


Solving the Trapezoidal Fuzzy Transportation Problems via New Heuristic: The Dhouib-Matrix-TP1

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ABSTRACT

The transportation problem is a one of the principal topics in operational research where goods are initially stored at different sources and need to be livered to destinations in such a way that the total transportation cost is minimum. In this paper, the authors consider the transportation problem in a trapezoidal fuzzy environment and they introduce the column-row heuristic Dhouib-Matrix-TP1 to solve it in just p iterations (where p is the maximal number between the total number of sources and destinations). The Dhouib-Matrix-TP1 heuristic is enhanced with the robust ranking function and with a new operation for selection based on mean and min metrics. To justify the proposed method, several numerical experiments are given to show the effectiveness of the new technique in solving the trapezoidal fuzzy transportation problems.

KEYWORDS

Combinatorial Optimization, Fuzzy Systems, Heuristic, Operational Research, Robust Ranking Function, Transportation Problems, Trapezoidal Fuzzy Numbers

1. INTRODUCTION

The Transportation Problem (TP) plays an important role in many industrial real-life applications with the aim of ensuring the shipment of supplies from planned sources to specific destinations via minimal transportation costs.

The TP is widely studied in the literature. It was firstly designed by Hitchcock (1941). Dantzig (1951) solved the TP using the Simplex method. Dinagar and Palanivel (2009) studied the TP in trapezoidal fuzzy domain. Pandian and Natarajan (2010) designed the Fuzzy Zero Point method to optimize the fuzzy TP. Kaur and Kumar (2012) developed a new method to find an initial basic solution for TP where the transportation costs are denoted by generalized fuzzy numbers. Shanmugasundari and Ganesan (2013) introduced a fuzzy version of Vogel's and MODI methods in order to generate basic initial solution for TP with imprecise variables described by triangular fuzzy numbers. Beaulaa and Priyadharsini (2015) introduced an iterative method to find the optimal solution for the TP in trapezoidal intuitionistic fuzzy environment using the Stepping Stone method with indices based ranking methods. Ahmed et al. (2016) designed a new method entitled Allocation Table Method (ATM) to find an initial basic feasible solution for the TP and its efficiency is tested by solving several TP. Muruganandam and Srinivasan (2016) investigated a new heuristic with the graded means

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ranking function to optimize the fully trapezoidal fuzzy TP. Hunwisai and Kumam (2016) solved the trapezoidal fuzzy TP using the ATM and the Modified Distribution methods to find the optimal crisp solution. Moreover, Kumar (2016) developed the PSK technique for the TP under triangular and trapezoidal fuzzy numbers.

Uthra et al. (2017) proposed a new ranking measure to transform the costs in TP from trapezoidal intuitionistic fuzzy numbers to crisp ones. Furthermore, Kumar (2018) presented two techniques: the first one is based on Linear Programming technique and the second is based on the Distribution Method to optimize the balanced and unbalanced intuitionistic fuzzy TP. Kumar (2019) presented a mixed and fully triangular intuitionistic fuzzy solid TP and solved it using the PSK method with graphical representation of the found triangular intuitionistic fuzzy optimal solution. Ngastiti et al (2020) solved a fully trapezoidal fuzzy TP using the Zero Point and the Zero Suffix methods; then, the two methods are compared based on the value of the basic feasible solution and the number of iterations.

Moreover, Gargouri and Bouamama (2020) presented a rich literature review for the TP and its variants (multi-objective optimization, multimodal, optimization methods, case study, etc.). Mhaske and Bondar (2020) optimized triangular, pentagonal and heptagonal fuzzy transportation problems using the North West Corner method, the Matrix Minima method and the Vogel's approximation method; furthermore, they introduced a new function for nonagon and hendecagon fuzzy transportation cost numbers. Li et al. (2021) introduced a case study in a bike sharing system as a smart transportation system and then optimized it using the Deep Reinforcement Learning technique. Sikkannan (2021) solved the triangular fuzzy TP using an original heuristic based on the standard deviation metric and the magnitude ranking function.

Thus, there are different methods to solve the TP and very recently Dhouib (2021a) designs a new heuristic entitled Dhouib-Matrix-TP1 (DM-TP1) to solve the TP with crisp parameters in just p iterations (where p is the maximal number between the total number of sources and destinations). In this paper, the DM-TP1 heuristic is adapted to solve the trapezoidal fuzzy TP by using the robust ranking function developed by Yager (1981) and adding a new operation to select cities based on average and min metrics.

In the next following section, the proposed method for solving the trapezoidal fuzzy transportation problem is discussed. Section 2 includes some basic concepts about the generalized trapezoidal fuzzy set theory. Section 3 presents the mathematical formulation of the transportation problem. Section 4 describes in details the proposed DM-TP1 heuristic under fuzzy environment for the transportation problem. Section 5 gives several numerical examples to prove the effectiveness of the proposed heuristic. Finally, the conclusion and some perspectives are provided in the last section.

