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An Imperfect Inventory Model for Multi-Warehouses Under Different Discount Policies With Shortages

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ABSTRACT

A multi-warehouse inventory model with shortage has been developed where demand is taken as deterministic. In reality, during production, if the machine works for a long time, there may be a random breakdown, and the system shifts from controlled to out of control situation; in this time production system produces defective items. The retailer offers an all-unit quantity discount on the selling price of the item who in return gets a quantity-based discount on the purchase price of the item. The motive of this model is to study a multi-warehouse model for imperfect items under a quantity-based discount, where the defectives can be screened and then can be sold in a single batch where the decision variables are set as optimal ordering quantity and optimal shortage quantity to maximize the total profit of the retailer. A solution procedure is given to find the optimal solution, and a numerical example is given to illustrate this study. Sensitivity analysis is also performed to study the effect of the changes in parameter values on the optimal solution.

KEYWORDS

Defective Items, Multi-Warehouse, Quantity Discount, Shortages

1. INTRODUCTION

Inventory is an essential part of any firm as it constitutes the majority of the investment expenditure. So, inventory management plays an important role to operate a firm in a flexible manner. There are several functions that affect a firm like customer-demand, item deterioration, manufacturing defects, inventory shortage and also quantity-based discounts offered by the supplier. Management of these function can directly affect the total profit of retailer.

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Inventory consists of raw material, partially complete and finished goods. So, for these types of inventories, there are ordering costs, holding costs and shortage costs associated with them. Purchasing cost may also involve inventory cost but most of the models assumed it as constant unless there is quantity discount involved. The quantity and time of placing the order are the two most important decisions in inventory management. These decisions will help a firm to increase total profit.

It is a common practice for a supplier to offer a discount on a large lot size. There are several reasons, why a supplier might choose to offer a retailer a discount. These reasons include the following: to increase cash flow, to decrease inventory levels of the products, to boost market share, or simply to retain retailers (Papachristos & Skouri, 2003). For retailers, ordering costs can be reduced on large quantity orders meanwhile holding costs will increase. These costs can manage through mathematical models so that retailers can earn more profit.

It is assumed that items are produced perfect during manufacturing but practically this is not possible due to some breakdown by a machine when running for a long time.(Sarkar, 2012a). So, it is necessary for the retailer to inspect imperfect items and separate imperfect and perfect items.

Furthermore, a retailer has a limited space to keep an inventory. In this situation, the retailer needs another space, named rent-warehouse, to keep inventory. Rent-warehouse has unlimited storing space with better-preserving facilities (Bhunia et al., 2014). Hence, rent-warehouse has higher holding costs than own-warehouse. As demand increases, shortages may occur. Also, imperfection of items could be the cause of shortages. (Aastha. et al.,2020, Priyanka & Pareek, 2020, Taha, 2017).

This paper deals with a company that sell items in a multi-warehouse environment with deterministic demand. Here, we assume all-units discounts, imperfect items and shortages. The purpose of this study is to optimize quantity and backorder items that maximize the total profit of the inventory. This model becomes more realistic by considering quantity discounts. This is an important feature for small retailers. Here, we use a procedure to obtain optimal solution and numerical examples are also provided to validate the results. To study the effect of parameters on the optimal solution, sensitivity analysis is also carried out.

The organization of this paper is as follows. Notation and assumption are introduced in the next section of this paper and then model formulation is defined in the following section. Based on the formulation, a solution procedure is mentioned in section 5 to find the optimal solution. In section 6, a numerical example is provided along with its optimal solution. Sensitivity analysis on different parameters is provided in section 7 and the conclusion is explained in the last section.

