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# Exploring Candidates' Initial Images of Similar Figures

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## ABSTRACT

This article investigates preservice candidates' knowledge of similar figures while engaged in teacher-related mathematical tasks (TRMT) with the primary goal of developing a lesson plan on similar figures. This investigation was part of a larger study investigating candidate knowledge of what and how to teach proportional reasoning concepts while engaged in TRMT. The data collection and TRMT cycle included individual discussions and reflections, group planning and discussions, and group presentations. Individual candidates also wrote their teaching philosophies and math autobiographies, which were reviewed. While candidates appeared to grasp many of the aspects of the concept of similar figures and similarity, there was a lack of precision and strength in their representations. The knowledge base of the candidates in both mathematics and teaching varied. As the study progressed, candidates relied heavily on their subject matter knowledge to justify their images and ideas.



## KEYWORDS

Candidate Knowledge, Content Knowledge, Mathematical Knowledge, Pedagogical Content Knowledge, Similarity, Subject Matter Knowledge, Teacher Education, Teacher Related Tasks


## INTRODUCTION

The exploration of pre-service candidates and their journey in education has been a longstanding pursuit. Over the years, much of the research in this area of education has revolved around two concepts: knowledge and practice (Oliveira & Henriques, 2021; Charalambous et al., 2020; Da Ponte & Chapman, 2015; Ball et al., 2008; Ma, 1999; Even, 1990; Shulman, 1986). What has been learned is that candidates need to be engaged in activities that enhance their understanding of *what* and *how* to teach mathematics. Stylianides and Stylianides (2006) define teacher-related mathematical tasks (TRMT) as encompassing this dual role through a series of activities that focus on a primary goal. Engaging in activities, such as TRMT is seen as crucial for candidates, as they strive to cultivate their knowledge and skills for effective teaching (Oliveira & Henriques, 2021; Akar, 2015; Da Ponte & Chapman, 2015; Ball, et al., 2008; Stylianides & Stylianides, 2006).

This investigation of pre-service candidate teachers' knowledge of similar figures was part of a larger study that investigated candidate knowledge of *what* and *how* to teach proportional reasoning concepts, while engaged in TRMT. The TRMT involved a combination of teaching and research methods. The research was based on standards from the National Council of Teachers of Mathematics' (NCTM) which provide a vision for the teaching of mathematics (NCTM, 2000; NCTM, 2007).

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During the explorations of TRMT, researchers focused on understanding candidates' initial images of similar figures, and with this focus, formulated the following research question:

*When given opportunities to discuss and plan a lesson, what are candidates' images of what and how to teach similar figures?*

Therefore, this study has the potential to enhance the body of research concerning candidates' images of similar figures while engaged in TRMT. It accomplishes this through a conceptual framework that supports candidates' subject matter knowledge (SMK) and pedagogical content knowledge (PCK). While the primary focus is on a group of initial licensure candidates, this research also provides insights into a teacher education course that implements TRMT.

## CONCEPTUAL FRAMEWORK

Stylianides and Stylianides (2006) define TRMT as having a dual purpose. They are mathematics tasks that are connected to teaching, which also support the learning of mathematics and *what* is important to teach and help teachers to see *how* the mathematics relate to teaching. A "task" is described as a series of related activities focusing on ideas and aims promoting a goal (Stylianides & Stylianides, 2006). Lappan and Even (1989) state that to support students' mathematical empowerment, "teachers themselves need to know mathematics and experience learning in ways that build a deep and flexible understanding of what mathematics is and what it means to do mathematics" (p.22). TRMT can be seen as a hybrid of two types of teaching experiments: classroom experiments (Borba & Confrey, 1993; Cobb & Steffe, 1983; Steffe, 1991) and candidate teacher development (Simon, 1995).