2. THE TRAPEZOIDAL FUZZY NUMBERS

The generalized trapezoidal fuzzy number \tilde{F} is denoted by $\tilde{F}(f_1, f_2, f_3, f_4; w)$, where f_1 and f_4 are respectively the left and right widths of \tilde{F} .

A fuzzy number \tilde{F} has a mapping $\mu_{\tilde{F}}(x) : \mathbb{R} \rightarrow [0,1]$ with the following proprieties:

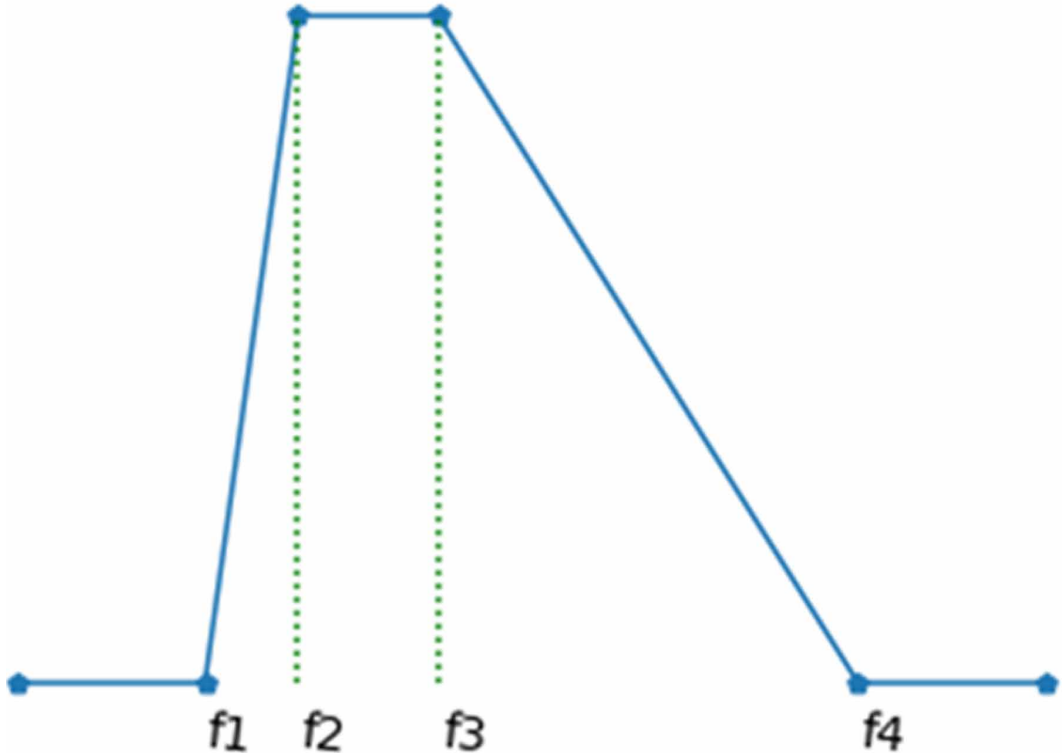
- $\mu_{\tilde{F}}(x) = 0$ outside the interval $[f_1, f_4]$
- $\mu_{\tilde{F}}(x) = w$ in $[f_2, f_3]$
- $\mu_{\tilde{F}}(x)$ is a monotonic increasing function on $[f_1, f_2]$
- $\mu_{\tilde{F}}(x)$ is a monotonic decreasing function on $[f_3, f_4]$

The membership function $\mu_{\tilde{F}}(x)$ for a trapezoidal fuzzy number \tilde{F} is defined by:

$$\mu_{\tilde{F}}(x) = \begin{cases} w \frac{(x - f_1)}{(f_2 - f_1)}, & \text{if } f_1 \leq x \leq f_2, \\ w, & \text{if } f_2 \leq x \leq f_3, \\ w \frac{(f_4 - x)}{(f_4 - f_3)}, & \text{if } f_3 \leq x \leq f_4, \\ 0, & \text{otherwise.} \end{cases}$$

Figure 1. Trapezoidal membership function defined by $\tilde{F}(20, 30, 45, 90; 1)$

Trapezoidal Fuzzy Numbers



Consequently, we can define the order between any two generalized fuzzy trapezoidal numbers $\tilde{F}(f_1, f_2, f_3, f_4; w)$ and $\tilde{E}(e_1, e_2, e_3, e_4; w)$ by:

- $\tilde{F} \prec \tilde{E} \Leftrightarrow R(\tilde{F}) \prec R(\tilde{E})$
- $\tilde{F} \succ \tilde{E} \Leftrightarrow R(\tilde{F}) \succ R(\tilde{E})$
- $\tilde{F} \approx \tilde{E} \Leftrightarrow R(\tilde{F}) \approx R(\tilde{E})$
- $\tilde{F} = -\tilde{E} \Leftrightarrow R(\tilde{F}) + R(\tilde{E}) = 0$