2. LITERATURE REVIEW

This model is based on a multi-warehouse environment with discount and shortage situations in imperfect items. A two-warehouse model was described bySett et al., (2012) with time-varying deterioration and increasing demand. A two-warehouse model with deterioration and inflation was developed by Jaggi et al., (2015) where demand was based on selling price. Jaggi et al., (2017) proposed a model with two-warehouse under deteriorating items with credit financing policy. Also, they assumed imperfect items in this model. By assuming stock dependent demand a two-warehouse model was developed by Shaikh et al., (2019) under inflation conditions. Another two-warehouse model was proposed by Mashud et al., (2020)with the assumption of the discount facility and non-instantaneous deterioration rate. Partial shortages were also there with the credit financing policy. Mittal et al., (2020) developed a two-warehouse model with the effect of human error on inventory and shortages. Priyanka & Pareek, (2020)developed a two-warehouse inventory model by including non-instantaneous deterioration with stochastic demand. They also assumed credit financing in this model.

A model was developed by Jaggi et al. (2017) with imperfect and deteriorated items under two -warehouse environment. An inventory model was developed by Dhaka et al. (2019)where items are imperfect. They assumed stock dependent demand with credit financing policy. By imposing the assumption of imperfect items another model was created by Gilotra et al. (2020). In addition, they imposed carbon emission and human errors in the model. The impact of imperfect items was described in an inventory model which was developed by Aastha. et al. (2020). They assumed shortages under a two-warehouse environment. This model was based on profit-maximizing the retailer with deterministic demand.

Mandal & Giri (2017) proposed a two-warehouse model with imperfect items where demand was based on stock. In that model, a discount was offered on quantity. An EOQ model was developed by Shah & Naik, (2018)with price-sensitive demand and discount policy. Another inventory model was generated by Aastha et al. (2020) which was based on quantity discounts under credit financing policy. In this model, authors assumed non-instantaneous deterioration with demand depends on advertisement and selling price.

This paper develops a mathematical model for a multi-warehouse environment with imperfect items, quantity discounts and shortages. The decision variables are set as optimal order quantity and backorder quantity to find the optimal solution. We assume that customers purchase more quantity when offered a discount, this in return, help suppliers to earn more profit and also hold customers. Table 1 shows some related work of this research. Quantity discount with multi-warehouse and imperfect items differ this model from others. This model demonstrates the real problem for small retailers.

Table 1. Some research works related to inventory models

Author	Produced items	Warehouses	Discount	Shortage
Papachristos & Skouri, 2003	Perfect	One	Yes	Yes
Kevin Hsu & Yu, 2009	Imperfect	One	Yes	No
Chung et al., 2009	Imperfect	Two	No	No
Cárdenas-Barrón, 2009	Perfect	One	Yes	No
Cárdenas-Barrón et al., 2010	Perfect	One	Yes	No
Jaggi et al., 2011	Imperfect	One	No	No
Sarkar, 2012	Imperfect	One	No	No
Jaggi et al., 2013	Imperfect	One	No	Yes
Jaggi et al., 2014	Perfect	Two	No	Yes
Zhou et al., 2015	Imperfect	One	Yes	No
Jaggi et al., 2015	Perfect	Two	No	Yes
Jaggi et al., 2017	Imperfect	Two	No	No
Mittal et al., 2017	Imperfect	One	No	No
Chung et al., 2018	Perfect	One	Yes	No
Jayaswal et al., 2019	Imperfect	One	No	No
Manna et al., 2020	Imperfect	One	Yes	No
Gilotra et al., 2020	Imperfect	One	No	No
Priyanka & Pareek, 2020	Perfect	Two	No	Yes
Aastha et al., 2020	Imperfect	Two	No	Yes
Aastha et al., 2020	Perfect	One	Yes	No
This model	Imperfect	Two	Yes	Yes

3. ASSUMPTIONS AND NOTATION

3.1 Notation

Table 2. Notation

$\mathbf{y}_1$	**lot size** $units\ /\ cycle$
W	limited space of OW $\text{units} / \text{cycle}$
$y_1 - W$	unlimited space in RW $units\ /\ cycle$
D	demand rate $\text{unit} / \text{year}$
j	variable cost/ unit
A	ordering cost / order
p	proportion of imperfect items in y_1
$f\left(p\right)$	p.d.f of p
s	selling price of items with good quality
v	selling price of defective items, $\text{v} < \text{j}$
b	backordering cost per unit
$h_{r,}$ho	holding cost for the items in RW and OW, respectively h_r ho
m_i	i th price breaking point, i=1, 2, 3, …, n
$u_i = u_i\left(\mu\right)$	per unit material cost
x	rate of screening
d	cost for screening
t_r	screening rate in RW
t_ω	screening rate in OW
t_1	time for RW when all the units used
t_2	time for OW when all the units used