Throughout the years, teacher education programs took on the responsibility for much of the research, training, and support of candidates' SMK and PCK. Teachers' knowledge of the subject matter needed for teaching is seen as diverse and multidimensional (Grossman et al., 1989; Wilson et al., 1987). Given the possible repercussions teachers' knowledge of subject matter may have on their pedagogy, many teacher educators considered ways to incorporate discussions of subject matter into teacher education programs (Ball et al., 2008; Grossman et al., 1989). Ma (1999) stated that it is during teacher education programs that candidates have one of only a few opportunities to cultivate their SMK of school mathematics and that "their mathematical competence starts to be connected to a primary concern about teaching and learning school mathematics" (p.145). Several researchers worked with candidates to advance teacher education programs and develop courses promoting teachers' knowledge (Ball, 1988; Ball & McDiarmid, 1990; Ball et al., 2008; Lappan & Even, 1989). However, there is an additional concern as to the effect of too few university courses on teachers' knowledge when these courses are intended to replace some of the instilled beliefs about teaching and learning that candidates developed as students in their K-12 mathematics classes (Ball et al., 2008; Ball et al., 2001).

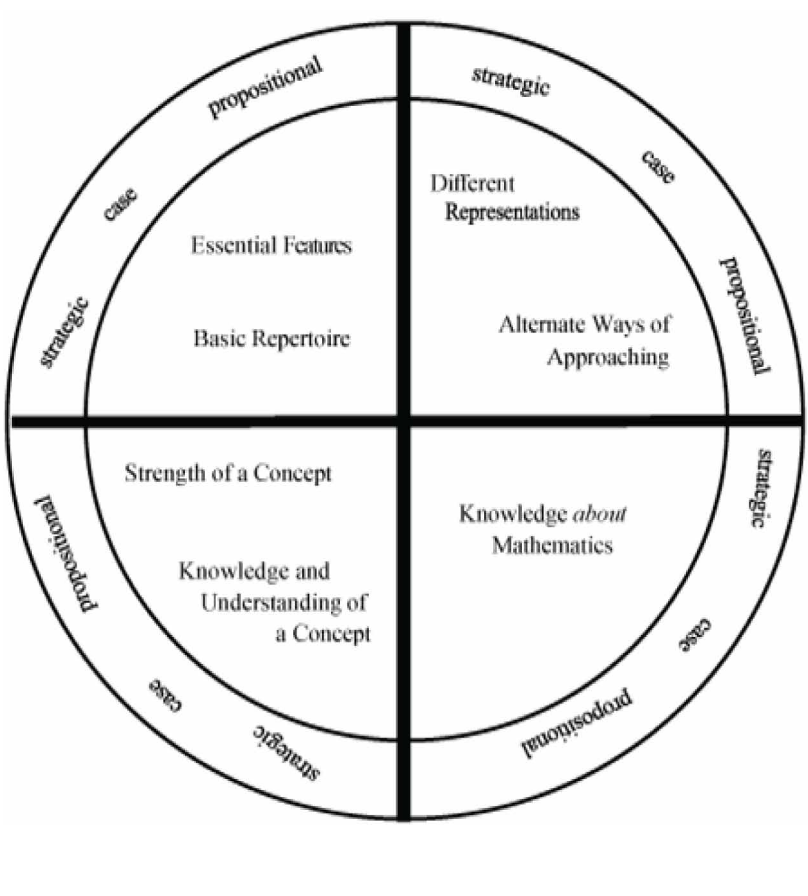
The concept of mathematical knowledge *for* teaching corresponds to and represents an effort to further refine Shulman's notions of SMK and PCK (Shulman, 1986). In his research, Shulman described three forms of knowledge that teachers use in practice: propositional knowledge, case knowledge, and strategic knowledge. Propositional knowledge is knowledge from examples of literature that contain useful principles about teaching. It is also considered the "wisdom of practice", empirical principles, norms, values, and the ideological and philosophical principles of teaching. Case knowledge is specific, well-documented, descriptive events of propositional knowledge. Cases are occurrences in practice that are detailed descriptions exemplifying theoretical claims and communicating principles of practice and norms. Strategic knowledge is used when a teacher cannot rely on propositional or case knowledge. During these times, the teacher must formulate a solution when no simple interpretation appears possible. Teachers' strategic knowledge goes beyond principles or specific experiences and is formulated using alternate approaches.

