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Impact of Inflation and Credit Financing Policy on the Supply Chain With Learning

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ABSTRACT

Technology drives many fields to improve the quality of items during the supply of the products. Despite proficient planning in the industrial system and the presence of sophisticated techniques, there may be some defective items in the lots. This paper deals with the inventory model that determines economic order quantity (EOQ) with learning effect for decaying defective quality items under the inflationary condition and credit financing policy. The objective of the work is to analyze the impact of credit financing policy, learning, and inflationary condition on the order quantity and retailer profits. Results revealed that the trade-credit policy will be beneficial for the retailer. Conclusively, sensitive analysis has been presented to understand the robustness of the models.

KEYWORDS

Defective Items, Deterioration, Inflation, Inspection, Learning Effects, Trade-Credit Financing

1. INTRODUCTION

1.1 Motivational Models

Many researchers have implemented the phenomenon of delay in paymentspolicy as an organization structure in their related studies. Whitin (1957) considered inspection the worsening and perish of stylish commodities and associated articles at the termination of a recommended interval of time. Ghare and Schrader (1963) studied and provided a mathematical execution for deteriorating stuffs which followed an exponential decay rate. Apart from various economic order quantity (EOQ) models that have covered some accurate assumptions related to all those formulated that the lots are not always of perfect quality. Porteus (1986) gave many extensive reviews on defective stuffs. Further, Goyal (1985) suggested an inventory model for deriving the quantity of the financial arrangement of the items for which the seller would permit a fixed delay in payments.

Later, a basic model for the inflationary conditions has been developed by Buzacott (1975) for deteriorating items under different policies. Datta and Pal (1991) have discussed effects of inflation

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and time-value of money on an inventory model with linear time-dependent demand rate and shortages. Sarker and Pan (1994) have discussed the effects of inflation and the time value of money on the order quantity and allowable shortages. Hariga (1995) also proposed an EOQ model for deteriorating items with shortages and time-varying demand. Hariga and Ben-Daya (1996) discussed optimal time varying lot-sizing model under inflationary conditions.

Further, Wright (1936) introduced the learning concept as a power function. Jaber and Bonney (1996) derived a mathematical model with shortages and backorder under the leaning effect. Jaggi et al. (2013) suggested a deterministic model for imperfect commodities with permissible delay in payment with shortages. Tiwari et al. (2018) proposed sustainable inventory management model with imperfect quality items and carbon emission is also considered to understand the environmental impact on the inventory model. Jayaswal et al. (2019) introduced concept of learning for imperfect items with delay in payments. They optimized order quantity and maximized retailer's profit. Barman et al.(2021) gave an economic production quantity (EPQ) model with inflation under cloudy fuzzy system for deteriorating items. Jayaswal et al. (2021) presented an inventory model with learning where demand is a function of credit period. Jayaswal et al. (2021) proposed an economic order quantity (EOQ) model for deteriorating defective quality items with the effect of leaning under credit period scheme. Singh et al. (2021) presented an optimal policy for deteriorating Items with generalized deterioration, trapezoidal-type demand, and shortages. Verma et al. (2022) discussed the impact of price-sensitive demand and premium payment scheme on bullwhip effect.

1.2 EOQ with Trade Credit Financing Models

There are many renowned researchers who worked on management and inventory control like, Shah (1993, 1993b).Goyal (1985) personalized model to accommodate the calculated stock formulation with constant decaying rate. Shinn et al. (1996) enhanced Goyal (1985) model by considering the quantity repayment for the belonging's expenditure. Jamal et al. (1997) modified the work of Aggarwal and Jaggi (1995) to provide accommodation allowances for shortages. An optimal duration of credit financing for goods and the related products that the trader put up for sales to the brokers for maximizing the seller's income (Kim et al., 1995). Cheung and Hausman(1997) have calculated both preventive maintenance and safety stocks in an unreliable production environment. Chung (1998) suggested the repayment flow of practical move towards the organized exploration of the preeminent stock policy in trade financing.

Researchers like Chu et al. (1998), Jamal et al. (2000) invented models for determining the optimal time for payments. Chang et al. (2003) derived economic order quantity (EOQ) sample for decaying items in which the length of delay in payments is directly connected to the order quantity. Shinn and Hwang (2003) formulated the retailer's optimized prices. Huang and Chung (2003) improved Goyal (1985) prototype further to study the renewal of cash approach whose intention is to trim down the yearly total average cost under delay in payments.

All the above discussed models considered one-level trade-credit financing but on the contrary, in the majority of dealing of businesses, this assumption is impractical as well as unrealistic. Generally, the supplier provides a finance period to the retailer and the retailer eventually takes that for his clients. In recent times, researchers have developed the inventory models that considered two-level credit financing strategy. Huang (2003) demonstrated the mathematical model for the seller which introduces a financing period of time to the consumer which is comparatively shorter than the one offered by the merchant, for the reason to encourage the necessity. Teng et al. (2006) formulated an economic production quantity model where the producer receives a trade-credit from the dealer.

A two-level trade-credit policy which includes credit-linked demand has been proposed by Jaggi et al. (2008). On the converse, Teng and Chang (2009) amended Huang's (2007) model by giving reduction to the assumption that delay in cash policy is somewhat longer than the one presented by the vendor. Chen and Kang (2009) devised the combined inventory model for the two-level delay payment policy with price sensitive demand under a negotiation scheme. Tripathi and Misra (2012) developed an optimal inventory policy for items having constant demand and constant deterioration rate with trade credit. Tiwari et al. (2017) applied lot-sizing strategies for perishable items with time-dependent demand and trade-credit policy. Jaggi et al. (2017) extended credit financing in economic ordering policies for non-instantaneous decaying items with price dependent demand and two storage facilities. Jaggi et al. (2015) analyzed the effect of deterioration on a two-warehouse inventory model for imperfect quality items.Further, Nobil et al. (2019) considered a multi-item single machine production structure having imperfect products with shortages, rework, and scrapped considering inspection, dissimilar deficiency levels. Sangal et al. (2017) customized the non-instantaneous optimal policy for defective products with partial shortages and learning impact. Nobil et al. (2018) improved the economic production quantity model considering the warm-up time in a cleaner manufacture situation.

Tiwari et al. (2018) developed a two-echelon inventory model for perishable products and demand rate is supposed to be retailer's selling price dependent and displayed stock level. Joint pricing and inventory model for deteriorating items with expiration dates and partial backlogging under two-level partial trade-credit has been developed by Tiwari et al. (2018). Tiwari et al. (2016) suggested a two-warehouse model with inflationary conditions and partially backlogged shortages. Tiwari et al. (2018) developed a green production quantity model with trade-credit policy, random imperfect products and failure in reworking. Optimal trade-credit and lot size policies in economic production quantity models with learning curve production costs have been studied by Teng et al. (2014). Shin et al. (2018) suggested two-echelon supply chain models with inspection errors and trade-credit effect. Sarkar (2016) illustrated a channel coordination and quantity discount policy with single-setup multi-delivery (SSMD). Kumar et al. (2019) proposed a model with new product launch. Yadav et al. (2018) proposed inventory model in which end demand was price sensitive.

Jaber et al. (2008) extended the model with the help of learning concept for the imperfect items. Paitro et al. (2018)presented a mathematical model with concept of learning and rebate policy for imperfect decaying items under fuzzy environment. Khan et al. (2010) developed a production model under the impact of learning for imperfect items. Esmaeili and Nasrabadi (2021) presented supply chain model with trade-credit policy under inflationary situations for multi retailers. Akbar et al. (2021) extended an economic production model (EPQ) model with trade-credit scheme for deteriorating items under inflationary situation where demand rate is a function of selling price. Tripathi and Misra (2012) developed an optimal inventory policy for items having constant demand and constant deterioration rate with trade credit.

1.3 EOQ with Leaning Models

Anzanello and Fogliatto (2011) suggested an inventory model with a special kind of learning curve and compared the study. Konstantaras et al. (2011) proposed a production model for perishable products under learning effects and shortages. Aggarwal et al. (2016) proposed EOQ inventory model with the impact of learning and partial backlogging under fuzzy environment. Salameh et al. (1993) investigated the learning effect on optimal quantity to be ordered and minimum inventory cost. Salameh and Jaber (2000) extended the traditional models by considering imperfect quality products and also considered the issue of poor-quality items.The effect of learning has been studied by Argote et.al (1990) by collecting data from various organizations. Cunningham (1980) studied the learning curve as a tool for management. An EPQ model has been derived by Kumar et al. (2003) by considering fuzzy demand and rate of deterioration.

Dutton and Thomas (1984) studied the various progress functions in different fields theoretically. A mathematical model has been developed by Givi et al. (2015) which estimated the human errors and reliability over time. Jaber and Khan (2010) studied the effect of changing the learning curve in the rework and production. Jaber and Salameh (1995) developed the concept of learning with the shortages. Jaber and Bonney (2003) investigated the effect of forgetting and learning on the lot-sizing problem. Jaber and Guiffrida (2004, 2008) modified the learning curve studied by Wright (1936). Jayaswal et al. (2019) showed the effect of learning on the economy ordering policy for defective items under fuzzy environment with permissible delay in payment. Mittal et al. (2021) showed the effect of learning on the optimal ordering policy of inventory model for deteriorating items with shortages and trade-credit financing. Jayaswal et al. (2021) developed a fuzzy-based EOQ model with credit financing and backorders under human learning. Finally, contribution of selected authors is given in the Table 1.