continued on following page

Table 2. Continued

$\mathbf{y}_1$	**lot size** $units\ /\ cycle$
t_3	time period of shortages
B	backorder quantity in units
T	length of a cycle
$y_{1opt}^{\ *}$	optimal lot size
$TR\left(B,\mu\right)$	sale revenue
$TC\left(B,\mu\right)$	total cost
TP(B, μ)	total profit per cycle

3.2 Assumptions

1. A known, and constant demand rate is considered.
2. Replenishment rate is instantaneous.
3. Inventory to be sold in single batch only.
4. Proportion of defective items p follows a uniform distribution $\left[\alpha,\beta\right]$ where $\left[0 \le \alpha \le \beta \le 1\right]$ (Wee et al., 2007)
5. A fully backlogged shortage are allowed.
6. The purchase price is offered under the following, all-units quantity discount scheme:(Papachristos & Skouri, 2003)

$$u_i\left(\mu\right)=\begin{cases} u_1 \quad for\, m_1 < q \le m_2 \\ u_2 \quad for\, m_2 < q \le m_3 \\ \quad\quad . \\ \quad\quad . \\ \quad\quad . \\ u_n \quad for\, m_n < q \end{cases}$$

4. PROBLEM DEFINITION

Initially, at time $t = 0$, y_1 units come in the system. As the system has rent-warehouse too, W units are stored in own-warehouse and rest of the units are stored in rent-warehouse. But due to high holding costs in rent-warehouse, first units are consumed from rent-warehouse and then from own-warehouse. Generally, it is assumed that all the units come perfectly in the system but practically this is not possible due to improper transport, low quality of raw material. In this case, retailer must apply the screening process with the rate of x units per unit time. This paper assumes the p percent of imperfect items found in y_1 units, where p is a random variable whose p.d.f. is, $f\left(p\right)$, and its mean is $E\left(p\right)=p$. Thus, py_1 defective items and $\left(1-p\right)y_1$ non-defective items found in y_1 units. These imperfect items sold at v per unit salvage value, where $v < j$. After this screening process, the demand is fulfilled from rent warehouse by time t_1 and then upcoming demand is fulfilled from own warehouse. After t_3, shortages occur when both the warehouse is exhausted. (Figure. I)

5. MATHEMATICAL MODEL

The time horizon of total inventory is given by,

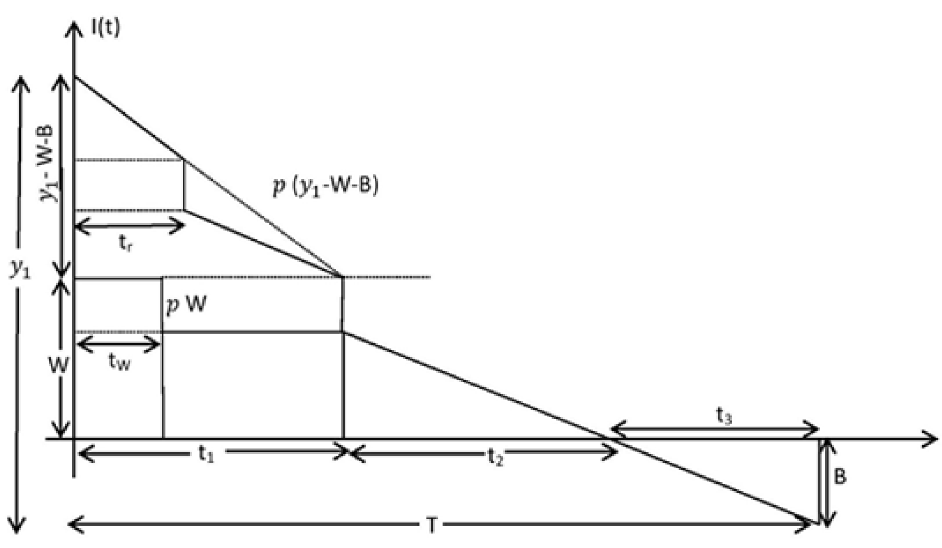
$$T=\left(1-p\right)\frac{y_1}{D} \tag{1}$$

