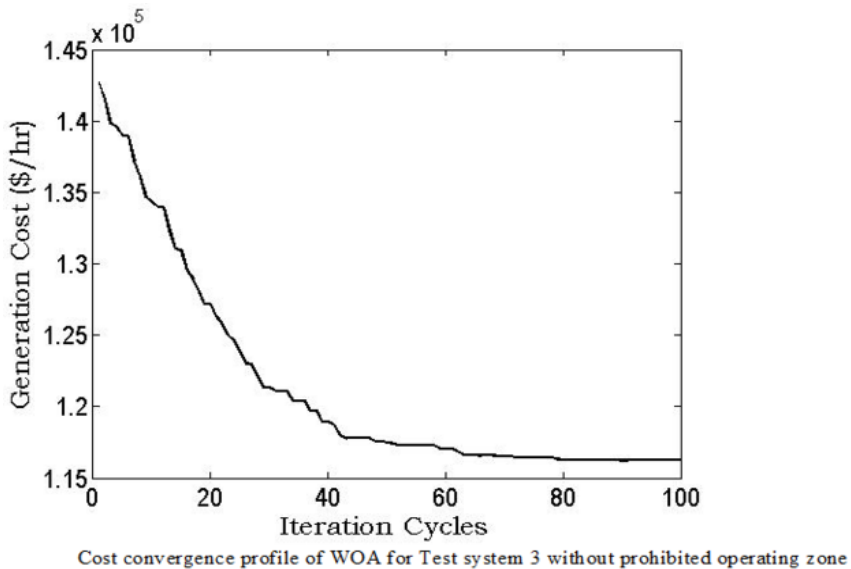
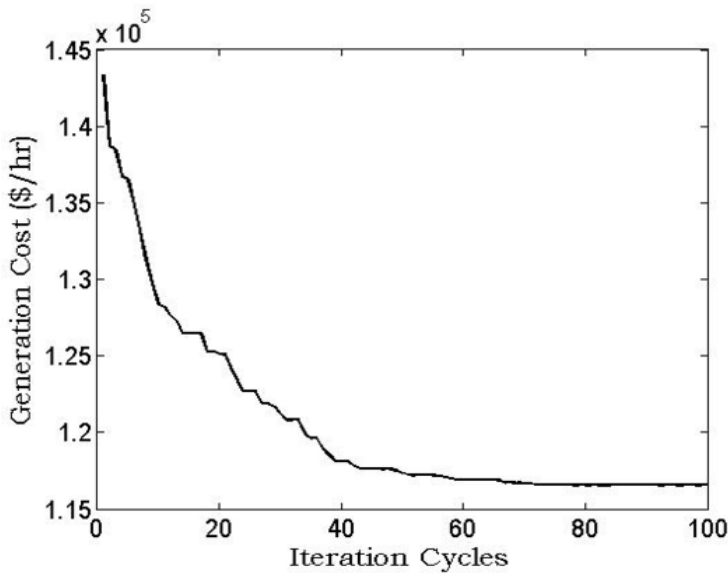


Figure 9.



Cost convergence profile of WOA for Test system 3 with prohibited operating zone

6. CONCLUSION

In this presentation, optimal dispatch framework is demonstrated for the combined heat and power economic dispatch problem. For the first time a new efficient evolutionary based whale optimization algorithm has been successfully implemented to solve CHPED problem with various constraints. Numerical simulation is done on three testing systems in order to observe the efficiency and feasibility of WOA algorithm. The hunting behaviour of the whales of proposed algorithm enhances the optimization performances. To judge the performances of proposed algorithm, firstly it has been tested on 7 units system for both prohibited and without prohibited zone. The obtained generation cost of test system 1, for prohibited zone is 10098.4554 \$/hr and for without prohibited zone the cost is 10094.2091 \$/hr. Further the study has been extended with 24 units, where generation cost of test system 2, for prohibited zone is 57997.0697 \$/hr and for without prohibited zone the cost is 57898.6023 \$/hr. Finally, the proposed system is implemented on several type of unit combination with 48 units to judge the effectiveness of WOA algorithm. Here the obtained generation cost of test system 3, for prohibited zone is 116530.6922 \$/hr and for without prohibited zone the cost is 116239.7747 \$/hr. The results obtained by the proposed WOA method for these systems have been compared with other settled methods reported in the literature. The simulation results show as the proposed method succeeded in achieving the goal of reducing generation costs. These features of WOA, presented in this paper evidently corroborate this method as an appropriate tool which can be used to address the acceptable solutions of many practical power system problems in future.