1.4 Learning Curve

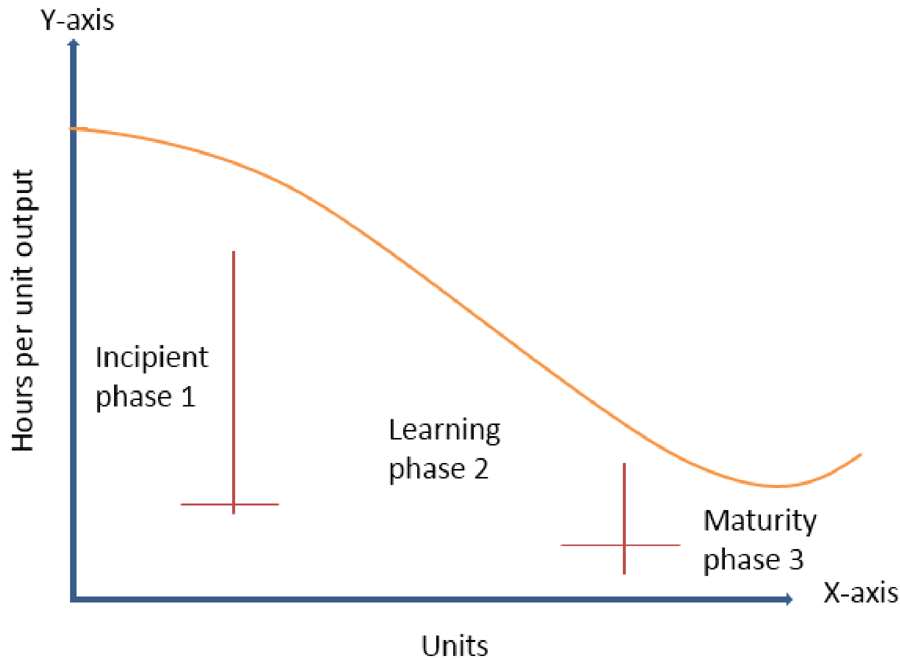
The learning effect acts as a considerable function for cost reduction and maximizing the profit. Some authors discussed the impact of the learning shape in the same direction asper Wright (1936), Jordon (1958) and Carlson (1973). From the figure 1, three unlike phases can be seen, where the first phase is called segment stage, the second is called learning phase and finally, the last phase is called the maturity phase as shown below in the figure.

The number of imperfect items presented in each batch is assumed by an S-shape logisticlearning curve and graphically shown below in figure 2.

Table 1. Author's contribution table

Author(s)	Impact of learning	Screening	Trade-credit	Deterioration	Defective items	Inflation
Wright (1936)	✓					
Cunnigham (1980)	✓					
Dutton (1984)	✓					
Argote et al. (1990)	✓					
Salameh et al. (1993)	✓	✓				
Jaber et al. (1996)	✓	✓			✓	
Salameh and Jaber (2000)		✓			✓	
Jaber et al. (2008)	✓	✓			✓	
Khan et al. (2010)	✓	✓			✓	
Anzanello and Fogliatto (2011)	✓					
Jaggi et al. (2011)		✓	✓	✓	✓	✓
Jaggi et al. (2013)		✓	✓		✓	
Jaggi et al. (2017)		✓	✓	✓	✓	✓
Jayaswal et al. (2018)	✓	✓	✓		✓	✓
Patro et al. (2018)	✓	✓		✓		
Nobil et al. (2019)					✓	✓
Akbar et al. (2021)			✓	✓		✓
Esmaeili and Nasrabadi (2021)			✓	✓		✓
Barman et al. (2021)				✓		✓
Jayaswal et al. (2021)	✓		✓		✓	
Jayaswal et al. (2021)	✓	✓	✓	✓	✓	
This Paper	✓	✓	✓	✓	✓	✓

Figure 1. Three stages of learning



$$P(n) = \frac{a}{e^{bn}} + g$$

Where a , g are positive number, n is number of shipments and b is learning rates

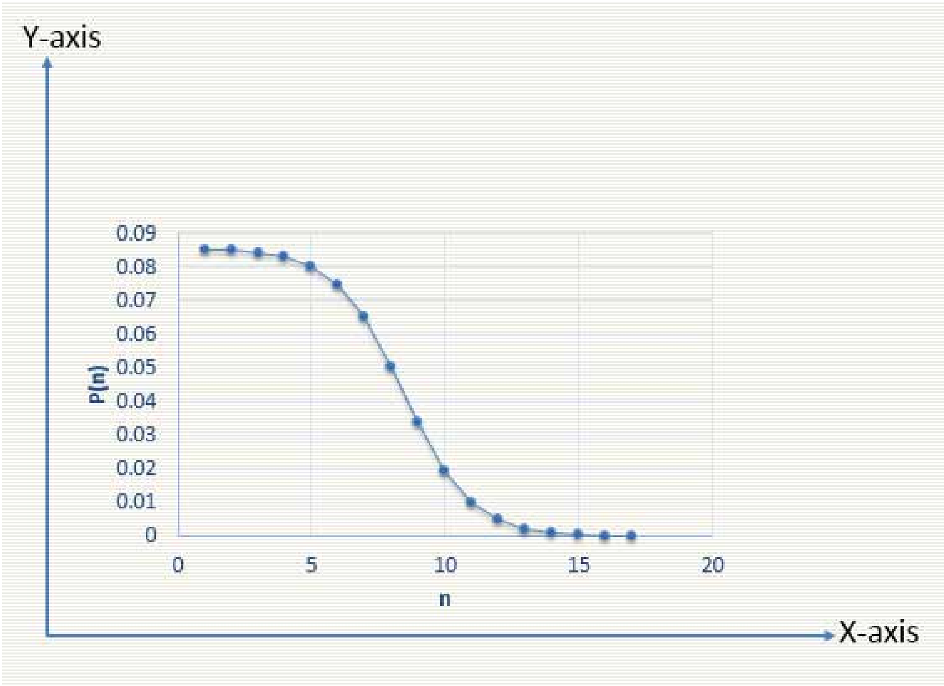
2. ASSUMPTIONS AND NOTATIONS

Assumptions and notations are given below.

2.1 Assumption

1. The continuity of replacement is allowed
2. Shortages and lead time are not included in this model
3. The credit financing policy is allowed as per Jaggi et al. (2013)
4. The screening rate is greater than demand rate (Jaggi et al., 2013 and Jaggi et al., 2011a)
5. The time horizon plane has been considered finite
6. The learning effect is involved in holding and ordering cost
7. Lots have some percentage of defectives items (Salameh and Jaber, 2000)
8. Imperfect quality items follow the S-shape learning curve suggested by Jaber et al. (2008)
9. Imperfect items are sold at discounted price after the inspection process
10. Lots have a constant deterioration rate
11. The inflation rate is constant

Figure 2. S-shape learning curve



2.2 Notation

z Order quantity has been taken as decision variable (units)
D Order size (units/year)
M Credit period from seller side (year)
C_k Ordering cost ($/ cycle)
C_p Unit purchasing cost ($/unit)
p Unit selling cost of perfect quality items ($/units)
P Percentage of defective items are presents in z
$P(n)$ Imperfect quality items are following S-shape learning curve
c_s Unit selling price of defective quality items, $c_s < p$ ($)
C_s Screening cost ($/units)
θ Deterioration cost (per year)
C_k Carrying cost ($/unit/year)
λ Screening rate, $\lambda > D$ (unit/year)
t_n Inspection time (year)
T_n Cycle length (year)
I_e Interest earned ($/unit)
I_p Interest paid ($/unit)
$I_1(t)$ Stock level at $t \in [0, t_n]$
$I_2(t)$ Stock level at $t \in [t_n, T_n]$

SR_i Total sales revenue for different case
TC_i Total cost with cases, $i = 1,2,3$ (\$)
$\Psi_i\left(T_n\right)$ Total profit for cases, $i = 1,2,3$ (\$)
r Discount rate, representing the time value of money
i inflation rate
R $(= r - i)$ rebate rate of inflation which is constant

2.3 Some Concepts Related to Assumptions

The impact of learning presented in the holding cost, and this can be represented mathematically as,

$$C_h\left(n\right) = C_{ho} + \frac{C_{h1}}{n^{\alpha}}, C_{ho}, C_{h1} > 0$$

Where n represents the number of orders, α is a learning factor, C_{ho}, partially fixed holding cost and C_{h1}, partially fixed holding cost in each shipment.

The impact of learning presented in the ordering cost and mathematically it can be represented as,

$$C_k\left(n\right) = C_{ko} + \frac{C_{k1}}{n^{\beta}}, C_{ko}, C_{k1} > 0$$

Where n is represent the number of orders, β is a learning factor, C_{ko}, partially fixed ordering cost and C_{k1}, partially fixed ordering cost in each shipment.