Furthermore, the arithmetic operation between \tilde{F} and \tilde{E} are given below:

- $\tilde{F} \oplus \tilde{E} = (f_1 + e_1, f_2 + e_2, f_3 + e_3, f_4 + e_4; w = \min\{w_1, w_2\})$
- $\tilde{F} \ominus \tilde{E} = (f_1 - e_4, f_2 - e_3, f_3 - e_2, f_4 - e_1; w = \min\{w_1, w_2\})$
- $\lambda \geq 0, \lambda \tilde{F} = (\lambda f_1, \lambda f_2, \lambda f_3, \lambda f_4; w)$
- $\lambda \leq 0, \lambda \tilde{F} = (\lambda f_4, \lambda f_3, \lambda f_2, \lambda f_1; w)$

3. MATHEMATICAL FORMULATION FOR THE FUZZY TRANSPORTATION PROBLEM

Let us suppose an industrial company with the purpose of minimizing the total transportation cost of products from designed source i to affected destination j in uncertain environment. The uncertainty of the decision maker is mathematically presented as a fuzzy number in the travelling cost \tilde{c}_{ij} ($\tilde{c}_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij})$ for the trapezoidal form).

Here, the objective is to find the optimal value of x_{ij} that will minimize the total transportation cost (see Eq. 1) while satisfying the supply and demand restrictions (see Eq. 2). The fuzzy TP type-1 is mathematically formulated as follows by:

$$\text{Minimize: } z = \sum_{i=1}^m \sum_{j=1}^n \tilde{c}_{ij} x_{ij} \quad (1)$$

Subject to:

$$\begin{aligned} \sum_{j=1}^n x_{ij} &= a_i ; i = 1, 2, \dots, m \\ \sum_{i=1}^m x_{ij} &= b_j ; j = 1, 2, \dots, n \\ x_{ij} &\geq 0 \text{ for all } i \text{ and } j \end{aligned} \quad (2)$$

The notation of the TP is:

- m total number of supplies (sources)
- n total number of demands (destinations)
- a_i Amount of supply at source i
- b_j Amount of demand at destination j
- \tilde{c}_{ij} Trapezoidal fuzzy transportation cost from supply i to demand j
- x_{ij} Amount to be shipped from source i to destination j

4. THE MODIFIED DHOUIB-MATRIX-TP1 METHOD: DM-TP1

This section describes the proposed method, DM-TP1, to solve the TP in trapezoidal fuzzy environment. In fact, Dhoub (2021a) designs the DM-TP1 heuristic to solve the TP with crisp parameters. Whereas, in this paper, we enrich the DM-TP1 method with two techniques: At first, a

new operation for selection based on average and min metrics is used and at second, the application of the robust ranking technique for the representative value of the trapezoidal fuzzy number is applied.

Very recently, we design and develop a new concept namely Dhoub-Matrix to solve combinatorial problems. This concept is based on constructive methods using descriptive statistical metrics to generate the optimal or a near optimal solution. For the travelling salesman problems, a deterministic heuristic named Dhoub-Matrix-TSP1 (DM-TSP1) is introduced by Dhoub (2021b) followed by a stochastic version entitled Dhoub-Matrix-TSP2 (DM-TSP2) proposed in Dhoub (2021c). Then, a comparative study between DM-TSP1 and DM-TSP2 is presented in Dhoub (2021d). Moreover, to solve very large instances of the travelling salesman problems, a novel metaheuristic namely Dhoub-Matrix-TSP4 is illustrated in Dhoub (2021e). Furthermore, the DM-TSP1 heuristic is modified to solve the travelling salesman problems in fuzzy environments in Dhoub (2021f), Dhoub and Dhoub (2021) and Miledi et al. (2021).

In this paper, we modify the DM-TP1 heuristic to solve the trapezoidal fuzzy TP by dividing it into two parts: The first part deals with constructing the entire crisp travelling cost matrix, where the second part deals with finding the optimal allocation. In this work, the robust ranking technique from Yager (1981) is used to convert the trapezoidal fuzzy numbers into crisp ones by the following formula:

$$R(\tilde{a}) = \int_0^1 (0.5)(\alpha_\alpha^L, \alpha_\alpha^U) d\alpha$$

Where:

$$(\alpha_\alpha^L, \alpha_\alpha^U) = [\{(b-a)\alpha + a, d - (d-c)\alpha\}, \{(f-e)\alpha + e, h - (h-g)\alpha\}]$$

The proposed DM-TP1 heuristic generates the optimal or a near optimal solution just with p iterations (where p is the maximum between sources and destinations: $p = \max(m, n)$).