Even's (1990) aspects of SMK, along with Shulman's (1986) forms of knowledge were reviewed for potential images in the context of *what* and *how* to teach while engaged in the TRMT. It is within discussions and presentations to introduce the topic of similarity that a candidate's knowledge of similarity can be explored. Figure 1 represents the conceptual framework using Even's Aspects in Examining SMK and Shulman's Three Forms of Knowledge.

Shulman's forms of knowledge (propositional, case, and strategic) will be used in conjunction with Even's (1990) aspects in examining the candidates' SMK. The candidates' examples, definitions, activities, and explanations are examined to see if they are using propositional, case, or strategic knowledge. If the candidates use approaches that are grounded in theory, such as cooperative learning, then it may be possible they are accessing their propositional knowledge. When the candidates have been previously exposed to an approach, definition, or example and use it in their explanations, examples, or activity, they may be accessing their case knowledge. The candidates' strategic knowledge may be looked at if they seem to encounter a situation that they are unfamiliar with and resolve it with a new approach that they had not seen or used before. It is important to note that the learners, as well as the context, determine the type of knowledge being used.

Researching the *what* and *how* of teaching in the context of TRMT further explores the relationship between PCK and SMK. Even's Framework (1990) noted that while each aspect of the framework has been defined independently, there is an interconnectedness among the aspects of SMK. The aspects of essential features and basic repertoire will be examined to seek candidates' initial insights, definitions, and examples used to initially describe *what* is similarity. The aspects of different representations and alternate ways of approaching will be examined to look at *how* the candidates are approaching

**Figure 1. Framework using Even's examination of SMK and Shulman's three knowledge forms**



the topic and the examples they use to define the concept of similarity. The strength of a concept along with knowledge and understanding of a concept will be examined when candidates provide insight into *what* relationships exist between procedural and conceptual knowledge of similarity, along with *how* candidates use opportunities to explore similarity. Candidates' knowledge about *what* mathematics is, is general knowledge that gives inquiry, truth, and construction to *how* knowledge and understanding is used in mathematics. Again, Shulman's (1986) forms of knowledge (propositional, case, and strategic) will be explored in relation to the aspects within the framework.

Heaton (1992) raised concerns regarding the significance of mathematical SMK and the responsibilities of both teachers and professional development programs in the context of reforming mathematics teaching and learning. Her primary concern was directed toward programs that aimed to promote meaningful mathematics instruction but lacked a strong foundation in mathematical content. Several researchers share these apprehensions, not only regarding professional development programs for educators but also in terms of reinforcing SMK within teacher education preparation programs (Swafford et al., 1997; Shulman, 1986).

Swafford, Jones, and Thornton (1997) conducted a study focusing on the impact of an intervention program designed to enhance middle-grade teachers' understanding of geometry and student cognitive processes. They emphasized the intrinsic connection between student achievement and instructional practices, emphasizing the adage that "the more a teacher knows about a subject and the way students learn, the more effective that individual will be in nurturing mathematical understanding" (p. 467). Their findings indicated that the intervention program effectively improved teachers' mathematical SMK, particularly in terms of instructional techniques and student cognition. The teachers in their study not only increased the time dedicated to teaching geometry but also improved the quality of instruction in the subject. Moreover, the researchers discovered that not only did students' comprehension of geometry-related concepts advance, but the participating teachers also expanded their knowledge of the concepts they were expected to teach.

These research studies, along with numerous others, provide substantial support for our conceptual framework, which centers on the "what" and "how" of teaching similar figures within the context of TRMT.

## METHODOLOGY

The study used convenience sampling. Convenience sampling chooses individuals who are conveniently available for a study (Lankshear & Knobel, 2004). Participants for this study were individuals in an initial licensure program at a university in the southeastern United States and enrolled in a mathematics methods course.

Data was collected from each TRMT cycle focusing on a question prompt. During a semester, there were two question prompts explored during a six-week cycle. Each prompt focused on a topic related to proportional reasoning found within the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Data from the TRMT was collected between the second and fourteenth week of the semester. Each TRMT consisted of three parts: individual discussion and reflection, group discussions, and presentation.