The percentage of imperfect quality items presented in each lot follows the behavior of learning, which is shown in the mathematical formula given below,

$$P\left(n\right) = a/e^{bn} + g$$

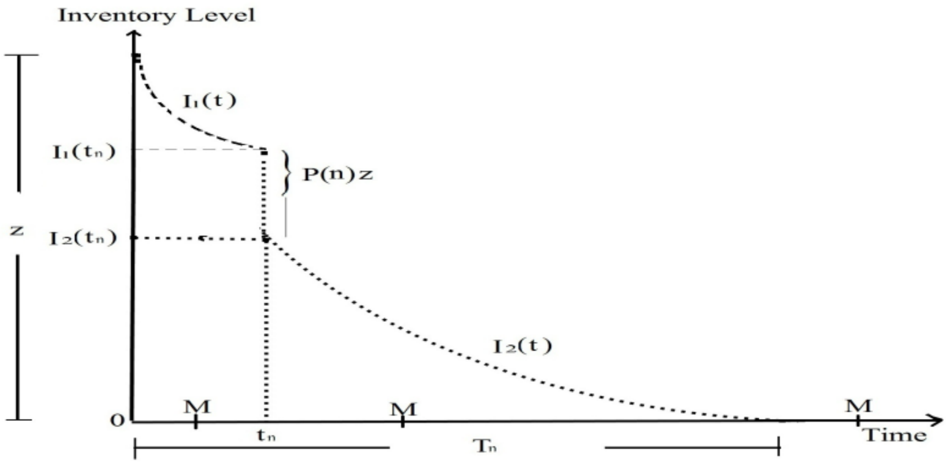
Where a, g are positive number, n is number of shipments and b is learning rates

In this paper, Jaggi et al. (2011) work is extended by introducing the learning concept. The impact of learning is determined with trade-credit financing under inflationary conditions.

3. MATHEMATICAL FORMULATION

According to assumption, the inventory level z is the inventory level at $t = 0$ which has defective and non-defective items. The entire lot has been inspected at a constant rate of λ units/year. After the inspections process, items are separated into defective and non-defective items. Further, it is also assumed that the inspection time, $t_n = \frac{z}{\lambda}$. After inspection the defective quality items have been sold at discounted price, c_s. To avoid the shortages, it is assumed $\left(1 - P\left(n\right)\right)z \geq Dt_n$, which infers that, $P\left(n\right) \leq 1 - \frac{D}{\lambda}$. From the figure 3, $I_1\left(t\right)$ be the inventory level in the interval $\left[0, t_n\right]$ which

Figure 3. Representation of inventory process with learning and trade-credit period



reduces with time due to demand. After inspection, $I_2\left(t\right)$ be the perfect quality items during the interval $\left[t_n, T_n\right]$.

$$\begin{aligned} &\frac{dI_1\left(t\right)}{dt} + \theta I_1\left(t\right) = -D, \quad t \in \left[0, t_n\right] \\ &\quad at \;\; t = 0, \;\; I_1\left(0\right) = z. \\ &I_1\left(t\right) = ze^{-\theta t} + \frac{D}{\theta}\left[e^{-\theta t} - 1\right] \end{aligned} \tag{1}$$

Now, the stock at $\left(t = t_s\right)$

$$\begin{aligned} IEl &= I_1\left(t_s\right) - P\left(n\right)z = ze^{-\theta t_s} + \frac{D}{\theta}\left[e^{-\theta t_s} - 1\right] - P\left(n\right)z \\ &= \left(1 - P\left(n\right)\right)z - Dt_s. \end{aligned} \tag{2}$$

Where, *IEL* represents effective inventory level.

Now, it is considered that $I_2\left(t\right)$ is an inventory level in the time interval $\left[t_n,\, T_n\right]$. Finally, the governing differential equations for the inventory model are given below,

$$\begin{aligned} &\frac{dI_2\left(t\right)}{dt} + \theta I_2\left(t\right) = -D, \quad t \in \left[t_n, T_n\right] \\ &\quad I_2\left(t_s\right) = IEl = \left(1 - P\left(n\right)\right)z - Dt_s, \;\; I_2\left(T_n\right) = 0. \\ &I_2\left(t\right) = \frac{D}{\theta}\left[e^{\theta\left(t_n - t\right)} - 1\right] + \left[\left(1 - P\left(n\right)\right)z - Dt_n\right]e^{\theta\left(t_n - t\right)} \end{aligned} \tag{3}$$

where,

$$t_n = \frac{z}{\lambda}. \tag{4}$$

For calculating the order quantity, from the equation (3), as we know that $I_2\left(T_n\right) = 0$, then after solving equation (3), the value of z will be given in equation (5),

$$z = \frac{D\left(e^{\theta T_n} - 1\right)}{\theta\left(1 - p\left(n\right)e^{\theta T_n}\right)} \tag{5}$$

The retailer's total profit is given below and represented by Ψ_i $\Psi_i =$ Total revenue$\left(SR\right)$– Ordering cost$\left(C_k\right)$ -Inspection cost$\left(I\,C\right)$ -Purchasing cost$\left(P\,C\right)$-Holding cost$\left(IHC\right)$ +Interest gained$\left(IE\right)$ - Interest charged$\left(IP\right)$ (6)

The components of equation (6) can be calculated and are given below,

(a) Total revenue in the period$\left[0, T_n\right]$, let us say SR_1 and revenue by selling the of imperfect quality items on the salvage cost will be SR_2 .

Total revenue (SR)=SR_1+ SR_2

$$\begin{aligned}
SR_1 &= \int_0^{T_n} p\,D\,e^{-Rt}dt = \frac{pD}{R}\left[1 - e^{-RT_n}\right] \\
SR_2 &= c_s P(n) z\, e^{-Rt_n} \\
SR &= SR_1 + SR_2 \\
SR &= \frac{pD}{R}\left[1 - e^{-RT_n}\right] + c_s P(n) z\, e^{-Rt_n}
\end{aligned} \tag{7}$$

Further, ordering cost, inspection cost, purchasing cost and holding cost calculations are given below,

(b) Ordering cost $C_k = C_o + \dfrac{C_{k_1}}{n^{\alpha}}$

(c) Inspection cost$\left(IC\right) = C_s z$

(d) Purchasing cost$\left(P\,C\right) = C_p z$

(e) Holding cost (IHC) will be given below

$$IHC = C_h\left[\int_0^{t_n} I_1(t)e^{-Rt}dt + \int_{t_n}^{T_n} I_2(t)e^{-Rt}dt\right]$$

$$C_h\left[\frac{z}{(\theta+R)}\left[1-e^{-(\theta+R)T_n}\right] + \frac{D}{\theta}\left[\frac{1-e^{-(\theta+R)T_n}}{(\theta+R)} + \frac{e^{-RT_n}-1}{R}\right] + \frac{P(n)z}{R}\left(e^{-RT_n}-e^{-Rt_n}\right)\right] \quad (8)$$

Further, the total cost (TC) will be the sum of all above calculated costs which is give below,

$$TC = C_k + C_s z + C_p z + IHC \quad (9)$$

and the total profit is,

$$\Psi_i = SR - TC + IE - IP \quad (10)$$

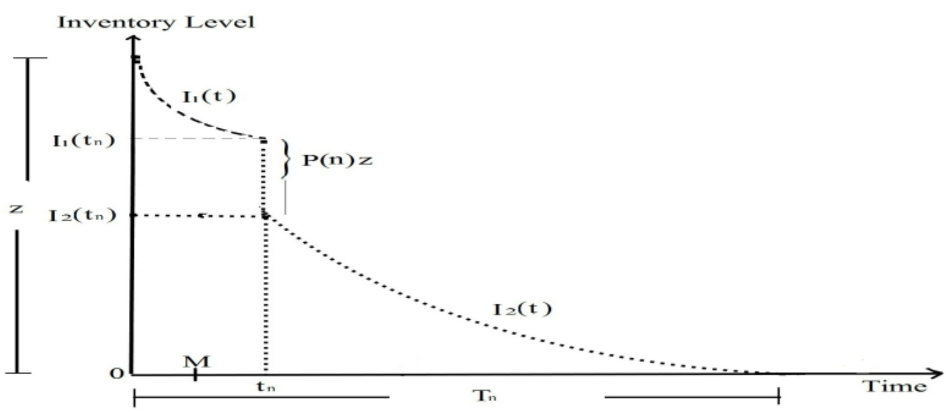
The calculation of interest earned (IE) and interest paid (IP) can be determined as per cases which are given below in detail.

Case1: $0 \le M \le t_n \le T_n$

From figure 4, the retailer earns profit on the income which is earned by selling the inventory for the period $[0, M]$ and it is equal to $pI_e D\left[\frac{1-e^{-RM}}{R^2} - \frac{Me^{-RM}}{R}\right]$. The retailer must pay interest on the unsold items for the time period M to T_n which is equal to

$$C_p I_p\left[\frac{z}{\theta+R}\left[e^{-(\theta+R)M} - e^{-(\theta+R)T_n}\right] + \frac{D}{\theta}\left[\frac{e^{-(\theta+R)M} - e^{-(\theta+R)T_n}}{\theta+R} + \frac{e^{-RT_n} - e^{-RM}}{R}\right] + \frac{P(n)z}{R}\left[e^{-RT_n} - e^{-Rt_n}\right]\right]$$