Sale revenue $\left(TR\left(p,y_1\right)\right)$ is the sum of sales of imperfect and the perfect items.

$$TR\left(p,y_1\right)=sy_1\left(1-p\right)+vpy_1 \tag{2}$$

And Total cost ($TC\left(B,y_1\right)$) = ordering cost + purchase cost + holding cost + backordering cost + screening cost

Figure 1. Inventory system with cycle time



$$TC(B,y_1)) = u_i y_1 + A + jy_1 + dy_1 + h_r\left(\left(\frac{1}{2}t_1(y_1 - W)(1-p)\right) + t_r p(y_1 - W)\right) + h_o\left(Wt_W + W(t_1 - t_W)(1-p) + \frac{(W(1-p) - B)(t_2 - t_1)}{2}\right) + \frac{Bbt_3}{2} \tag{3}$$

Since

Then Eq. (3)

$$TC(B,y_1) = u_i y_1 + A + jy_1 + dy_1 + h_r\left(\frac{1}{2D}(y_1 - W)^2(1-p)^2 + \frac{p(y_1 - W)^2}{x}\right) + h_o\left(\frac{W(y_1 - W)(1-p)^2}{D} + \frac{W^2 p}{x} + \frac{1}{2D}\left((W(1-p) - B)^2 - W(y_1 - W)(1-p)^2 + B(1-p)(y_1 - W)\right)\right) + \frac{B^2 b}{2D} \tag{4}$$

The total profit per unit of time $\left(TPU(B,y_1)\right)$ is calculated with the help of total revenue per unit of time and total cost per unit of time. Hence, $TPU(B,y_1) = \dfrac{TR(p,y_1) - TC(B,y_1)}{T}$ (5)

$$= D\left(s - v + h_r\frac{(y_1 - W)^2}{xy_1} + h_o\frac{W^2}{xy_1}\right) + \left(\frac{D}{1-p}\right)\left(v - u_i - \frac{A}{y_1} - j - d - h_r\frac{(y_1 - W)^2}{xy_1} - h_o\frac{W^2}{xy_1}\right) - h_r(y_1 - W)^2\left(\frac{1-p}{2y_1}\right) - h_o\left(\frac{W(y_1 - W)(1-p)}{2y_1} + \frac{W^2(1-p)}{2y_1} - \left(\frac{WB}{y_1}\right) + \frac{B(y_1 - W)}{2y_1}\right) - \left(\frac{1}{1-p}\right)\left(\frac{h_o B^2}{2y_1} - \frac{B^2 b}{y_1}\right) \tag{6}$$

Since p is a random variable that follows a uniform distribution with known p.d.f. f(p) then the expected value of total profit $\left(ETPU(B,y_1)\right)$ is

$$ETPU(B,y_1) = D\left(s - v + h_r\frac{(y_1 - W)^2}{xy_1} + h_o\frac{W^2}{xy_1}\right) + E\left(\frac{1}{1-p}\right)D\left(v - u_i - \frac{A}{y_1} - j - d - h_r\frac{(y_1 - W)^2}{xy_1} - h_o\frac{W^2}{xy_1}\right) - h_r(y_1 - W)^2\left(\frac{1 - E(p)}{2y_1}\right) - h_o\left(\frac{W(y_1 - W)(1 - E(p))}{2y_1} + \frac{W^2(1 - E(p))}{2y_1} - \left(\frac{WB}{y_1}\right) + \frac{B(y_1 - W)}{2y_1}\right) - E\left(\frac{1}{1-p}\right)\left(\frac{h_o B^2}{2y_1} - \frac{B^2 b}{y_1}\right) \tag{7}$$