Table 5. Simulation results achieved by various approaches of test system 3 (without prohibited zone)

Control Variables	Algorithms					Control Variables	Algorithms				
	CPSO	TVAC-PSO	TLBO	GSO	WOA		CPSO	TVAC-PSO	TLBO	GSO	WOA
P ₁ (MW)	359.0392	538.5587	538.5693	448.9126	538.5654	P ₃₁ (MW)	10.0002	10.0031	10.5480	10.0212	11.1687
P ₂ (MW)	74.5831	75.1340	225.3021	150.5151	299.6639	P ₃₂ (MW)	56.7153	35.0000	52.7180	37.7288	42.7073
P ₃ (MW)	74.5831	75.1340	229.9473	80.7660	224.9998	P ₃₃ (MW)	109.1877	95.4799	82.1522	92.0380	81.1562
P ₄ (MW)	139.3803	140.6146	159.1352	160.0923	109.9632	P ₃₄ (MW)	65.6006	54.9235	52.0606	50.4524	50.6704
P ₅ (MW)	139.3803	140.6146	160.0561	109.9592	109.7700	P ₃₅ (MW)	109.18	95.4799	82.7394	95.2834	81.5446
P ₆ (MW)	139.3803	140.6146	109.7821	159.8520	159.7417	P ₃₆ (MW)	65.6006	54.9235	45.7398	52.3657	53.6799
P ₇ (MW)	139.3803	140.6146	159.6609	160.1104	109.8777	P ₃₇ (MW)	10.6158	23.4981	10.0075	10.0683	11.7770
P ₈ (MW)	139.3803	140.6146	159.6492	159.8453	159.7935	P ₃₈ (MW)	60.5994	54.0882	30.0332	45.7741	45.2178
P ₉ (MW)	139.3803	140.6146	109.9660	160.0219	159.7709	H ₂₇ (MWth)	111.4458	108.1177	105.0678	109.8046	104.8286
P ₁₀ (MW)	74.7998	112.1998	40.3726	114.9957	40.5504	H ₂₈ (MWth)	125.6898	88.9006	78.9162	83.3599	84.7591
P ₁₁ (MW)	74.7998	112.1998	77.5821	115.1906	78.3533	H ₂₉ (MWth)	111.4458	108.1177	104.8270	104.961	104.8958
P ₁₂ (MW)	74.7998	74.7999	92.2489	92.6482	55.0864	H ₃₀ (MWth)	125.6898	88.9006	119.6006	80.8014	86.8420
P ₁₃ (MW)	74.7998	74.7999	55.1755	55.0420	92.4059	H ₃₁ (MWth)	40.0001	40.0013	40.2345	39.9976	40.5006
P ₁₄ (MW)	679.8810	269.2794	448.6854	269.4783	538.575	H ₃₂ (MWth)	29.8706	20.0000	28.0508	21.2295	23.5031
P ₁₅ (MW)	148.6585	299.1993	149.4238	299.4636	224.5933	H ₃₃ (MWth)	120.6188	112.9260	105.4339	110.9901	104.8813
P ₁₆ (MW)	148.6585	299.1993	224.7173	299.7175	76.4069	H ₃₄ (MWth)	97.0997	87.8827	85.40864	84.0301	84.2087
P ₁₇ (MW)	139.0809	140.3973	109.9355	159.9635	109.9693	H ₃₅ (MWth)	120.6188	112.9260	105.7694	112.7913	105.1043
P ₁₈ (MW)	139.0809	140.3973	159.9052	159.8998	160.1869	H ₃₆ (MWth)	97.0997	87.8827	79.9447	85.6985	86.8075
P ₁₉ (MW)	139.0809	140.3973	159.7255	159.7568	109.9289	H ₃₇ (MWth)	40.2639	45.7849	40.0001	40.0211	40.7615
P ₂₀ (MW)	139.0809	140.3973	159.7820	60.0218	159.9116	H ₃₈ (MWth)	31.6361	28.6765	17.7401	24.8763	24.6434
P ₂₁ (MW)	139.0809	140.3973	60.0777	160.0075	109.9527	H ₃₉ (MWth)	357.9456	433.9113	394.6160	458.7095	445.3680
P ₂₂ (MW)	139.0809	140.3973	110.0689	159.9142	160.3647	H ₄₀ (MWth)	59.9916	60.0000	59.9300	59.9975	59.9927
P ₂₃ (MW)	74.7998	74.7998	77.6818	114.9146	78.0628	H ₄₁ (MWth)	59.9916	60.0000	59.9578	60.0000	59.9955
P ₂₄ (MW)	74.7998	74.7998	40.2707	40.1116	40.0001	H ₄₂ (MWth)	120.0000	120.0000	118.5797	119.9632	119.6935
P ₂₅ (MW)	112.1993	112.1997	92.4108	93.8700	92.4613	H ₄₃ (MWth)	120.0000	120.0000	118.3425	119.9990	119.7236
P ₂₆ (MW)	112.1993	112.1997	55.0956	93.6315	55.8619	H ₄₄ (MWth)	370.6214	415.9741	480.6566	422.7929	443.9983
P ₂₇ (MW)	92.8423	86.9119	81.4882	89.9223	81.0572	H ₄₅ (MWth)	59.9999	60.0000	59.9346	59.9792	59.9954
P ₂₈ (MW)	98.7199	56.1027	44.5478	49.6516	51.3086	H ₄₆ (MWth)	59.9999	60.0000	59.9810	59.9974	59.9930