Figure 4. Inventory representation for Case 1



The total revenue is represented by SR_3 for case1

$$SR_3 = SR + pI_e D\left[\frac{1-e^{-RM}}{R^2} - \frac{Me^{-RM}}{R}\right] \tag{11}$$

and the total cost for this case is represented by TC_1,

$$TC_1 = TC + IP$$
$$TC_1 = C_k + C_s z + C_p z + IHC + IP \tag{12}$$

The retailer's total profit will be equal to

$$\Psi_1\left(T_n\right) = SR_3 - TC_1$$

After putting the values from equation (11) and (12), the retailer's total profit is equal to,

$$\Psi_1\left(T_n\right) = \left[\frac{pD}{R}\left[1-e^{RT_n}\right] + c_s p\left(n\right) z e^{-Rt_n} + pIeD\left[\frac{1-e^{-RM}}{R^2} - \frac{Me^{-RM}}{R}\right]\right]$$
$$-\left[\begin{array}{l} C_k + C_s z + C_p z + \frac{C_h z}{\theta+R}\left[1-c^{-(\theta+R)T_n}\right] + \frac{C_h D}{\theta}\left[\frac{1-e^{-(\theta+R)T_n}}{(\theta+R)} + \frac{e^{-RT_n}-1}{R}\right] + \\ \frac{C_h p\left(n\right) z}{R}\left[e^{-RT_n} - e^{-Rt_n}\right] + c_p I_p\left[\frac{z}{(\theta+R)}\left[e^{-(\theta+R)M} - e^{-(\theta+R)T_n}\right] + \frac{D}{\theta}\left[\frac{e^{-(\theta+R)M} - e^{-(\theta+R)T_n}}{\theta+R} - \frac{e^{-RT_n} - e^{-RM}}{R}\right]\right] \\ + \frac{p\left(n\right) z}{R}\left[e^{-RT_n} - e^{-Rt_n}\right] \end{array}\right] \tag{13}$$

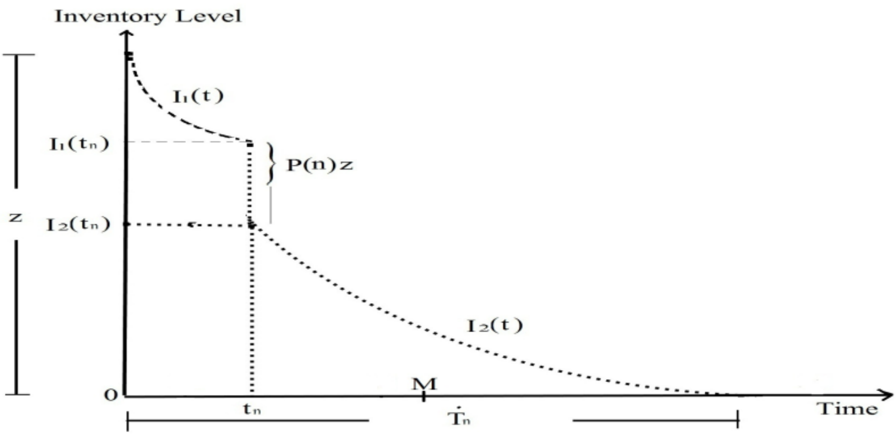
Case2: $0 \le t_n \le M \le T_n$

From figure 5, the retailer earns profit on the total profit at M and for imperfect quality items for the time $\left(M - t_n\right)$ which is equal to $pI_e D\left[\frac{1-e^{RM}}{R^2} - \frac{Me^{-RM}}{R}\right] + e^{-Rt_n} c_s P\left(n\right) I_e z\left(M - t_n\right)$ and interest paid after this period, which is equal to,

$$C_p I_p\left[\frac{z}{\theta+R}\left[e^{-(\theta+R)M} - e^{-(\theta+R)T_n}\right] + \frac{D}{\theta}\left[\frac{e^{-(\theta+R)M} - e^{-(\theta+R)T_n}}{\theta+R} + \frac{e^{-RT_n} - e^{-RM}}{R}\right] + \frac{P(n)z}{R}\left[e^{-RT_n} - e^{-RM}\right]\right]$$

The total sales revenue for the case2, which is represented by,

Figure 5. Inventory representation for case 2



$$SR_4 = SR + IE$$

$$= SR + pI_eD\left[\frac{1-e^{RM}}{R^2} - \frac{Me^{-RM}}{R}\right] + e^{-Rt_n}c_sP(n)I_ez(M-t_n)$$

$$TC_2 = TC + IP$$

$$= C_k + C_sz + C_pz + IHC$$

$$+C_pI_p\left[\begin{array}{l}\dfrac{z}{\theta+R}\left[e^{-(\theta+R)M} - e^{-(\theta+R)T_n}\right] + \dfrac{D}{\theta}\left[\dfrac{e^{-(\theta+R)M} - e^{-(\theta+R)T_n}}{\theta+R} + \dfrac{e^{-RT_n} - e^{-RM}}{R}\right]\\ + \dfrac{P(n)z}{R}\left[e^{-RT_n} - e^{-RM}\right]\end{array}\right]$$

The retailer's total profit for this case,

$$\Psi_2(T_n) = SR_4 - TC_2$$

$$\Psi_2(T_n) = \left[\frac{pD}{R}\left[1-e^{RT_n}\right] + c_sP(n)z\,e^{-Rt_n} + pIeD\left[\frac{1-e^{-RM}}{R^2} - \frac{Me^{-RM}}{R}\right] + e^{-Rt_n}p(n)C_sIe(M-t_n)\right] -$$
$$\left[\begin{array}{l}C_k + C_sz + C_pz + \dfrac{C_hz}{\theta+R}\left[1-c^{-(\theta+R)T_n}\right] + \dfrac{C_hD}{\theta}\left[\dfrac{1-e^{-(\theta+R)}}{(\theta+R)} + \dfrac{e^{-RT_n}-1}{R}\right] +\\ \dfrac{C_hp(n)z}{R}\left[e^{-RT_n} - e^{-Rt_n}\right] + c_pI_p\left[\dfrac{z}{(\theta+R)}\left[e^{-(\theta+R)M} - e^{-(\theta+R)T_n}\right] + \dfrac{D}{\theta}\left[\dfrac{e^{-(\theta+R)M} - e^{-(\theta+R)T_n}}{\theta+R} - \dfrac{e^{-RT_n} - e^{-RM}}{R}\right]\right]\\ +\dfrac{p(n)z}{R}\left[e^{-RT_n} - e^{-RM}\right]\end{array}\right] \quad (14)$$

Case3: $0 \le t_n \le T_n \le M$

From figure 6, retailers earn profit till period M on the items sold, defective and non-defective items. The total gain is equal to,

$$pI_eD\left[\frac{1-e^{RM}}{R^2}-\frac{Me^{-RM}}{R}\right]+e^{-Rt_n}c_sP\left(n\right)I_ez\left(M-t_n\right)+pI_eDT_ne^{-RT_n}.$$

In this case, interest paid will be zero.
The total sales revenue for this case and which is represented by

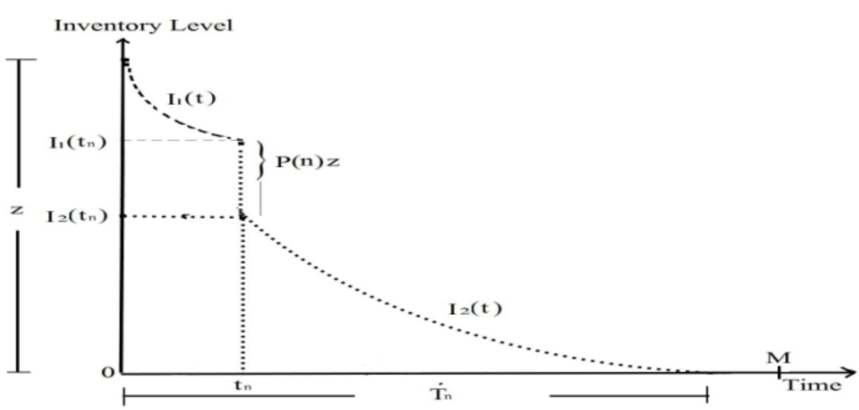
$$SR_5=SR+IE$$
$$=SR+pI_eD\left[\frac{1-e^{RM}}{R^2}-\frac{Me^{-RM}}{R}\right]+e^{-Rt_n}c_sP\left(n\right)I_ez\left(M-t_n\right)+pI_eDT_ne^{-RT_n}$$

$$TC_3=IP+TC$$
$$=C_k+C_sz+C_pz+IHC+IP$$

The retailer's total profit for this case will be equal to,

$$\Psi_3\left(T_n\right)=SR_5-TC_3$$

Figure 6. Inventory representation Case 3

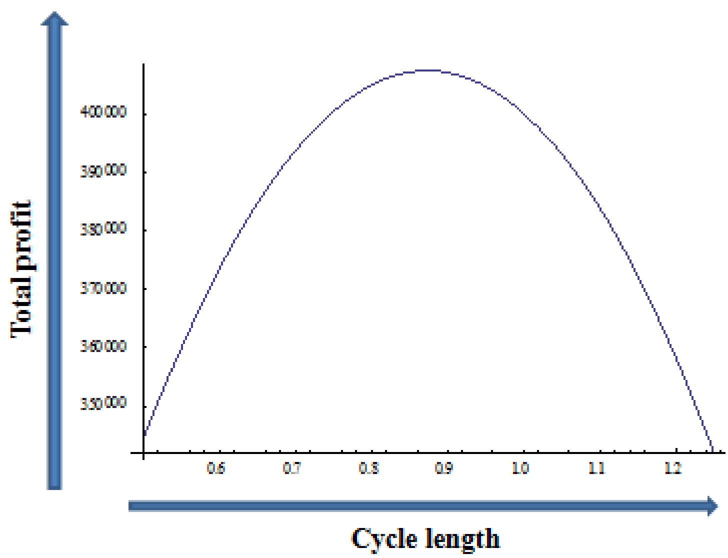


$$\Psi_3\left(T_n\right)=\begin{bmatrix}\dfrac{pD}{R}\left[1-e^{RT_n}\right]+c_sP\left(n\right)z\,e^{-Rt_n}+pIeD\left[\dfrac{1-e^{-RM}}{R^2}-\dfrac{Me^{-RM}}{R}\right]+e^{-Rt_n}p\left(n\right)C_sIe\left(M-t_n\right)\\ +pIep\left(n\right)z\left(M-t_n\right)e^{-Rt_n}\end{bmatrix}$$
$$-\begin{bmatrix}C_k+C_sz+C_pz+\dfrac{C_hz}{\theta+R}\left[1-c^{-(\theta+R)T_n}\right]+\dfrac{C_hD}{\theta}\left[\dfrac{1-e^{-(\theta+R)}}{\left(\theta+R\right)}+\dfrac{e^{-RT_n}-1}{R}\right]+\dfrac{C_hp\left(n\right)z}{R}\left[e^{-RT_n}-e^{-Rt_n}\right]\\ \end{bmatrix} \tag{15}$$