Optimal lot size is $y_1^* = \sqrt{W^2 + \dfrac{u_i + \mathrm{A} + \dfrac{B^2 b}{D} + h_o\left(\dfrac{pW^2}{x} + \dfrac{B^2}{2D} - \dfrac{3WB(1-p)}{2D}\right)}{h_r\left(\dfrac{(1-p^2)}{2D} + \dfrac{p}{x}\right)}}$ (8)

6. SOLUTION PROCEDURE

The purpose of this model is to optimize order quantity (y_1^*) for both the warehouses and optimal shortages B^*, from these we get expected total profit $ETPU(B,y_1)$, therefore the necessary condition for expected profit to be optimal are $\dfrac{\partial ETPU\left(B,y_1\right)}{\partial y_1}$ and $\dfrac{\partial ETPU\left(B,y_1\right)}{\partial B}$ which can be calculated by the eq. (7) and setting the result to zero

$$\frac{\partial ETPU\left(B,y_1\right)}{\partial B}=-h_o\left(\frac{-W}{y_1}+\frac{1}{2}\frac{y_1-W}{y_1}\right)-E\left(\frac{1}{1-p}\right)\left(\frac{h_oB}{y_1}-\frac{2Bb}{Dy_1}\right)=0 \tag{9}$$

$$\frac{\partial ETPU\left(B,y_1\right)}{\partial y_1}=D\left(\frac{2h_r\left(y_1-W\right)}{xy_1}-\frac{h_r\left(y_1-W\right)^2}{xy_1^{\,2}}-\frac{h_WW^2}{xy_1^{\,2}}\right)+E\left(\frac{1}{1-p}\right)D\left(\frac{A}{y_1^{\,2}}-\frac{2h_r\left(y_1-W\right)}{xy_1}+\frac{h_r\left(y_1-W\right)^2}{xy_1^{\,2}}+\frac{h_oW^2}{xy_1^{\,2}}\right)\frac{h_r\left(y_1-W\right)\left(1-E\left(p\right)\right)}{y_1}+\frac{h_r\left(y_1-W\right)^2\left(1-E\left(p\right)\right)}{2y_1^{\,2}}-h_o\left(\frac{1}{2}\frac{W\left(1-E\left(p\right)\right)}{y_1}-\frac{1}{2}\frac{W\left(y_1-W\right)\left(1-E\left(p\right)\right)}{y_1^{\,2}}-\frac{1}{2}\frac{W^2\left(1-E\left(p\right)\right)}{y_1^{\,2}}+\frac{WB}{y_1^{\,2}}+\frac{B}{2y_1}-\frac{B\left(y_1-W\right)}{2y_1^{\,2}}\right)-E\left(\frac{1}{1-p}\right)\left(-\frac{h_oB^2}{2y_1^{\,2}}+\frac{B^2b}{y_1^{\,2}}\right)=0 \tag{10}$$

Eqs. (9) and (10) can be solved simultaneously for B* and y_1 * using Maple. Here, the sufficient condition for expected total profit to be concave. First taking the second derivative

$$\frac{\partial^2 ETPU\left(B,y_1\right)}{\partial B^2}=-E\left(\frac{1}{1-p}\right)\left(\frac{h_o}{y_1}-\frac{2b}{y_1}\right) \tag{11}$$

$$\frac{\partial^2 ETPU\left(B,y_1\right)}{\partial y_1^{\,2}}=D\left(\frac{2h_r}{xy_1}-\frac{4h_r\left(y_1-W\right)}{xy_1^{\,2}}+\frac{2h_r\left(y_1-W\right)^2}{xy_1^{\,3}}+\frac{2h_oW^2}{xy_1^{\,3}}\right)+E\left(\frac{1}{1-p}\right)D\left(-\frac{2A}{y_1^{\,3}}-\frac{2h_r}{xy_1}+\frac{4h_r\left(y_1-W\right)}{xy_1^{\,2}}+\frac{2h_r\left(y_1-W\right)^2}{xy_1^{\,3}}-\frac{2h_oW^2}{xy_1^{\,3}}\right)-\frac{h_r\left(1-E\left(p\right)\right)}{y_1}+\frac{2h_r\left(y_1-W\right)\left(1-E\left(p\right)\right)}{y_1^{\,2}}-\frac{h_r\left(y_1-W\right)^2\left(1-E\left(p\right)\right)}{y_1^{\,3}}-h_o\left(\frac{-W\left(1-E\left(p\right)\right)}{y_1^{\,2}}+\frac{W\left(y_1-W\right)\left(1-E\left(p\right)\right)}{y_1^{\,3}}+\frac{W^2\left(1-E\left(p\right)\right)}{y_1^{\,3}}-\frac{2WB}{y_1^{\,3}}-\frac{B}{y_1^{\,2}}+\frac{2\left(y_1-W\right)}{y_1^{\,3}}\right)-E\left(\frac{1}{1-p}\right)\left(\frac{h_oB^2}{y_1^{\,3}}-\frac{2B^2b}{y_1^{\,3}}\right) \tag{12}$$