Our DM-TP1 method is composed only of 9 steps:

- Step 1: use the robust ranking function to convert the trapezoidal fuzzy numbers to crisp numbers
- Step 2: balance the transport matrix.
- Step 3: compute the difference between the average and the min for each row and place it at the last column entitled: Average Min Supply Row (AMSR).
- Step 4: apply the same operation on each column. Compute the difference between the average and the min for each column and place these difference values at the last row entitled: Average Min Demand Column (AMDC).
- Step 5: identify the highest element among the AMSR and the AMDC, if it belongs to SDR elements then select the minimal element (x_{ij}) of its corresponding row else check the minimal element (x_{ij}) of its corresponding column.
- Step 6: if $a_i \leq b_j$ then allocate the a_i number of units to the x_{ij} , affect $b'_j = b_j - a_i$ and discard the row i .
- Step 7: if $a_i > b_j$ then allocate the b_j number of units to the x_{ij} , affect $a'_i = a_i - b_j$ and discard the column j .
- Step 8: repeat step 5-6-7 until all columns are discarded.

Step 9: calculate the minimal transportation cost $z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}$.

5. COMPUTATIONAL RESULTS

In this section, some examples are provided to illustrate the potential application of the proposed DM-TP1 heuristic.

5.1 Example 1

Let us consider a Trapezoidal fuzzy transportation problem (Figure 2) proposed by Ngastiti et al. (2020). Our method DM-TP1, solves this TP in just 5 iterations (the maximum between 4 sources and 5 destinations). Whereas, Ngastiti et al. (2020) solved this TP by the zero-point and the zero-suffix methods respectively by 6 and 7 iterations.

Figure 2. Fully trapezoidal fuzzy TP

Source	Destination					Inventory
	1	2	3	4	5	
1	(5,7,8,11)	(1,6,7,12)	(2,4,5,7)	(2,5,7,9)	(7,9,10,12)	(20,35,45,60)
2	(5,8,9,12)	(2,5,7,9)	(1,6,7,12)	(5,7,8,11)	(5,8,9,12)	(15,25,35,45)
3	(1,6,7,12)	(5,8,9,12)	(7,9,10,12)	(1,6,7,12)	(2,5,7,9)	(10,15,25,30)
4	(2,5,7,9)	(5,7,8,11)	(5,7,8,11)	(5,8,9,12)	(1,6,7,12)	(5,8,12,15)
Demand	(15,25,25,45)	(15,25,35,45)	(8,14,16,22)	(10,15,25,30)	(2,4,6,8)	

The first iteration starts by converting the trapezoidal fuzzy numbers to crisp numbers using the robust ranking function:

$$R(\tilde{a}) = \int_0^1 (0.5)(\alpha_\alpha^L, \alpha_\alpha^U) d\alpha$$

Where:

$$(\alpha_\alpha^L, \alpha_\alpha^U) = [\{(b - a)\alpha + a, d - (d - c)\alpha\}, \{(f - e)\alpha + e, h - (h - g)\alpha\}]$$

Therefore,

$$\begin{aligned} R(\tilde{c}_{11}) &= R(5, 7, 8, 11) \\ &= \int_0^1 (0.5)(-\alpha + 16) d\alpha = 7.75 \end{aligned}$$

Similarly, all the trapezoidal fuzzy numbers are converted to crisp numbers and the AMSR and the AMDC indicators are computed. Now, find the maximum between AMSR and AMDC (2.56) and select the minimal element of its corresponding column which is 4.5 at position d_{13} (see Figure 3). Thus, affect to x_{13} a value of 15 and then discard column 3.

Next, compute the AMSR and the AMDC indicators, find the maximum between them (1.88) and select the minimal element of its corresponding row which is 5.75 at position d_{22} (see Figure 4). Then, affect to x_{22} a value of 30 and discard row2 and column 2.

Figure 3. Compute the AMSR and AMDC indicators

Source	Destination					Inventory	AMSR
	1	2	3	4	5		
1	7.75	6.5	4.5	5.75	9.5	40	2.30
2	8.5	5.75	6.5	7.75	8.5	30	1.65
3	6.5	8.5	9.5	6.5	5.75	20	1.60
4	5.75	7.75	7.75	8.5	6.5	10	1.50
Demand	30	30	15	20	5		
AMDC	1.38	1.38	2.56	1.38	1.81		

Figure 4. Discard column 3 and select the highest element between AMSR and AMDC

Source	Destination					Inventory	AMSR
	1	2	3	4	5		
1	7.75	6.5		5.75	9.5	25	1.63
2	8.5	5.75		7.75	8.5	30	1.88
3	6.5	8.5		6.5	5.75	20	1.06
4	5.75	7.75		8.5	6.5	10	1.38
Demand	30	30		20	5		
AMDC	1.38	1.38		1.38	1.81		

Hence, calculate the AMSR and the AMDC indicators, find the maximum between them (1.92) and find the minimal element of its corresponding row which is 5.75 at position d_{14} (see Figure 5). Then, affect to x_{14} a value of 20 and discard column 4.