Since the TRMT was seen as part of the candidates' methods course, they needed to have an idea of some of the content and substantive structures involved with similarity and similar figures. In some instances, the researcher needed to probe the candidates to further their explanations of these ideas. The researcher was a faculty member and instructor of the methods course in question and was a participant observer throughout the sessions. TRMT are a hybrid action-research teaching experiment because the researcher was also the teacher of the methods courses and organized the experiments for the candidates, as well as assumed the responsibility for the instruction.

During the first step of the TRMT, known as "Step 1: Individual Discussion and Reflection," the researcher tried to get an understanding of what the candidates knew about a topic and what they initially understood. This portion of the TRMT cycle took approximately two weeks, depending upon the number of students in the course. The protocol for this step consisted of two major components: discussion and write-up. During the discussion, participants were given a question prompt that presented the topic to be addressed and the participants immediately discussed what they recalled previously about the topic and any methods or "strategies" used to teach them. The reflection component was an assignment immediately following the discussion. During the reflection, which mirrored the discussion, participants were given an hour to write up and submit their ideas about the topic and teaching it. They were asked not to use any resources outside of their ideas and examples. They were also instructed that there were no right or wrong ways to complete this task.

The purpose of the individual discussion and reflection was to ascertain the candidates' initial understanding of the topic and how they might teach the topic. The candidates' knowledge of material related to the topic is reflected in their images of *what* and *how* to teach. This allowed the researcher to begin to infer the candidates' initial images of the concept along with representations about teaching the concept, before any influences took place by group members or the instructor during later steps within the TRMT.

"Step 2: Group Discussions", occurred about one week after the individual discussions and reflections. Groups were formed so the candidates could discuss their answers to the prompts and present a decided response, as a lesson, to their classmates. Depending on the number of candidates enrolled in the course, groups consisted of three to four members and were formed, again, for convenience. Groups were formed based on common potential meeting times for candidates. This was done to avoid time conflicts and provide candidates with predetermined meeting times, which they established, to develop their lesson presentation. Candidates came together to meet for an initial hour and began planning their presentation.

Group discussions allowed time for the candidates to share their ideas about answering the probe. It was during this time that the candidates' ideas of *what* and *how* to teach became clearer and some began to make and have different images about teaching the topic. In some instances, when discussing what they did in the individual reflection, some candidates came to realize other ways to answer the probe and developed and kept these alternative ways of teaching the topic for themselves.

In "Step 3: Presentations," the groups developed presentations, discussing the prompt and how to teach the concept to students. The presentation was for other candidates in their methods course after the groups had two weeks to meet and develop their presentations. The presentations were scheduled to take up to 20 minutes with an additional 30 minutes allowed for feedback from other students and the instructors. Candidates were told that presentation times were going to be closely monitored and to be mindful of their times. After the groups presented their ideas, they received feedback from the instructor and their classmates. Since the presentations and discussions were up to 50 minutes in length, the presentation helped to reinforce any ideas that they had and any new ideas that had been developed.

## DATA AND ANALYSIS

The data sources included videos of the individual interviews, group interviews, and group presentations, and transcripts of these discussions. Written artifacts such as the individual and group discussion notes, and any other written materials or manipulatives were also analyzed. The instructor's feedback was reviewed as part of the data during the group presentation. Lastly, at the beginning of the course, candidates write their "math autobiographies" and philosophies of teaching. These were also examined for any background information to the research study.

The conceptual framework used was instrumental in the analysis of the data. Analysis of the data for the research involved coding, sorting into categories of significance, and establishing patterns among the categories. Data from interviews and the lesson plans were coded, extracted, and examined.

Candidates' initial images of similarity were assessed within Even's (1990) aspects of SMK, in conjunction with Shulman's (1986) forms of practicing knowledge. The candidates' examples, definitions, activities, and explanations are looked at to see if they are using propositional, case, or strategic knowledge. If the candidates used approaches that are grounded in theory, such as cooperative learning, then it may be possible they are accessing their propositional knowledge. When the candidates have been previously exposed to an approach, definition, or example and use it in their explanations, examples, or activity, they may be accessing their case knowledge. The candidates' strategic knowledge may be looked at if they seem to encounter a situation that they are unfamiliar with and resolve it with a new approach they had not seen or used before. It is important to note that the learners, as well as the context, determine the type of knowledge being used.