$$\frac{\partial^2 ETPU\left(B,y_1\right)}{\partial y_1\partial B}=-h_o\left(\frac{W}{y_1^{\,2}}+\frac{1}{2y_1}-\frac{1}{2}\frac{y_1-W}{y_1^{\,2}}\right)-E\left(\frac{1}{1-p}\right)\left(\frac{-h_oB}{y_1^{\,2}}+\frac{2bB}{y_1^{\,2}}\right) \tag{13}$$

$$\left(\frac{\partial^2 ETPU\left(B,y_1\right)}{\partial B^2}\right)\times\left(\frac{\partial^2 ETPU\left(B,y_1\right)}{\partial y_1^{\,2}}\right)-\left(\frac{\partial^2 ETPU\left(B,y_1\right)}{\partial y_1\partial B}\right)^2\geq 0 \tag{14}$$

$$\text{And }\left(\frac{\partial^2 ETPU\left(B,y_1\right)}{\partial B^2}\right)\leq 0,\left(\frac{\partial^2 ETPU\left(B,y_1\right)}{\partial y_1^{\,2}}\right)\leq 0 \tag{15}$$

By necessary and sufficient condition, the above equations prove that the function *ETPU (B,* y_1 *)* is strictly concave with a negative-deðnite Hessian matrix with optimal *(B*,* y_1 **)* values. With the optimal B* and y_1 * values known, the net proðt can be derived from Eq. (7).

7. NUMERICAL EXAMPLE

Some parameters are needed for evaluate the mention inventory system and these parameters are obtained from (Aastha et al., 2020)

$$D = 50{,}000\,unit\ /year; W = 800\,units\ /\ cycle; \mathrm{A} = \$100\ /cycle;$$

$$j = \$20\ /unit; h_r = \$7\ /unit\ /\ year; h_o = \$5\ /\ unit\ /\ year; x = 1unit\ /\ min; d = 0.5\ /\ unit; u_i = 5\ /\ unit; s = \$50\ /\ unit; v = \$20\ /unit; b = 10\ /\ units$$

In this example, the inventory model works on an $8\,hours\ /\ day$ for whole year, therefore the screening rate per year, $x = 1 * 60 * 8 * 365 = 175200\,units\ /\ year$.

The proportion of imperfect random variable, p, follows uniformly distribution with p.d.f. as

$$f(p) = \begin{cases} 25, & 0 \le \rho \le 0.04 \\ 0, & otherwise \end{cases}$$

From this $E(p) = 0.02$ and $E\left(\frac{1}{1-p}\right) = 1.02055$.

By solving the equation (15) and (16), the optimal solution is

As per the results the profit of current work is greater than existing work. Hence, practically this work is best fitted.

8. SENSITIVITY ANALYSIS

The sensitivity analysis is performed for the above example. The effects of fraction defective items ([p]), ordering cost, selling price of defective items and discount on the expected total profit $\left(ETPU\left(B^*, y_1^*\right)\right)$ are shown in Figure 1, Figure 2, Figure 3 and Figure 4, respectively.