Now, compute the AMSR and the AMDC indicators, find the maximum between them (1.50) and find the minimal element of its corresponding column which is 5.75 at the position d_{35} (see Figure 6). Then, affect to x_{35} a value of 5 and discard column 5.

Finally, affect to x_{11} a value of 5, affect to x_{31} a value of 15, affect to x_{41} a value of 10 and discard column 1.

So that, the minimal total cost is:

Figure 5. Discard row 2 and column 2

Source	Destination					Inventory	AMSR
	1	2	3	4	5		
1	7.75			5.75	9.5	25	1.92
2							
3	6.5			6.5	5.75	20	0.50
4	5.75			8.5	6.5	10	1.17
Demand	30			20	5		
AMDC	0.92			1.17	1.50		

Figure 6. Discard column 4 and select the highest element between AMSR and AMDC

Source	Destination					Inventory	AMSR
	1	2	3	4	5		
1	7.75				9.5	5	0.88
2							
3	6.5				5.75	20	0.38
4	5.75				6.5	10	0.38
Demand	30				5		
AMDC	0.92				1.50		

$$\begin{aligned}
 Z &= \sum_i^m \sum_j^n c_{ij} x_{ij} \\
 &= (c_{13}x_{13}) + (c_{22}x_{22}) + (c_{14}x_{14}) + (c_{35}x_{35}) + (c_{11}x_{11}) + (c_{31}x_{31}) + (c_{41}x_{41}) \\
 &= (4,5*15) + (5,75*30) + (5,75*20) + (5,75*5) + (7,75*5) + (6,5*15) + (5,75*10) = 577.5
 \end{aligned}$$

The found crisp optimal solution is 577,5 with $x_{11}=5$, $x_{13}=15$, $x_{14}=20$, $x_{22}=30$, $x_{31}=15$, $x_{35}=5$, $x_{41}=10$.

To compare DM-TP1 to the methods proposed by Ngastiti et al. (2020), two criteria are used the value of the basic feasible solution and the number of iterations. All the methods find the optimal solution 577.5 with different numbers of iterations. In fact, the Zero Point method needs 7 iterations,

Figure 7. Discard column 1

Source	Destination					Inventory	AMSR
	1	2	3	4	5		
1	7.75					5	0
2							
3	6.5					15	0
4	5.75					10	0
Demand	30						
AMDC	0.92						

the Zero suffix needs 6 iterations and our method DM-TP1 needs only 5 iterations (which corresponds to the maximum between supplies and demands).

5.2 Example 2

A second example from Muruganandam and Srinivasan (2016) is used to test the performance of the modified DM-TP1 (see Figure 8). Our method DM-TP1 heuristic solves this problem in just 4 iterations (the maximum between 3 sources and 4 destinations).

Figure 8. Fully trapezoidal fuzzy TP

Source	Destination				Inventory
	1	2	3	4	
1	(1,2,3,4)	(1,3,4,6)	(9,11,12,14)	(5,7,8,11)	(1,6,7,12)
2	(0,1,2,4)	(-1,0,1,2)	(5,6,7,8)	(0,1,2,3)	(0,1,2,3)
3	(3,5,6,8)	(5,8,9,12)	(12,15,16,19)	(7,9,10,12)	(5,10,12,17)
Demand	(5,7,8,10)	(1,5,6,10)	(1,3,4,6)	(1,2,3,4)	

At first, the trapezoidal fuzzy numbers are converted to crisp numbers using the robust ranking function and the AMSR and AMDC indicators are computed. Then, find the maximum element between AMSR and the AMDC (4.75) and select the minimal element which is 1.5 at position d_{24} (see Figure 9). Thus, affect to x_{24} a value of 1.5 and then discard the row 2.

Next, calculate the AMSR and the AMDC indicators, find the maximum between them (4.25) and find the corresponding minimal element which is 5.5 at the position d_{31} (see Figure 10). Then, affect to x_{31} a value of 7.5 and then discard column 1.

Figure 9. Compute the AMSR and AMDC indicators

Source	Destination				Inventory	AMSR
	1	2	3	4		
1	2.5	3.5	11.5	7.75	6.5	3.81
2	1.75	0.5	6.5	1.5	1.5	2.06
3	5.5	8.5	15.5	9.5	11	4.25
Demand	7.5	5.5	3.5	2.5		
AMDC	1.50	3.67	4.67	4.75		

Figure 10. Discard row 2 and select the highest element between AMSR and AMDC

Source	Destination				Inventory	AMSR
	1	2	3	4		
1	2.5	3.5	11.5	7.75	6.5	3.81
2						
3	5.5	8.5	15.5	9.5	11	4.25
Demand	7.5	5.5	3.5	1		
AMDC	1.50	2.50	2.00	0.88		

Moreover, compute the AMSR and the AMDC indicators, find the maximum between them (4.08) and find minimal element of its corresponding row which is 3.5 at the position d_{12} (see Figure 11). Then, affect to x_{12} a value of 5.5 and then discard column 2.