Finally, the retailer's total profits for all the three cases in the combined form, $\Psi\left(T_n\right)$

$$\Psi_i\left(T_n\right)=\begin{cases}\Psi_1\left(T_n\right) & 0\le M\le t_n\le T_n \quad case-1\\ \Psi_2\left(T_n\right) & 0\le t_n\le M\le T_n \quad case-2\\ \Psi_3\left(T_n\right) & 0\le t_n\le T_n\le M \quad case-3\end{cases} \tag{16}$$

Figure 7. Concavity of the total profit function for case 1



3.2 Solution Method

For maximizingtotal profit,the necessary condition will be, $\frac{d\Psi_i\left(T_n\right)}{dT_n} = 0, \forall\, i = 1, 2 \text{and}\, 3$ for each case.It is assumed, $T_n = T_1$ with the help of mathematical software and the value of $T_n = T_1$ (suppose) after that, second derivative will be calculated, $\frac{d^2\Psi_i\left(T_n\right)}{dT_n^{\,2}}$ $\forall\, i = 1, 2 \text{and}\, 3$ and put the value of $T_n = T_1$ in the second derivative, and then if $\frac{d^2\Psi_i\left(T_1\right)}{dT_n^{\,2}} \le 0, \forall\, i = 1, 2 \text{and}\ 3$, then $T_n = T_1$ is the maximum value of T_n which is represented by T^* is the optimal cycle length. Further, since the derivatives of total profit functions are very complicated and mathematically it is very difficult to prove the concavity, thus, concavity has been shown graphically in figure 7 for the best case. (Appendix A)

3.3 Algorithm

All the steps have been considered with the help of algorithm provided by the Shin et al. (2016).

Step1: Put all the values of the parameters related to inventory model, $[D, I_e, I_p, C_p, C_s, C_k, C_h, n, \alpha, \theta, P\left(n\right), M, p, c_s, \lambda, R]$ in equation (16).

Step2: Now, find the value of $T_n^{\,*} = T_1$ with the help of necessary condition and substituting in equations (5) and (4),then find out z^* and t_n. If $0 \le M \le t_n \le T_n$,then find out the retailer's profit for case1 from equation (13).

Step3: If step 2 not satisfied then put $T_n^{\,*} = T_1$ with the help of solution method and substituting in equations (5) and (4) then calculate z^* and t_n. If $0 \le t_n \le M \le T_n$, then find the total retailer's profit.

Step4: If step 3 not satisfied then put $T_n^{\,*} = T$ in equation (20) and substituting in equation (5) and (4) then to calculate z and t_n. If $0 \le t_n \le T_n \le M$, then find the retailer's profit related to case3 from equation (15).

Step5: In this stage, all the cases are compared, and best case will be selected in which profit is maximized with respect to the optimal cycle length.

3.4 Numerical Example

Some of the parameters are considered from Jaggi et al. (2011) and other parameters are assumed. Case1 has considered for calculation of retailer's total profit according to the algorithm.

$D = 50000$ units per year, $\lambda = 175000$ units per year, $p = \$50$ per unit, $C_s = \$0.5$ per unit, $C_p = \$25$ per unit, $C_{ho} = \$4$ per unit per year, $C_{h1} = \$1$ per unit per year, $C_{ko} = \$90$ per cycle, $C_{k1} = \$10$ per cycle, $\alpha = 0.2, \beta = 0.2, a = 40, g = 999, b = 0.7932, M = 0.15$ year, $R = 0.06$ $I_e = 0.10$ per year, $I_p = 0.15$ per year, $\theta = 0.1$ per year, $n = 5$, $p\left(5\right) = 0.0651$,

Now, all input inventory parameters are inserted in the retailer's total profit function and cycle length also calculated with the help of solution procedure and algorithm. Now, if we take for this case.

Table 2. Impact of learning rate under learning effect on inspection time, cycle length, lot size and whole profit

Learning rate b	Inspection time t_n (year)	Cycle length T_n (year)	Lot size z (units)	Total Profit for retailer $\Psi_1\left(T_n^*\right)$ ($)
0.79	0.3086	0.8735	54073	407371
0.80	0.3087	0.8739	54088	407588
0.90	0.3101	0.8810	54338	411671
1.00	0.3122	0.8917	54711	417842
1.10	0.3152	0.9066	55223	426468
1.20	0.3185	0.9252	55850	437294
1.30	0.3226	0.9455	56520	449185
1.40	0.3261	0.9646	57135	460475

$\frac{d\Psi_1\left(T_n\right)}{dT_n} = 0,$ then it got the value of cycle length, $T_n = 0.8735\,\text{year}$ provided $\frac{d^2\Psi_1\left(0.8735\right)}{dT_n^{\,2}} = -41.54 \le 0,$ it means that $T_n^* = 0.8735\,\text{year}$ is the optimal value of cycle length. After that, the optimal order quantity, $z^* = 54073\,\text{unit}$, inspection time, $t_n^* = 0.3086$ year and retailer's total profit, $\Psi_1\left(T_n^*\right) = \$\,407371$ .Numerical example is suggesting that when leaning rate is 0.79, credit financing period is 0.15year, demand rate is 50000 unit per year and inspection time is 175200 per year, take then cycle length, 0.8735year, inspection time, 0.3086year, order quantity,54073 units and retailer's profit, 407371 dollars will exit. It means that the present mode will be beneficial for the practitioners if leaning rate, from 0.79 to 1.40, number of shipments, 5 and financing period, 0.15 year. The present model will behave differently when learning rates will be out of the range from 0.79 to 1.40.

Table 3. Effects of the shipments on the inspection time, cycle length, lot size and total profit

Shipments $\left(n\right)$	Percentages defective items p(n)	Inspection time, t_n	Cycle length T_n (year)	Lot size z (units)	Retailer's total profit $\Psi_1\left(T_n^*\right)$ ($)
1	0.0699	0.3003	0.8486	52620	394939
2	0.0698	0.3033	0.8548	53143	398240
3	0.0695	0.3050	0.8600	53450	400506
4	0.0688	0.3060	0.8655	53730	403115
5	0.0675	0.3086	0.8735	54073	407371

Table 4. Effects of credit period under impact of learning on inspection time, cycle time, lot size and total profit

Fixed credit period M (year)	Inspection time t_n (year)	Cycle length T_n (year)	Lot size z (Units)	Total Profit for retailer $\Psi_1\left(T_n^*\right)$ ($)
0.013	0.2939	0.8339	51491	316584
0.027	0.2954	0.8380	51758	325781
0.041	0.2969	0.8421	52025	334997
0.068	0.2998	0.8499	52533	352822
0.12	0.3054	0.8649	53511	387342
0.15	0.3086	0.8735	54073	407371

Table 5. Impact of deteriorating rate under learning effect on inspection time, cycle length, lot size and total profit

Deterioration rate θ	Inspection time t_n (in year)	Cycle time T_n (in year)	Lot size z (in units)	Total Profit for retailer $\Psi_1\left(T_n^*\right)$ ($)
0.10	0.3086	0.8735	54073	407371
0.15	0.2781	0.7729	48734	370482
0.20	0.2531	0.6932	44358	340808
0.25	0.2363	0.6283	41404	316417
0.30	0.2145	0.5746	37590	296013

Table 6. Effects of inflation rate on total profit, lot size, cycle time and inspection time

Net discount rate of inflation rate $\left(R = r - i\right)$	Screening time t_n	Cycle length T_n (year)	Lot size z (units)	Total Profit for retailer $\Psi_1\left(T_n^*\right)$ ($)
0.10	0.2818	0.8014	49382	350372
0.08	0.2962	0.8359	51621	374833
0.06	0.3086	0.8735	54043	407371
0.04	0.3240	0.9147	56774	452080
0.02	0.3410	0.9599	59752	577624

The effect of the learning rate has been briefly explained in the sensitive analysis section in Table 2. This study reveals that impact of learning is very effective tool with permissible delay in payment and inflation rate in this model under the influence of learning for decaying articles with defective quality. The effect of cycle length, inspection time, order quantity as well as profit over the number of shipments, learning rate, trade credit, deterioration rate, total discount rate, and percentage defective has been studied in the next section 4.

4. SENSITIVITY ANALYSIS

The sensitive analysis has been shown on the model parameters which aregiven below.