continued on next page

Table 5. Continued

Control Variables	Algorithms					Control Variables	Algorithms				
	CPSO	TVAC-PSO	TLBO	GSO	WOA		CPSO	TVAC-PSO	TLBO	GSO	WOA
P ₂₉ (MW)	92.8423	86.9119	81.0560	81.2954	81.1713	H ₂₇ (MWth)	119.9856	119.9989	117.8207	120.0000	119.8226
P ₃₀ (MW)	98.7199	56.1027	91.6819	46.6966	53.7235	H ₂₈ (MWth)	119.9856	119.9989	119.1898	120.0000	119.6815
Statistical analysis											
	CPSO			TVAC-PSO		TLBO	GSO		WOA		
Minimum cost (\$/hr)	119708.8818			117824.8956		116739.3640	116457.9578		116239.7747		
Mean cost (\$/hr)	-			-		116756.0057	116463.6522		116242.3856		
Maximum cost (\$/hr)	-			-		116825.8223	116473.2183		116247.0892		
Computational time (Sec)	-			-		10.38	9.51269		8.54		

Table 6. Simulation results achieved by various approaches of test system 3 (with prohibited zone)

Control Variables	Algorithms		Control Variables	Algorithms	
	GSO	WOA		GSO	WOA
P ₁ (MW)	179.8745	538.5612	P ₃₁ (MW)	10.1191	10.5036
P ₂ (MW)	360.0000	147.6867	P ₃₂ (MW)	35.1879	31.0917
P ₃ (MW)	150.7185	148.9331	P ₃₃ (MW)	96.8952	81.4073
P ₄ (MW)	60.0000	163.1960	P ₃₄ (MW)	44.8817	44.7380
P ₅ (MW)	60.0648	110.2650	P ₃₅ (MW)	86.3425	81.1155
P ₆ (MW)	159.8784	61.3060	P ₃₆ (MW)	44.8670	41.7898
P ₇ (MW)	160.3713	110.4222	P ₃₇ (MW)	10.0624	12.9038
P ₈ (MW)	177.5771	160.7780	P ₃₈ (MW)	35.1607	56.5551
P ₉ (MW)	120.0893	111.2300	H ₂₇ (MWth)	112.5046	104.7206
P ₁₀ (MW)	115.373	115.0342	H ₂₈ (MWth)	78.4728	88.6995
P ₁₁ (MW)	114.9535	43.5281	H ₂₉ (MWth)	112.6499	105.5462
P ₁₂ (MW)	94.5954	92.9978	H ₃₀ (MWth)	79.0427	76.8772
P ₁₃ (MW)	55.1880	93.2851	H ₃₁ (MWth)	40.0200	40.2151
P ₁₄ (MW)	628.9382	628.2326	H ₃₂ (MWth)	20.0605	18.2231
P ₁₅ (MW)	360.0000	225.2170	H ₃₃ (MWth)	113.5695	105.0157
P ₁₆ (MW)	299.4804	153.2682	H ₃₄ (MWth)	79.1803	79.0794
P ₁₇ (MW)	122.0289	110.9468	H ₃₅ (MWth)	107.6768	104.8589
P ₁₈ (MW)	110.0491	160.3005	H ₃₆ (MWth)	79.2272	76.5449
P ₁₉ (MW)	60.0000	160.5245	H ₃₇ (MWth)	40.0098	41.2405