Hence, compute the AMSR and AMDC indicators, find the maximum between them (3.00) and find the minimal element of its corresponding row which is 9.5 at the position d_{34} (see Figure 12). Then, affect to x_{34} a value of 1 and then discard column 4.

Finally, affect to x_{13} a value of 1, affect to x_{33} a value of 2.5 (see Figure 13) and discard column 3. So that, the minimal total cost is:

$$\begin{aligned}
 Z &= \sum_i^m \sum_j^n c_{ij} x_{ij} \\
 &= (c_{24} x_{24}) + (c_{31} x_{31}) + (c_{12} x_{12}) + (c_{34} x_{34}) + (c_{13} x_{13}) + (c_{33} x_{33}) \\
 &= (1.5 * 1.5) + (5.5 * 7.5) + (3.5 * 5.5) + (9.5 * 1) + (11.5 * 1) + (15.5 * 2.5) = 122.5
 \end{aligned}$$

Figure 11. Discard column 1 and select the highest element between AMSR and AMDC

Source	Destination				Inventory	AMSR
	1	2	3	4		
1		3.5	11.5	7.75	6.5	4.08
2						
3		8.5	15.5	9.5	3.5	3.50
Demand		5.5	3.5	1		
AMDC		2.50	2.00	0.88		

Figure 12. Discard column 2 and select the highest element between AMSR and AMDC

Source	Destination				Inventory	AMSR
	1	2	3	4		
1			11.5	7.75	1	1.88
2						
3			15.5	9.5	3.5	3.00
Demand			3.5	1		
AMDC			2.00	0.88		

Moreover, the minimal total cost found by DM-TP1 is 122.5 with $x_{24}=1.5, x_{31}=7.5, x_{12}=5.5, x_{34}=1, x_{13}=1, x_{33}=2.5$. The result found by our method DM-TP1 presents an improvement of 3.67% compared to the optimal solution found by Muruganandam and Srinivasan (2016) which is 127 (see Figure 14).

5.3 Example 3

Let us consider the 3x3 mixt fuzzy transportation problem (see Figure 15) proposed by Kumar (2016). This problem describes the transportation of umbrella products from three different factories to three different retail stores with mixture numbers (crisp and fuzzy).

At first, all the mixture parameters are converted to trapezoidal fuzzy numbers, then, to crisp numbers. Figure 16, depicts the stepwise application of the DM-TP1 heuristic to find the optimal solution (176) in just 3 iterations. The generated solution is same as the optimal solution obtained by Kumar (2016) using the PSK method (which is same as solutions found by VAM, MODI, Zero Point Methods).

Figure 13. Discard column 4

Source	Destination				Inventory	AMSR
	1	2	3	4		
1			11.5		1	0
2						
3			15.5		2.5	0
Demand			3.5			
AMDC			2.00			