The above graphs show that, from Figure 1, as the fraction of imperfect items increases the expected total profit decreases significantly. From figure 2, as the purchase cost of items increases

Table 3. The optimal solution for equation (15) and (16)

y_1^*	B^*	$ETPU\left(B^*, y_1^*\right)\$$	t_2^* (Yr.)	t_3^* (Yr.)	T^* (Yr.)
1373	167	12130.342	0.012	0.001	0.027

Table 4. A comparison table between the results of existing work and current work

Parameters	y_1	B	Time	Total profit
Current work	1374	168	0.027	12130.342
Existing work	1425	113	0.027	11675.162

Table 5. Sensitivity analysis with different parameters of the inventory system

Parameter	Value	T (years)	t_1 (years)	t_2 (years)	y_1 (units/cycle)	B (units/cycle)	TP ($)
y_1	0.02	0.027	0.0112	0.012	$1,374$	168	12130.3
	0.03	0.026	0.0113	0.0122	1,384	164	12101.1
	0.04	0.0267	0.0114	0.0121	1,393	161	12071.3
	0.05	0.0267	0.0115	0.0120	1,403	158	12040.9
A	100	0.027	0.0112	0.012	$1,374$	168	12130.3
	125	0.026	0.0113	0.0122	1,384	164	12121.4
	150	0.0267	0.0114	0.0121	1,393	161	12113.3
	175	0.0267	0.0115	0.0120	1,403	158	12105.7
V	20	0.027	0.0112	0.012	$1,374$	168	12130.3
	25	0.026	0.0113	0.0122	1,384	164	12181.3
	30	0.0267	0.0114	0.0121	1,393	161	12232.3
	35	0.0267	0.0115	0.0120	1,403	158	12283.3
U	6	0.027	0.0112	0.012	$1,374$	168	11624.3
	5	0.026	0.0113	0.0122	1,384	164	12130.3
	4	0.0267	0.0114	0.0121	1,393	161	12640.5
	3	0.0267	0.0115	0.0120	1,403	158	13150.7

then the expected total profit increases. In this scenario, the buyer should be careful while ordering and the vendor should make his system more efficient to improve the quality of produced items.

From figure 3, as the selling price of defective item increases, the expected total profit increases significantly. From figure 4, as the offered discount decreases the expected total profit increases. In this scenario, if the buyer got some defective items, then the buyer must be careful about the selling price of defective items as well as the offered discount.

9. MANAGERIAL INSIGHTS

Here, a discount policy is used in this model in such a way the customer gets the discount based on the purchased quantity. This shows the significance of the model. It is useful for a small retailer who is facing a problem in purchasing quantity. By the discount feature, they will know how much they can earn.