P ₂₀ (MW)	87.0872	110.2795	H ₃₈ (MWth)	19.9447	29.7924
P ₂₁ (MW)	159.9218	159.8047	H ₃₉ (MWth)	435.2939	441.0491
P ₂₂ (MW)	60.0079	159.5464	H ₄₀ (MWth)	59.9635	59.9910

continued on next page

Table 6. Continued

Control Variable s	Algorithms		Control Variables	Algorithms	
	GSO	WOA		GSO	WOA
P ₂₃ (MW)	120.0000	49.6233	H ₄₁ (MWth)	59.9930	59.9719
P ₂₄ (MW)	40.0000	77.3790	H ₄₂ (MWth)	119.9950	119.3908
P ₂₅ (MW)	108.1572	93.3557	H ₄₃ (MWth)	119.8985	118.9276
P ₂₆ (MW)	93.5594	92.7388	H ₄₄ (MWth)	462.5644	472.1942
P ₂₇ (MW)	94.7653	81.0200	H ₄₅ (MWth)	59.9983	59.9941
P ₂₈ (MW)	44.0478	55.8776	H ₄₆ (MWth)	59.9987	59.9697
P ₂₉ (MW)	95.0884	82.3471	H ₄₇ (MWth)	120.0000	118.5364
P ₃₀ (MW)	44.6682	42.2101	H ₄₈ (MWth)	119.9359	119.1617
Statistical analysis					
		GSO			WOA
Minimum cost (\$/hr)		117098.4186		116530.6922	
Mean cost (\$/hr)		117103.0283		116534.9214	
Maximum cost (\$/hr)		117109.9737		116538.4239	
Computational time (Sec)		10.9758		8.93	

Table 7. Simulation results of all three test systems for with and without prohibited zone

Test systems	Minimum cost (\$/hr)		
	7 units system	24 units system	48 units system
Without Prohibited	10094.2091	57898.6023	116239.7747
With Prohibited	10098.4554	57997.0697	116530.6922

7. DISCUSSIONS ON FUTURE RESEARCH WORK

Some advanced algorithms may be applied to the proposed area in CHPED system to improve the system performances under the different non-linearity condition in future. In the present analysis, authors have considered three test systems to identify the effectiveness and robustness of the proposed algorithm. But, in actual practice power system is always experiences different uncertainties due to interconnection of renewable sources with the grid. So in future, we will make a scheduling of CHPED with such type of renewable sources to judge the superiority of the test areas of given power system.

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