Figure 14. Comparison chart

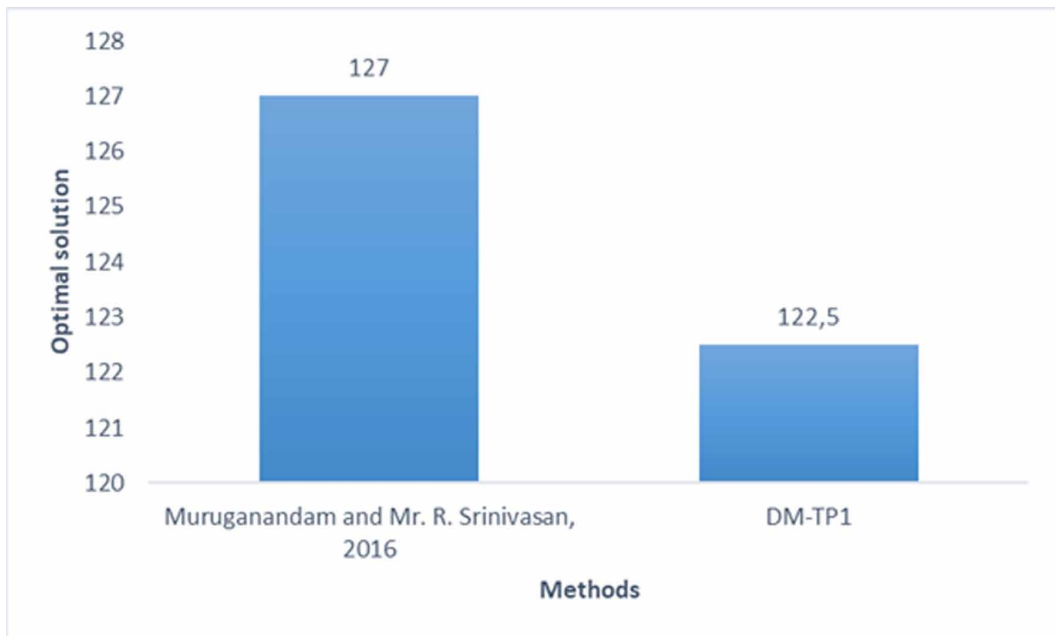


Figure 15. Mixt transport cost matrix

Source	Destination			Inventory
	1	2	3	
1	6	(1,2,3)	(5,7,9,11)	10
2	(1,2,3)	(4,8,16,20)	10	(10,16,24,30)
3	(1,3,5,7)	3	(6,8,10)	(8,16,24)
Demand	(11,22,30,41)	(5,8,8,11)	12	

Figure 16. The application of DM-TP1

a)

Source	Destination			Inventory
	1	2	3	
1	(6,6,6,6)	(1,2,2,3)	(5,7,9,11)	(10,10,10,10)
2	(1,2,2,3)	(4,8,16,20)	(10,10,10,10)	(10,16,24,30)
3	(1,3,5,7)	(3,3,3,3)	(6,8,8,10)	(8,16,16,24)
Demand	(11,22,30,41)	(5,8,8,11)	(12,12,12,12)	

b)

Source	Destination			Inventory	AMSR
	1	2	3		
1	6	2	8	10	3.33
2	2	12	10	20	6.00
3	4	3	8	16	2.00
Demand	26	8	12		
AMDC	2.00	3.67	0.67		

c)

Source	Destination			Inventory	AMSR
	1	2	3		
1	6	2	8	10	3.33
2					
3	4	3	8	16	2.00
Demand	6	8	12		
AMDC	1.00	0.50	0.00		

d)

Source	Destination			Inventory	AMSR
	1	2	3		
1	6			2	1.00
2					
3	4			16	2.00
Demand	6			12	
AMDC	1.00			0.00	

e)

Source	Destination			Inventory	AMSR
	1	2	3		
1				2	1.00
2					
3			10	2.00	
Demand			12		
AMDC			0.00		

$$z = (2*20)+(2*8)+(4*6)+(8*2)+(8*10)=176$$

6. CONCLUSION

The fuzzy theory set can be used to solve many real-life problems like the transportation problems. In this paper, the trapezoidal fuzzy transportation problems are explored using a new column-row heuristic namely the Dhouib-Matrix-TP1 with the robust ranking function for the representative value of the trapezoidal fuzzy numbers. The novel heuristic Dhouib-Matrix-TP1 is a new way to handle the uncertainty in the transportation problems with just p iterations (where p is the maximal number between the total number of sources and destinations) to find the optimal or a near optimal solution. In the future, the proposed heuristic Dhouib-Matrix-TP1 will be applied on real-world transportation problems in the intuitionistic and neutrosophic environments.