Managerial Insights

- It is observed from table 2, if the value of learning rate increases then cycle length, inspection time, lot size and the retailer's profit increase. The saturation level of retailer's total profit achieved when learning rate reaches to a certain level ($b = 0.7932$). Findings clearly suggest that the presence of learning has positive effect on the order quantity. When the learning will increase then the order quantity will increase which results in higher profit. Learning will make the system for efficient and effective and smooth the supply chain process.
- It can be analyzed from table 3, if the number of shipments increases, then the cycle length, inspection time, order quantity as well as retailer's total profit increase due to the learning effect in $C_h\left(n\right)$, $C_k\left(n\right)$ and $p\left(n\right)$. It indicates that retailer's total profit; cycle time and order quantity are affected by number of shipments. Number of shipments increase then it results in reduction in the holding cost, but transportation cost will increase. Percentage of defective increases marginally and cycle time and order quantity increase significantly.
- It can be observed from table 4, if the value of M increases, then the total profit is increasing as the interest earned will increase which leads more profit. Increase in the value of *M* facilitates the retailer to earn more profit and hold funds for more time and increase his interest earned part. Which results in more profit.
- From table 5, the value of deterioration rate increases, the order cycle time, inspection time, lot size and retailer's profit decrease. Due to deterioration the utility of the goods decreases. From results we can see that it is optimal for the retailer to order frequently in small lots which will help to reduce the deterioration impact on the items and increase the profit. Deterioration rate increase

then the items are more sensitive and deteriorate soon, retailer must order more frequently less quantity which will reduce the holding cost and increase the total profit.

- It is studied from the table 6, if the inflation rate decreases then the total profit and lot size increase. As under inflationary conditions the price of goods increases; therefore, the retailer would like to order large quantity for longer period which helps him to increases his profit. In case of high inflation rate it is advisable that retail should order more which leads to increase in the holding cost but this can be compensated from the sales.

5. CONCLUSION

In this paper an inventory model is developed with learning process and trade-credit financing under inflationary condition for imperfect quality items. The results reveal that learning effect increase the efficiency of supply chain. The total profit will be maximized with respect to optimal cycle length. Eventually, it is analyzed that the output of this present study demonstrated that the percentage defective items per shipment and cost reduces as learning parameters increases. Finally, supply chain partners will be benefit when the trade-credit financing policy is applied with inflationary condition in the present model. The present paper can be very helpful in the supply chain management for retailer's ordering policy in developing countries where inflationary situation becomes. Present work can be extended for credit dependent demand with carbon emission impact.

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REFERENCES

Agarwal, A., Sangal, I., & Singh, S. R. (2017). Optimal policy for non-Instantaneous decaying inventory model with learning effect and partial shortages. *International Journal of Computers and Applications*, *161*(10), 13–18. doi:10.5120/ijca2017913318

Aggarwal, S. P., & Jaggi, C. K. (1995). Ordering policies of deteriorating items under permissible delay in payments. *The Journal of the Operational Research Society*, *46*(5), 658–662. doi:10.1057/jors.1995.90

Anzanello, M. J., & Fogliatto, F. S. (2011). Learning curve models and applications: Literature review and research directions. *International Journal of Industrial Ergonomics*, *41*(5), 573–583. doi:10.1016/j.ergon.2011.05.001

Argote, L., Beckman, S. L., & Epple, D. (1990). The persistence and transfer of learning in industrial settings. *Management Science*, *36*(2), 140–154. doi:10.1287/mnsc.36.2.140

Buzacott, J. A. (1975). Economic order quantities with inflation. *The Journal of the Operational Research Society*, *26*(3), 553–558. doi:10.1057/jors.1975.113

Chang, C. T., Ouyang, L. Y., & Teng, J. T. (2003). An EOQ model for deteriorating items under supplier credits linked to ordering quantity. *Applied Mathematical Modelling*, *27*(12), 983–996. doi:10.1016/S0307-904X(03)00131-8

Chen, L. H., & Kang, F. S. (2010). Integrated inventory models considering the two-level trade credit policy and a price-negotiation scheme. *European Journal of Operational Research*, *205*(1), 47–58. doi:10.1016/j.ejor.2009.11.028

Cheung, K. L., & Hausman, W. H. (1997). Joint determination of preventive maintenance and safety stocks in an unreliable production environment. *Naval Research Logistics*, *44*(3), 257–272. doi:10.1002/(SICI)1520-6750(199704)44:3<257::AID-NAV2>3.0.CO;2-7

Chu, W. H. J., & Lee, C. C. (2006). Strategic information sharing in a supply chain. *European Journal of Operational Research*, *174*(3), 1567–1579. doi:10.1016/j.ejor.2005.02.053

Chung, K. J. (1998). A theorem on the determination of economic order quantity under conditions of permissible delay in payments. *Computers & Operations Research*, *25*(1), 49–52. doi:10.1016/S0305-0548(98)80007-5

Chung, K. J., & Huang, Y. F. (2003). The optimal cycle time for EPQ inventory model under permissible delay in payments. *International Journal of Production Economics*, *84*(3), 307–318. doi:10.1016/S0925-5273(02)00465-6

Cunningham, J. A. (1980). Using the learning curve as management tool. *IEEE Spectrum*, *17*(6), 43–48.

Datta, T. K., & Pal, A. K. (1991). Effects of inflation and time-value of money on an inventory model with linear time-dependent demand rate and shortages. *European Journal of Operational Research*, *52*(3), 326–333. doi:10.1016/0377-2217(91)90167-T

De, K. S., Kundu, P. K., & Goswami, A. (2003). An economic production quantity inventory model involving fuzzy demand rate and fuzzy deterioration rate. *Journal of Applied Mathematics & Computing*, *12*(1), 251–260.

Dutton, J. R., & Thomas, A. (1984). Treating progress function as a managerial opportunity. *Academy of Management Review*, *9*(2), 235–247. doi:10.2307/258437

Ghare, P. M., & Schrader, G. P. (1963). A model for exponentially decaying inventory model. *Journal of Industrial Engineering*, *14*(5), 238–243.

Givi, Z. S., Jaber, M. Y., & Neumann, W. P. (2015). Modelling worker reliability with learning and fatigue. *Applied Mathematical Modelling*, *39*(17), 5186–5199. doi:10.1016/j.apm.2015.03.038

Globerson, S., Levin, N., & Shtub, A. (1989). The impact of breaks on forgetting when performing a repetitive task. *IIE Transactions*, *21*(5), 376–381. doi:10.1080/07408178908966244

Goyal, S. K. (1985). Economic order quantity under conditions of permissible delay in payments. *The Journal of the Operational Research Society*, *36*(4), 335–338. doi:10.1057/jors.1985.56

Hariga, M. (1995). An EOQ model for deteriorating items with shortages and time-varying demand. *The Journal of the Operational Research Society*, *46*(3), 398–404. doi:10.1057/jors.1995.54

Hariga, M. A., & Ben-Daya, M. (1996). Optimal time varying lot-sizing models under inflationary conditions. *European Journal of Operational Research*, *89*(2), 313–325. doi:10.1016/0377-2217(94)00256-8

Huang, Y. F. (2003). Optimal retailer's ordering policies in the EOQ model under trade credit financing. *The Journal of the Operational Research Society*, *54*(9), 1011–1015. doi:10.1057/palgrave.jors.2601588

Jaber, M. Y., & Bonney, M. (1996). Optimal lot sizing under learning considerations: The bounded learning case. *Applied Mathematical Modelling*, *20*(10), 750–755. doi:10.1016/0307-904X(96)00072-8

Jaber, M. Y., & Bonney, M. (1996). Production breaks and the learning curve: The forgetting phenomenon. *Applied Mathematical Modelling*, *2*(20), 162–169. doi:10.1016/0307-904X(95)00157-F

Jaber, M. Y., & Bonney, M. (1997). A comparative study of learning curves with forgetting. *Applied Mathematical Modelling*, *21*(8), 523–531. doi:10.1016/S0307-904X(97)00055-3

Jaber, M. Y., & Bonney, M. (2003). Lot sizing with learning and forgetting in set-ups and in product quality. *International Journal of Production Economics*, *83*(1), 95–111. doi:10.1016/S0925-5273(02)00322-5

Jaber, M. Y., Goyal, S. K., & Imran, M. (2008). Economic production quantity model for items with imperfect quality subject to learning effects. *International Journal of Production Economics*, *115*(1), 143–150. doi:10.1016/j.ijpe.2008.05.007

Jaber, M. Y., & Guiffrida, A. L. (2004). Learning curves for processes generating defects requiring reworks. *European Journal of Operational Research*, *159*(3), 663–672. doi:10.1016/S0377-2217(03)00436-3

Jaber, M. Y., & Guiffrida, A. L. (2008). Learning curves for imperfect production processes with reworks and process restoration interruptions. *European Journal of Operational Research*, *189*(1), 93–104. doi:10.1016/j.ejor.2007.05.024

Jaber, M. Y., & Khan, M. (2010). Managing yield by lot splitting in a serial production line with learning, rework and scrap. *International Journal of Production Economics*, *124*(1), 32–39. doi:10.1016/j.ijpe.2009.09.004

Jaber, M. Y., & Salameh, M. K. (1995). Optimal lot sizing under learning considerations: Shortages allowed and backordered. *Applied Mathematical Modelling*, *19*(5), 307–310. doi:10.1016/0307-904X(94)00040-D

Jaggi, C. K., Goel, S. K., & Mittal, M. (2011). Pricing and replenishment policies for imperfect quality deteriorating items under inflation and permissible delay in payments. *International Journal of Strategic Decision Sciences*, *2*(2), 20–35. doi:10.4018/jsds.2011040102

Jaggi, C. K., Goel, S. K., & Mittal, M. (2013). Credit financing in economic ordering policies for defective items with allowable shortages. *Applied Mathematics and Computation*, *219*(10), 5268–5282. doi:10.1016/j.amc.2012.11.027