In presenting findings and drawing conclusions about the data, issues of credibility are of utmost concern. Steps were taken to ensure that the findings were accurate and credible. In looking at the credibility of the observations, several precautions were taken in dealing with the observed data. As the instructor of the methods course, the researcher tried to limit himself in his questioning of the candidates to only clarifications within the discussions. He presented the task verbatim from the instruction sheet and allowed the candidates to begin the discussion. All the observations were recorded and the data from each episode was compared to previous episodes when analyzing the data. By employing the use of peer debriefing, this researcher asked an outside, uninterested third party to review the present research report. This third party was allowed to review the materials and to identify strengths and weaknesses in the report. This opportunity lends credibility and support to the conclusions drawn. Additionally, the use of a member check allowed the candidates of the study to review any materials, and this report on their initial images of similarity. Their input as to how the data was looked at helps in establishing greater credibility to the conclusions. To lend credibility to the findings, the participants must be given the opportunity to help in the reconstruction of the events that took place during the study cycle.

## PRESENTATION OF FINDINGS

The SMK for teaching specific topics is formed by seven aspects: essential features, different representations, alternative ways of approaching, strength of the concept, basic repertoire, knowledge and understanding of a concept, and knowledge about mathematics (Even, 1990). These aspects are used to frame the results reported on the candidates' initial images on the topic of similar figures. Shulman's (1986) three forms of teachers' knowledge will be incorporated to lend support throughout Even's analytical framework. In the presentation of the findings, each feature of Even's framework will be given along with the interpretations of the data.

### Essential Features and Basic Repertoire

Even (1990) described essential features and basic repertoire as the image of the concept that pays attention to its essence repertoire and the easily accessible powerful examples that illustrate important properties. The essential features that the candidates use in recalling and working with similarity and similar figures will be their mental pictures of the topic and the properties that they associate with it. Candidates having a mature basic repertoire can provide insight and understanding of general and more complicated knowledge. A mature basic repertoire can be used as a reference to monitor ways of thinking and acting. These aspects can be different for different people. Typically features of a concept are influenced by analytical judgments and the prototypical examples a candidate uses. Candidates appeared to have three initial images about the essential features of similar figures: *Same shape*, *different sizes*, and *side lengths determined similar figures*.

### *Same Shape*

The first theme deals with the idea that similar figures must have the same shape. During many of the initial discussions, candidates mentioned that figures had to be the same shape to be similar to other figures: triangles could only be similar to other triangles, squares could only be similar to other squares, and circles could only be similar to other circles. This feature became evident as candidates discussed how they would point out that the shapes were the same.

*C1: If I am going to start with the definition of similar, I would want my students to see that these are the same shape. I would want them to notice that a triangle is similar to only another triangle and a square is only similar to another square. I would want them to see that a triangle can't be similar to a square.*

*C2: I would tell my students that similar figures have to be the same shape. All triangles are similar to triangles, all squares are similar to squares, and all circles are similar to circles, pentagons, etc., but triangles aren't similar to squares, circles, pentagons, or whatever, only triangles. Squares with squares, and so on.*

Candidates had the initial image that an essential feature of similar figures is that they had to have the same shape. Expanding on this idea, while focusing their discussion, led some candidates to further explain that similar figures were "the same shape but different sizes." It is important to note that several candidates mentioned how some figures could not be similar. It was as if they were providing counterexamples to support and explain their initial images.

### *Different Sizes*

Expanding on the theme of same shape, some candidates further explained that similar figures were different sizes. This feature of similarity became evident from their discussions and incorporation of examples.

*C2: Okay, I would state that similar figures have the same shape but not necessarily the same size; they are either enlargements or reductions of the original shape. If they have a triangle, a similar triangle can be bigger or smaller than the first triangle. They are the same shape but different sizes.*

Building on the idea that similar figures meant that the figures have "the same shape but different sizes" was discussed, in part, by most candidates. Building on this idea along with what they recalled about similar figures shows what the candidates meant by "the same." The transitions that the candidates made from generalizations about "the same" when discussing similar triangles, to the exactness of what they meant by "the same" led to another theme within the essential features.