Figure 2. Effect of fraction of defective items on expected total profit

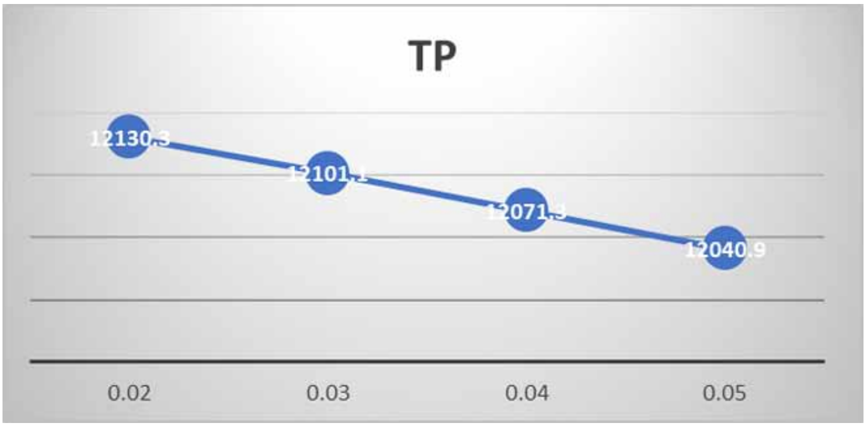


Figure 3. Effect of ordering cost on expected total profit

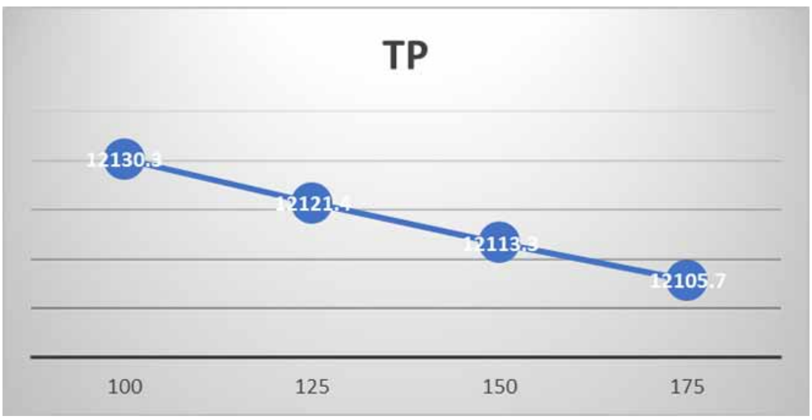


Figure 4. Effect of selling price of defective items on expected total profit

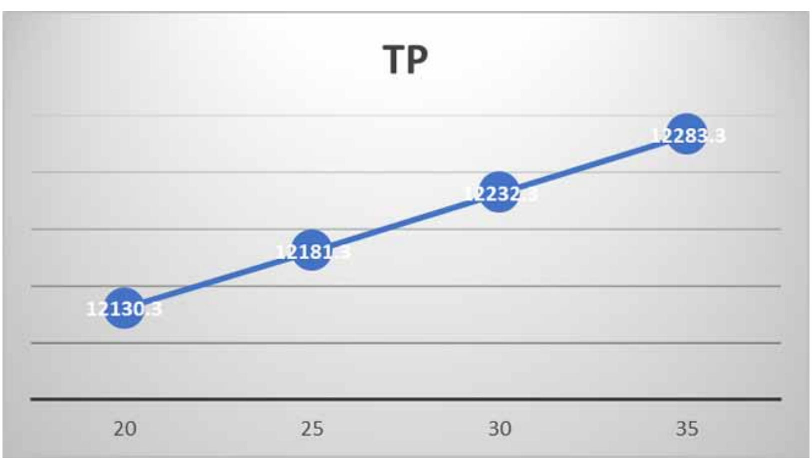
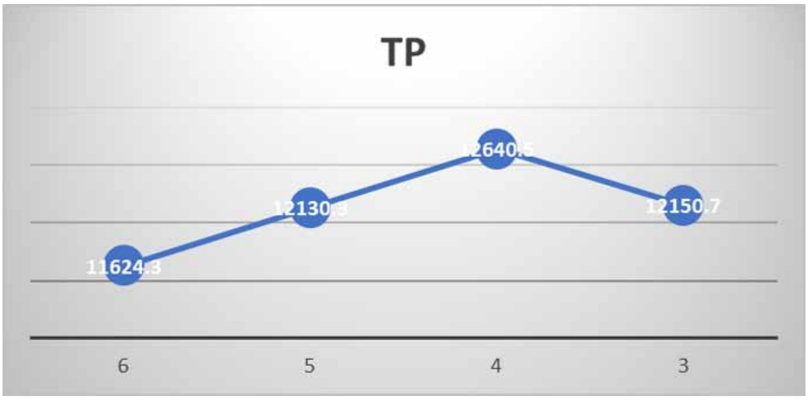


Figure 5. Effect of offered discount on expected total profit



10. CONCLUSION

This paper presents an EOQ model for defective items with shortages under a multi-warehouse environment to determine expected total profit when a discount is offered. It is demonstrated as if the buyer got defective items in the inventory, then the buyer can earn profit from defective items too. Also, to earn more profit, one can offer a discount on quantity. By this model, a businessperson must check the purchase cost of any item and the offered discount, these directly affect the expected total profit. This paper is very useful for retail and manufacturing trades. This will also be helpful for business industries. This model can be extended in several forms. It can be extended under carbon emission constraints with types of demand by using credit policy. If this model can be extended under different marketing scenarios taken by Sarkar et al., (2021) then it will be more realistic. Further, this research can be extended by considering the smart autonomation inspection process which is used by Dey et al., (2021). Also, this can extend with the SSMD policy used by Dey et al., (2019).

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