Jaggi, C. K., Goyal, S. K., & Goel, S. K. (2008). Retailer's optimal replenishment decisions with credit-linked demand under permissible delay in payments. *European Journal of Operational Research*, *190*(1), 130–135. doi:10.1016/j.ejor.2007.05.042

Jaggi, C. K., Khanna, A., & Mittal, M. (2011a). Credit financing for deteriorating imperfect-quality items under inflationary conditions. *International Journal of Services Operations and Informatics*, *6*(4), 292–309. doi:10.1504/IJSOI.2011.045560

Jaggi, C. K., Tiwari, S., & Goel, S. K. (2017). Credit financing in economic ordering policies for non-instantaneous deteriorating items with price dependent demand and two storage facilities. *Annals of Operations Research*, *248*(1), 253–280. doi:10.1007/s10479-016-2179-3

Jaggi, C. K., Tiwari, S., & Shafi, A. (2015). Effect of deterioration on two-warehouse inventory model with imperfect quality. *Computers & Industrial Engineering*, *88*, 378–385. doi:10.1016/j.cie.2015.07.019

Jamal, A. M. M., Sarker, B. R., & Wang, S. (1997). An ordering policy for deteriorating items with allowable shortages and permissible delay in payment. *The Journal of the Operational Research Society*, *48*(8), 826–833. doi:10.1057/palgrave.jors.2600428

Jamal, A. M. M., Sarker, B. R., & Wang, S. (2000). Optimal payment time for a retailer under permitted delay of payment by the wholesaler. *International Journal of Production Economics*, *66*(1), 59–66. doi:10.1016/S0925-5273(99)00108-5

Jayaswal, M., Sangal, I., Mittal, M., & Malik, S. (2019). Effects of learning on retailer ordering policy for imperfect quality items with trade credit financing. *Uncertain Supply Chain Management*, *7*(1), 49–62. doi:10.5267/j.uscm.2018.5.003

Jayaswal, M. K., Mittal, M., Sangal, I., & Tripathi, J. (2021). Fuzzy-Based EOQ Model with Credit Financing and Backorders Under Human Learning. *International Journal of Fuzzy System Applications*, *10*(4), 14–36. doi:10.4018/IJFSA.2021100102

Jayaswal, M. K., Sangal, I., & Mittal, M. (2019). Effect of Learning on the Economy Ordering Policy for Defective Items under Fuzzy Environment with Permissible Delay in Payment. *Investigação Operacional*, *40*(5), 574–599.

Khan, M., Jaber, M. Y., & Wahab, M. I. M. (2010). Economic order quantity model for items with imperfect quality with learning in inspection. *International Journal of Production Economics*, *124*(1), 87–96. doi:10.1016/j.ijpe.2009.10.011

Kim, S. J., Hwang, H., & Shinn, W. S. (1995). An optimal credit policy to increase wholesaler's profit with price dependent demand function. *Production Planning and Control*, *6*(1), 45–50. doi:10.1080/09537289508930252

Konstantaras, I., Skouri, K., & Jaber, M. Y. (2012). Inventory models for imperfect quality items with shortages and learning in inspection. *Applied Mathematical Modelling*, *36*(11), 5334–5343. doi:10.1016/j.apm.2011.12.005

Kumar, V., Sarkar, B., Sharma, A. N., & Mittal, M. (2019). New product launching with pricing, free replacement, rework, and warranty policies via genetic algorithmic approach. *International Journal of Computational Intelligence Systems*, *12*(2), 519–529. doi:10.2991/ijcis.d.190401.001

Mittal, M., Jayaswal, M. K., & Kumar, V. (2021). Effect of learning on the optimal ordering policy of inventory model for deteriorating items with shortages and trade-credit financing. *International Journal of System Assurance Engineering and Management*, 1-11.

Nobil, A. H., Afshar Sedigh, A. H., Tiwari, S., & Wee, H. M. (2019). An imperfect multi-item single machine production system with shortage, rework, and scrapped considering inspection, dissimilar deficiency levels and non-zero setup times. *Scientia Iranica*, *26*(1), 557–570.

Nobil, A. H., Tiwari, S., & Tajik, F. (2019). Economic production quantity model considering warm-up period in a cleaner production environment. *International Journal of Production Research*, *57*(14), 4547–4560. doi:10.1080/00207543.2018.1518608

Patro, R., Acharya, M., Nayak, M. M., & Patnaik, S. (2018). A fuzzy EOQ model for deteriorating items with imperfect quality using proportionate discount under learning effects. *International Journal of Management and Decision Making*, *17*(2), 171–198. doi:10.1504/IJMDM.2018.092557

Porteus, E. L. (1986). Optimal lot sizing, process quality improvement and setup cost reduction. *Operations Research*, *34*(1), 137–144. doi:10.1287/opre.34.1.137

Salameh, M. K., Abdul-Malak, M. A. U., & Jaber, M. Y. (1993). Mathematical modelling of the effect of human learning in the finite production inventory model. *Applied Mathematical Modelling*, *17*(11), 613–615. doi:10.1016/0307-904X(93)90070-W

Salameh, M. K., & Jaber, M. Y. (2000). Economic production quantity model for items with imperfect quality. *International Journal of Production Economics*, *64*(1-3), 59–64. doi:10.1016/S0925-5273(99)00044-4

Sangal, I., Agarwal, A., & Rani, S. (2017). A fuzzy environment inventory model with partial backlogging under learning effect. *International Journal of Computers and Applications*, *137*(6), 25–32. doi:10.5120/ijca2016908793

Sarkar, B. (2016). Supply chain coordination with variable backorder, inspection and discount policy for fixed lifetime products. *Mathematical Problems in Engineering*, *20*(2), 1–14. doi:10.1155/2016/6318737

Sarker, B. R., & Pan, H. (1994). Effects of inflation and the time value of money on order quantity and allowable shortage. *International Journal of Production Economics*, *34*(1), 65–72. doi:10.1016/0925-5273(94)90047-7

Shah, N. H. (1993a). A lot-size model for exponentially decaying inventory when delay in payments is permissible. *Cahiers Du CERO*, *35*(5), 115–123.

Shah, N. H. (1993b). A probabilistic order level system when delay in payments is permissible. *Journal of Korea*, *18*(4), 175–182.

Shin, D., Mittal, M., & Sarkar, B. (2018). Effects of human errors and trade credit financing in two echelon supply chain models. *European Journal of Industrial Engineering*, *12*(4), 465–503. doi:10.1504/EJIE.2018.093574

Shinn, S. W., & Hwang, H. (2003). Optimal pricing and ordering policies for retailers under order-size-dependent delay in payments. *Computers & Operations Research*, *30*(1), 35–50. doi:10.1016/S0305-0548(01)00076-4

Shinn, S. W., Hwang, H., & Sung, S. P. (1996). Joint price and lot size determination under conditions of permissible delay in payments and quantity discounts for freight cost. *European Journal of Operational Research*, *91*(3), 528–542. doi:10.1016/0377-2217(94)00357-2

Singh, T., Sethy, N. N., Nayak, A. K., & Pattnaik, H. (2021). An Optimal Policy for Deteriorating Items With Generalized Deterioration, Trapezoidal-Type Demand, and Shortages. *International Journal of Information Systems and Supply Chain Management*, *14*(1), 23–54. doi:10.4018/IJISSCM.2021010102

Teng, J. T., & Chang, C. T. (2009). Optimal manufacturer's replenishment policies in the EPQ model under two levels of trade credit policy. *European Journal of Operational Research*, *195*(2), 358–363. doi:10.1016/j.ejor.2008.02.001

Teng, J. T., Lou, K. R., & Wang, L. (2014). Optimal trade credit and lot size policies in economic production quantity models with learning curve production costs. *International Journal of Production Economics*, *155*(5), 318–323. doi:10.1016/j.ijpe.2013.10.012

Tiwari, S., Ahmed, W., & Sarkar, B. (2018). Multi-item sustainable green production system under trade-credit and partial backordering. *Journal of Cleaner Production*, *204*, 82–95. doi:10.1016/j.jclepro.2018.08.181

Tiwari, S., Cárdenas-Barrón, L. E., Goh, M., & Shaikh, A. A. (2018). Joint pricing and inventory model for deteriorating items with expiration dates and partial backlogging under two-level partial trade credits in supply chain. *International Journal of Production Economics*, *200*, 16–36. doi:10.1016/j.ijpe.2018.03.006