### *Side Lengths Determine Similar Figures*

Another theme in candidates' essential features of similarity deals with the role of side lengths in determining similar figures. In reviewing many of the first examples discussed by the candidates; side lengths were dominant in their common examples of similar figures.

In recalling what his teacher taught about similar figures, a candidate stated:

*C1: If we are doing similarity, my teacher would put the basic triangle and would put this three, four, five triangle and say that it was similar to this, say, four, five six triangle.*

While relying on case knowledge, C1 focused on the side lengths, while not considering the mathematical correctness of his example. Another candidate in the group drew diagrams and pointed out the multiplicative relationship between the figures:

*C3: Similar figures. Similar triangles have a number that can be multiplied or divided to find the sides of the other triangle. Here are three similar triangles, the first one has sides three, three, and … four. The second one is similar, and we know two sides: six and eight, and another side x. The third one is smaller, and we know it has a side of two, and two x's."*

*I: Are these x's the same?*

*C3: Well… no let's say this is x and these other two are y's. Then we could find the missing sides using multiplication or division. This x would be six because we are multiplying each side by two and the y's would be 1.5 because we are dividing each side by two [points at paper] see four divided by two is two and… three divided by two is 1.5, yeah.*

Other candidates pointed out the importance of the lengths of sides with almost all candidates mentioning a multiplicative relationship. Some also mentioned "proportions."

When asked what she recalled from similar figures C4 used ideas that matched C3 and others:

*C4: Then he'd [his teacher] do sides and measures, and then do the proportions and ratios. If this one is six and this one is 12 and this one is twice this one or this one is half this one. See the sides are proportional to each other.*

In looking at the last two examples, these candidates, and others, connected the idea of similar figures with multiplicative relationships with proportional sides. When asked, "What do you recall about similar figures?" responses focused only on the side lengths of similar triangles as an indicator for being similar. In the beginning, the ideas about the importance that candidates placed on angle measures were nonexistent. This is reflective of what Ball (1988) noted earlier, "prospective [elementary] teachers' focus on the surface differences… suggests their understanding is comprised of remembering the rules for specific cases, not a web of interconnected ideas" (p. 22). It seems evident that the candidates, at first, may not have "a web of interconnected ideas" when it comes to the concept of similar figures or similarity. At the beginning of the study, the candidates' essential features and basic repertoire, while limited, used their propositional knowledge to establish a definition and providing examples to exemplify the definition of similar figures. These generalizations focused on the same shape of figures, but different sizes, and using side lengths to determine similarity. It is important to note that as the study progressed, candidates did include a more accurate representation of mathematical similarity. Additionally, there appeared to be limited access to case knowledge in the beginning conversations with candidates. Some candidates felt uncertain about their responses, while others provided examples that were mathematically incorrect.

## Different Representations and Alternate Ways of Approaching

Even (1990) discussed the importance of using different representations when discussing a topic. Different representations give more insight into the topic and allow for a deeper understanding of a concept. By using more than one representation, one can abstract and grasp properties, concepts, and the common qualities of a topic.

The representations that the candidates used in explaining similarity involved looking at combinations of triangles. In the student activity part of their lessons, the candidates devised activities that used triangles and were asked to categorize them or deduce why they are considered similar.

Each of the candidates used different approaches to their discussion of similar triangles, but the representations that they used were focused on the examples of right triangles, isosceles triangles, and whether they were similar.

This is important to note because the candidates' use of only right and isosceles triangles, may not help their future students grasp properties, concepts, and common qualities of other types of similar triangles. Again, in the beginning, candidates accessed their case knowledge and relied on representations they felt they could explain fully.

As candidates participated further in the TRMT cycle, a few explored different ways to present the concept of similarity. This became evident when they discussed how they would present this concept to their future students. Even (1990) stressed the need for alternative ways of approaching a concept. It is very helpful to see a concept in various forms, such as in different divisions of mathematics, other disciplines, or everyday life. Using their substantive and syntactic knowledge, many of the candidates chose an approach that has influenced their SMK and PCK. Many of the candidates devised a student activity that would help their students to see patterns and relationships that they could then focus on and come up with what it meant for two triangles to be similar. In analyzing candidates' approaches, two themes were appeared: anticipating student difficulties and responses, and establishing patterns to generalize.