REFERENCES

- Ahmed, M. M., Khan, R. A., Uddin, S. M., & Ahmed, F. (2016). A new approach to solve transportation problems. *Open Journal of Optimization*, 5(01), 22–30. doi:10.4236/ojop.2016.51003
- Beaulaa, T., & Priyadharsini, M. (2015). Fuzzy Transportation Problem with the Value and Ambiguity Indices of Trapezoidal Intuitionistic Fuzzy Numbers. *Malaya Journal of Matematik*, 2, 427–437.
- Dantzig, G.B. (1951). Application of the simplex method to a transportation problem. *Activity Analysis of Production and Allocation*, 359-373.
- Dhouib, S. (2021a). A Novel Heuristic for the Transportation Problem: Dhouib-Matrix-TP1. *International Journal of Recent Engineering Science*, 8(4), 1–5.
- Dhouib, S. (2021b). A New Column-Row Method for Traveling Salesman Problem: The Dhouib-Matrix-TSP1. *International Journal of Recent Engineering Science*, 8(1), 6–10.
- Dhouib, S. (2021c). Stochastic Column-Row Method for Travelling Salesman Problem: The Dhouib-Matrix-TSP2. *International Journal of Engineering Research & Technology (Ahmedabad)*, 10(3), 524–527.
- Dhouib, S. (2021d). Minimizing the Total Distance for the Supply Chain Problem Using Dhouib-Matrix-TSP2 Method. *International Journal of Advanced Research in Engineering and Technology*, 12(5), 1–12.
- Dhouib, S. (2021e). *Multi-Start Constructive Method Through Descriptive Statistics Metrics to Solve Travelling Salesman Problem: The Dhouib-Matrix-TSP4-A Metaheuristic*. Submitted.
- Dhouib, S. (2021f). Haar Dhouib-Matrix-TSP1 Method to Solve Triangular Fuzzy Travelling Salesman Problem. *Research Journal of Recent Sciences*, 10(3), 1–3.
- Dhouib, Sa., & Dhouib, S. (2021). Optimizing the Trapezoidal Fuzzy Travelling Salesman Problem Through Dhouib-Matrix-TSP1 Method Based on Magnitude Technique. *International Journal of Scientific Research in Mathematical and Statistical Sciences*, 8(2), 1–4.
- Dinagar, D. S., & Palanivel, K. (2009). The transportation problem in fuzzy environment. *International Journal of Algorithms Computing and Mathematics*, 2, 65–71.
- Gargouri, M. M., & Bouamama, S. (2020). Multi-Objective Optimization Methods for Transportation Network Problems: Definition, Taxonomy, and Annotation. *International Journal of Operations Research and Information Systems*, 11(1), 1–36. doi:10.4018/IJORIS.2020010101
- Hitchcock, F. L. (1941). The distribution of a product several sources to numerous localities. *Journal of Mathematics and Physics*, 20(1-4), 224–230. doi:10.1002/sapm1941201224
- Hunwisai, D., & Kumam, P. (2017). A method for solving a fuzzy transportation problem via Robust ranking technique and ATM. *Cogent Mathematics*, 4(1), 1–11. doi:10.1080/23311835.2017.1283730
- Kaur, A., & Kumar, A. (2012). A new approach for solving fuzzy transportation problems using generalized trapezoidal fuzzy numbers. *Applied Soft Computing*, 12(3), 1201–1213. doi:10.1016/j.asoc.2011.10.014
- Kumar, S. P. (2016). PSK Method for Solving Type-1 and Type-3 Fuzzy Transportation Problems. *International Journal of Fuzzy System Applications*, 5(4), 121–146. doi:10.4018/IJFSA.2016100106
- Kumar, S. P. (2018). Linear Programming Approach for Solving Balanced and Unbalanced Intuitionistic Fuzzy Transportation Problems. *International Journal of Operations Research and Information Systems*, 9(2), 1–38. doi:10.4018/IJORIS.2018040104
- Kumar, S. P. (2019). PSK Method for Solving Mixed and Type-4 Intuitionistic Fuzzy Solid Transportation Problems. *International Journal of Operations Research and Information Systems*, 10(2), 20–53. doi:10.4018/IJORIS.2019040102
- Li, G., Cao, N., Zhu, P., Zhang, Y., Zhang, Y., Li, L., Li, Q., & Zhang, Y. (2021). Towards Smart Transportation System: A Case Study on the Rebalancing Problem of Bike Sharing System Based on Reinforcement Learning. *Journal of Organizational and End User Computing*, 33(3), 35–49. doi:10.4018/JOEUC.20210501.oa3

- Mhaske, S. A., & Bondar, L. K. (2020). Fuzzy Transportation Problem by Using Triangular, Pentagonal and Heptagonal Fuzzy Numbers With Lagrange's Polynomial to Approximate Fuzzy Cost for Nonagon and Hendecagon. *International Journal of Fuzzy System Applications*, 9(1), 1–18. doi:10.4018/IJFSA.2020010105
- Miledi, M., Dhoub, S., & Loukil, T. (2021). Dhoub-Matrix-TSP1 Method to Optimize Octagonal Fuzzy Travelling Salesman Problem Using α -Cut Technique. *International Journal of Computer and Information Technology*, 10(3), 130–133. doi:10.24203/ijcit.v10i3.105
- Muruganandam, S., & Srinivasan, R. (2016). A New Algorithm for Solving Fuzzy Transportation Problems with Trapezoidal Fuzzy Numbers. *International Journal of Recent Trends in Engineering & Research*, 2(03), 428–437.
- Ngastiti, B. T. P., Surarso, B., & Sutimin, S. (2020). Comparison Between Zero Point and Zero Suffix Methods in Fuzzy Transportation Problems. *Jurnal Matematika MANTIK*, 6(1), 38–46. doi:10.15642/mantik.2020.6.1.38-46
- Pandian, P., & Natarajan, G. (2010). A new algorithm for finding a fuzzy optimal solution for fuzzy transportation problems. *Applied Mathematical Sciences*, 4, 79–90.
- Sikkannan, P. K., & Shanmugavel, V. (2021). Sorting Out Fuzzy Transportation Problems via ECCT and Standard Deviation. *International Journal of Operations Research and Information Systems*, 12(2), 1–14. doi:10.4018/IJORIS.20210401.oa1
- Uthra, G., Thangavelu, K., & Umamageswari, R. M. (2017). An Optimal Solution for Generalized Trapezoidal Intuitionistic Fuzzy Transportation Problem. *Advances in Fuzzy Mathematics*, 12(3), 763–770.
- Yager, R. R. (1981). A procedure for ordering fuzzy subsets of the unit interval. *Information Sciences*, 24(2), 143–161. doi:10.1016/0020-0255(81)90017-7

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