APPENDIX A – ADDITIONAL EQUATIONS

Calculation for Case1:

$$\frac{d\Psi_1\left(T_n\right)}{dT_n} = pDe^{-RT_n} + p\,p\left(n\right)z'e^{-Rt_n} - \left[\begin{array}{l} C_p z' + C_s z' + \frac{C_h z'}{\theta+R}\left[1-e^{-\left(\theta+R\right)T_n}\right] + C_h z e^{-\left(\theta+R\right)T_n} \\ + \frac{C_h D}{\theta}\left[e^{-\left(\theta+R\right)T_n} - e^{-RT_n}\right] + \frac{C_h p\left(n\right)z'}{R}\left[e^{-RT_n} - e^{-Rt_n}\right] \\ -C_h p\left(n\right) z e^{-RT_n} + c_P I_p \left[\begin{array}{l} \frac{z'}{\theta+R}\left[e^{-\left(\theta+R\right)T_n} - e^{-RT_n}\right] \\ + z e^{-\left(\theta+R\right)T_n} \end{array}\right] + \\ c_p I_p \frac{D}{\theta}\left[e^{-\left(\theta+R\right)T_n} - e^{-RT_n}\right] + \frac{p\left(n\right)z'}{R}\left[e^{-RT_n} - e^{-Rt_n}\right] \\ -zp\left(n\right)e^{-RT_n} \end{array}\right] \quad \text{(i)}$$

$$\frac{d^2\Psi_1\left(T_n\right)}{dT_n^{\,2}} = -pRDe^{-RT_n} + p\,p\left(n\right)z''e^{-Rt_n} - \left[\begin{array}{l} C_p z'' + C_s z'' + \frac{C_h z''}{\theta+R}\left[1-e^{-\left(\theta+R\right)T_n}\right] + C_h z' e^{-\left(\theta+R\right)T_n} \\ + C_h z' e^{-\left(\theta+R\right)T_n} \\ -C_h z\left(\theta+R\right)e^{-\left(\theta+R\right)T_n} + \frac{C_h D}{\theta}\left[\begin{array}{l} -e^{-\left(\theta+R\right)T_n}\left(\theta+R\right) \\ + \mathrm{Re}^{-RT_n} \end{array}\right] \\ + \frac{C_h p\left(n\right)z''}{R}\left[e^{-RT_n} - e^{-Rt_n}\right] - \frac{C_h p\left(n\right)z'}{R}\left[\mathrm{Re}^{-RT_n}\right] \\ -C_h p\left(n\right)z'e^{-RT_n} + C_h R\,p\left(n\right)ze^{-RT_n} \\ + c_P I_p \left[\begin{array}{l} \frac{z''}{\theta+R}\left[e^{-\left(\theta+R\right)T_n} - e^{-RT_n}\right] + 2z'e^{-\left(\theta+R\right)T_n} \\ -z\left(\theta+R\right)e^{-\left(\theta+R\right)T_n} \end{array}\right] + \\ c_p I_p \frac{D}{\theta}\left[-e^{-\left(\theta+R\right)T_n}\left(\theta+R\right) + \mathrm{Re}^{-RT_n}\right] + \\ \frac{p\left(n\right)z''}{R}\left[e^{-RT_n} - e^{-Rt_n}\right] - \frac{p\left(n\right)z'}{R}\left[\mathrm{Re}^{-RT_n}\right] \\ -z'p\left(n\right)e^{-RT_n} + zRp\left(n\right)e^{-RT_n} \end{array}\right] \quad \text{(ii)}$$

The first and second derivative of the profit function for the case2:

$$\frac{d\Psi_2\left(T_n\right)}{dT_n} = pDe^{-RT_n} + p\,p\left(n\right)z'e^{-Rt_n} + e^{-Rt_n}pIep\left(n\right)z'\left(M-t_n\right)$$
$$-\left[\begin{array}{l} C_p z' + C_s z' + \frac{C_h z'}{\theta+R}\left[1-e^{-\left(\theta+R\right)T_n}\right] + C_h z e^{-\left(\theta+R\right)T_n} \\ + \frac{C_h D}{\theta}\left[e^{-\left(\theta+R\right)T_n} - e^{-RT_n}\right] \\ + \frac{C_h p\left(n\right)z'}{R}\left[e^{-RT_n} - e^{-Rt_n}\right] - C_h p\left(n\right)ze^{-RT_n} \\ + c_P I_p\left[\frac{z'}{\theta+R}\left[e^{-\left(\theta+R\right)T_n} - e^{-RT_n}\right] + ze^{-\left(\theta+R\right)T_n}\right] + \\ c_p I_p \frac{D}{\theta}\left[e^{-\left(\theta+R\right)T_n} - e^{-RT_n}\right] + \frac{p\left(n\right)z'}{R}\left[e^{-RT_n} - e^{-Rt_n}\right] - zp\left(n\right)e^{-RT_n} \end{array}\right] \quad \text{(iii)}$$

$$\frac{d^2\Psi_2\left(T_n\right)}{dT_n^{\,2}} = -pRDe^{-RT_n} + p\,p\left(n\right)z''e^{-Rt_n} + e^{-Rt_n}pIez''\left(M-t_n\right)$$
$$-\begin{bmatrix} C_p z'' + C_s z'' + \frac{C_h z''}{\theta+R}\left[1-e^{-(\theta+R)T_n}\right] + C_h z' e^{-(\theta+R)T_n} \\ +C_h z' e^{-(\theta+R)T_n} - C_h z\left(\theta+R\right)e^{-(\theta+R)T_n} + \frac{C_h D}{\theta}\begin{bmatrix}-e^{-(\theta+R)T_n}\left(\theta+R\right) \\ +\mathrm{Re}^{-RT_n}\end{bmatrix} \\ +\frac{C_h p\left(n\right)z''}{R}\left[e^{-RT_n}-e^{-Rt_n}\right] - \frac{C_h p\left(n\right)z'}{R}\left[\mathrm{Re}^{-RT_n}\right] \\ -C_h p\left(n\right)z'e^{-RT_n} + C_h R\,p\left(n\right)ze^{-RT_n} \\ +c_P I_p\begin{bmatrix}\frac{z''}{\theta+R}\left[e^{-(\theta+R)T_n}-e^{-RT_n}\right]+2z'e^{-(\theta+R)T_n} \\ -z\left(\theta+R\right)e^{-(\theta+R)T_n}\end{bmatrix}+ \\ c_p I_p\frac{D}{\theta}\left[-e^{-(\theta+R)T_n}\left(\theta+R\right)+\mathrm{Re}^{-RT_n}\right]+\frac{p\left(n\right)z''}{R}\left[e^{-RT_n}-e^{-Rt_n}\right]-\frac{p\left(n\right)z'}{R}\left[\mathrm{Re}^{-RT_n}\right] \\ -z'p\left(n\right)e^{-RT_n}+zRp\left(n\right)e^{-RT_n}\end{bmatrix} \tag{iv}$$

The first and second derivatives of the profit function for the case3:

$$\frac{d\Psi_3\left(T_n\right)}{dT_n} = pDe^{-RT_n} + p\,p\left(n\right)z'e^{-Rt_n} + e^{-Rt_n}pIep\left(n\right)z'\left(M-t_n\right) + pIeDM\left[e^{-RT_n} - RT_n e^{-RT_n}\right]$$
$$-pIeD\left[2Te^{-RT_n} - T^2\,\mathrm{Re}^{-RT_n}\right] \tag{v}$$
$$-\begin{bmatrix} C_p z' + C_s z' + \frac{C_h z'}{\theta+R}\left[1-e^{-(\theta+R)T_n}\right] + C_h z e^{-(\theta+R)T_n} \\ +\frac{C_h D}{\theta}\left[e^{-(\theta+R)T_n}-e^{-RT_n}\right]+\frac{C_h p\left(n\right)z'}{R}\left[e^{-RT_n}-e^{-Rt_n}\right]-C_h p\left(n\right)ze^{-RT_n}\end{bmatrix}$$

$$\frac{d^2\Psi_3\left(T_n\right)}{dT_n^{\,2}} = -pRDe^{-RT_n} + p\,p\left(n\right)z''e^{-Rt_n} + e^{-Rt_n}pIez''\left(M-t_n\right) + pIeDM\left[-2\,\mathrm{Re}^{-RT_n} - R^2T_n e^{-RT_n}\right]$$
$$-\begin{bmatrix} C_p z'' + C_s z'' + \frac{C_h z''}{\theta+R}\left[1-e^{-(\theta+R)T_n}\right] + C_h z' e^{-(\theta+R)T_n} + C_h z' e^{-(\theta+R)T_n} - C_h z\left(\theta+R\right)e^{-(\theta+R)T_n} \\ +\frac{C_h D}{\theta}\begin{bmatrix}-e^{-(\theta+R)T_n}\left(\theta+R\right) \\ +\mathrm{Re}^{-RT_n}\end{bmatrix}+\frac{C_h p\left(n\right)z''}{R}\left[e^{-RT_n}-e^{-Rt_n}\right]-\frac{C_h p\left(n\right)z'}{R}\left[\mathrm{Re}^{-RT_n}\right] \\ -C_h p\left(n\right)z'e^{-RT_n}+C_h R\,p\left(n\right)ze^{-RT_n}\end{bmatrix} \tag{vi}$$

Where,

$$z' = \frac{D\,e^{\theta T_n}\left[1 - Dp\left(n\right)\right] + Dp\left(n\right)e^{2\theta T_n}\left(D-1\right)}{\left(1 - P\left(n\right)\right)^2}, t_n = \frac{z'}{\lambda}$$ in the first derivative.

$$z'' = \frac{\begin{array}{l}\left(1 - p(n)e^{\theta T_n}\right)^2 \theta\, D\, e^{\theta T_n}\left[1 - Dp\left(n\right)\right] + \\ 2\,\theta\, Dp\left(n\right)e^{2\theta T_n}\left(D-1\right)\left(1 - p\left(n\right)e^{\theta T_n}\right)^2 \\ -2p\left(n\theta\right)e^{\theta T_n}\left(1 - p\left(n\right)e^{\theta T_n}\right)\left[De^{\theta T_n}\left(1 - p\left(n\right)e^{\theta T_n}\right) + Dp\left(n\right)e^{2\theta T_n}\left(D-1\right)\right]\end{array}}{\left(1 - P\left(n\right)\,e^{\theta T_n}\right)^4}$$ and $t_n = \frac{z''}{\lambda}$ in the second derivative.

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