### *Anticipating Student Difficulties and Responses*

Candidates, during their planning, mentioned using students to explore examples and how the students may react during the example:

*C4: Oh, okay. I thought I would start with a triangle like this [holds up a paper with a triangle]. It has equal sides… I just used this one… and I will display it on the Smartboard, and I wrote down how many centimeters there are. Then 1 to 3 students, depending on the number, will come up and measure the sides on the Smartboard, to find out if they are the same sides – if that's true - and then, of course, they will come up with that it is longer, but then I wanted to see what they conclude from that. Then I was thinking well they might conclude that you add to each side in order to make it bigger or longer, so I made another triangle [points to another triangle] where the sides aren't equally, and do just the same thing so they see that it is not through addition that makes it similar.*

*Then I'll take this off after they make their conclusion and then I will put this on, and then do the same thing where they measure again the sides, and then see is it again that you add a certain amount to each side. I guess that then they will realize that conclusion is wrong and just kinda see if they can come up with a conclusion.*

During C4's activity, she felt that it was necessary to anticipate where students may make incorrect assumptions about similar triangles and provide examples to combat those assumptions. She wanted her students to notice qualities and then guide her students to abstract the qualities of the concept of similarity. She also anticipated students' additive thinking and included another example to try and avoid this type of thinking. It is interesting to note that she thought that one counterexample would be enough to show her students the correct way of reasoning (multiplicative) and she thought it was enough to persuade her students from thinking about proportions incorrectly (additively). This view is compatible with Simon et al.'s (2000) notion of perception-based perspective. This idea incorporated her strategic knowledge in planning this activity for her students.

### *Establishing Patterns to Generalize*

Several candidates, in their first individual lesson, used one example to show important concepts in similarity. Candidate C5, in her individual interview, devised an introductory activity for the concept of similarity. It appears that she felt as if it was important to use examples to allow her students to abstract the properties of similarity. This was a similar thought process for several candidates, including the example from above.

*C5: My other triangle, I didn't do too well, but I told them to get into groups and measure the angles and sides and to fill in the answers and see what patterns they notice.*

*And after they finish all of that, I am going to ask for volunteers of what they found or what they deduced. Then we will talk about did they notice that the angles match. I am sure that somebody will get that and that all the sides reduce down to nine. So they can say that the triangles are similar.*

In her activity, she used strategic knowledge to predict that volunteers would be able to abstract the qualities of congruent corresponding angles by noticing "the angles match" and proportional sides as "the sides reduce to nine over five." This type of thinking exemplifies candidates' need to be exposed to different activities that bring the activities of teaching into context such as TRMT.

### The Strength, Knowledge, and Understanding of Similarity

Even (1990) saw the strength of a concept as the new opportunities that can be explored or addressed by it. Also, the strength of any concept is seen as having important sub-topics and sub-concepts. The strength of a concept directly relates to the candidates' knowledge and understanding of similarity and involves both their procedural and conceptual knowledge and the relationships between them. If someone has strength, knowledge, and understanding of a concept, then they have a network of concepts and relationships at their disposal. An understanding of similarity includes a good intuitive feel for the concept to solve problems, generate answers, and understand the importance of the topic as it relates to other areas and concepts.

Many of the candidates sought to teach for understanding through activities and examples that helped their students develop their understanding of the concept of similarity for themselves. When recalling what they remembered about similarity, most of the candidates' responses were immature and of a procedural nature. However, as they planned the lessons their ideas became more concrete and confident. In planning their lessons, it seems as if they were searching for more than definitions and examples, but connections between ratio, proportion, and the strength and knowledge of similarity. Teaching that was focused on procedural knowledge heavily influenced their lessons, but several candidates did try to incorporate conceptual aspects of similarity.

In the beginning, one candidate went beyond using ratios, proportions, and only numerical representations of similar triangles and brought up models, scaling, and indirect measurement to find the height of objects:

*C6: Then I would ask why is this idea important or what we need this for with the ratios. And then I was just saying a lot of models are being used like in architecture or if you build an airplane, you first have your little model and just go from there*

In their lessons, several candidates intended to use conceptually centered activities for their students to discover the properties of similarity. One candidate had a set of 12 triangles which students categorized and hopefully came up with what similar triangles meant mathematically. Another candidate began his lesson with a series of geometric figures in which his students knew they were similar and expected them to find a pattern of properties that made them so. One candidate, in her lesson, allowed students to measure the angles and sides of two similar triangles to explain why they are considered similar. In one lesson, a candidate projected the picture of a triangle on an overhead screen and expected her students to measure and deduce why the triangles on the overhead screen were considered similar to those on the monitor screen. These activities can be considered conceptually-oriented and support the strength, understanding, and knowledge of similarity because they focus on "the relationships and interconnections of ideas that explain and give meaning to mathematical procedures" (Eisenhart et al., 1993. p.9). Although limited and procedural, some candidates intended to make connections and provide opportunities for their students to define and develop an understanding of the concept of similarity. This lack of implementation could be a result of their limited case knowledge in teaching for understanding.

### Knowledge About Mathematics

Even (1990) described knowledge *about* mathematics as the knowledge that gives inquiry, truth, and the construction and use of knowledge and understanding within mathematics. Knowledge about mathematics is seen to include the ways, means, and processes by which truths are established as well as different ideas (Ball, 1991; Even, 1990; Shulman, 1986).

The candidates' ideas about establishing truth in geometry were vague. Truth was established through an example, usually a singular example before a definition was provided.

*C5: After we do the problem, I will put the way to define if two triangles are similar is to put them in proportion to each other.*

In some of their lesson plans, the candidates did allow students to harvest their own knowledge, but the way this was handled afterward varied. Usually, after the candidates expected their students to derive the meaning of similarity, the candidate provided their students with the definition.

*C6: [putting down the triangles she created] After students have had a chance to explore what similar figures are, and come up with a definition, I would give them the definition of what it means for figures to be similar.*

The instances in lesson plans where definitions came before examples and then more definitions and examples were limited. However, in most of the lessons, the teacher remained the authority in establishing the truth about the content.

There were several instances where the candidates told the definitions and correct answers, reflecting the idea that, "teachers are inclined to tell and show students how to do mathematics instead of creating activities that help students construct understanding of content" (Ball & Mosenthal, 1990, p. 3). Also, Even (1993) suggested that the instances of relying on the textbook, when faced with contradictions in their understanding and situations of unfamiliarity, strongly reflect the prospective teachers' limited content knowledge.

## DISCUSSION

Subject matter preparation has a very important role in the development of pre-service teacher candidates. Topics such as similarity, which are powerful and useful, need to be explored by candidates. In the previous analysis, it seems that the candidates in question had a grasp of some of the qualities of the concept of similarity and some good ideas about teaching students. However, they seemed to lack exactness in their representations and strength in the concept of similarity for any topic other than indirect measurement. The knowledge base that the candidates used varied from their propositional knowledge, in which the norms that they have experienced guided them, to instances where their case knowledge as students and working as teachers yielded different practices. Within these interviews it is easy to see that the candidates began to change their ideas about similarity, thus causing them to rely more heavily on their SMK to explain examples, definitions, student responses, and create activities.

## CONCLUSION

Results from the TRMT support Towers' (2001) conjecture that students' understanding is partly determined by teacher interventions. Incorporating TRMT and activities may allow students to reflect on and explore their knowledge of topics (Akar, 2015; Stylianides & Ball, 2008; Davis & Simmt, 2006; Davis & Staley, 2002; Staley & Davis, 2001). This suggests that candidate development opportunities should be intentional in providing development that identifies and discusses their subject matter and pedagogical understandings. Through development and discussions, it may be possible for candidates to recognize any limitations and misconceptions they have about the knowledge needed to teach mathematics.

## CONFLICT OF INTEREST

The author of this publication declares there are no competing interests.

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## AUTHOR'S NOTE

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